Accretion of Small Moons at Saturn

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Problems

• Numerous small moons at Saturn have expected collisional lifetimes from comet impact much shorter than age of the solar system.

• Rings have apparent age much shorter than age of the solar system based on apparent purity of water ice in rings.

• Having this as a result of a one-way collisional cascade requires something with low probability happened recently or the end of the system as we know it in the near future, or both [Colwell and Esposito 1992, 1993, 2000].
Possible Solution

Slow the whole thing down with reaccretion.

But, isn’t accretion impossible due to tidal forces?
Roche Zone

• Classical Roche Limit:

\[
\frac{a}{R} = 2.456 \left( \frac{\rho_p}{\rho} \right)^{1/3}
\]

That is for a strengthless fluid body in hydrostatic equilibrium.

Roche Limits for solid particles:

\[
\frac{a}{R_p} = 2.29 \left( \frac{\rho_p}{\rho} \right)^{1/3} \quad \text{For identical particles in contact}
\]

\[
\frac{a}{R_p} = 1.44 \left( \frac{\rho_p}{\rho} \right)^{1/3} \quad \text{For small particle on a large particle}
\]
## Roche Zone

Accretion possible to the right of curves

<table>
<thead>
<tr>
<th>$a/R_p$</th>
<th>Feature</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.32</td>
<td>F Ring</td>
</tr>
<tr>
<td>2.27</td>
<td>A Ring Outer Edge</td>
</tr>
<tr>
<td>2.21</td>
<td>Orbit of Pan</td>
</tr>
<tr>
<td>1.96</td>
<td>Huygens Gap</td>
</tr>
<tr>
<td>1.45</td>
<td>C Ring Gap</td>
</tr>
</tbody>
</table>

Canup and Esposito (1995)
Some Terminology

• Roche lobe = Hill Sphere*
• Hill Sphere is region of space around an object in which the gravitational acceleration points inward.

* In Porco et al. (2007), but not in Wikipedia.
How to test if accretion is occurring?

1. **Shape**: Roche limit for small particle on large particle depends on *where* it is on the larger particle. In other words, the Hill Sphere is not a sphere.

   - More stable
   - Less stable

   Hill Sphere axis ratios are $A:B:C=3:2:2$
How to test (continued)?

2. **Density: Accretion lowers density.**
   Lower density shrinks Hill Sphere. At some point the Hill Sphere is full, and accretion stops:

   \[
   \rho_{\text{crit}} = \frac{3M_p}{\gamma a^3}
   \]

   \(\gamma a^3\) = volume of the moon

   For Hill Sphere:

   \[
   \gamma = \frac{2\pi \ln(2 + \sqrt{3})}{3\sqrt{3}} = 1.59
   \]
Observations

- Resolve shapes of moons:
  - Measure axis ratios
  - Measure volume
- Determine mass of moons from gravitational influence on rings
- Calculate densities.
Fig. 6. Gravitational topography for Prometheus, assuming its observed shape but two different interior models: homogeneous and core. The former assumes a uniform interior with critical density for Prometheus of 0.4 g cm$^{-3}$; the latter assumes a core of 0.9 g cm$^{-3}$ and mantle of 0.15 g cm$^{-3}$, with an overall density equal to its critical density. The range of gravitational topography in kilometers is given below each. The observed shape and mass alone do not discriminate between these two interior models.
N-body Simulations

View from Saturn  Leading Face  North Face

Simulation axis ratios $A:B:C = 6:4:3$ instead of $6:4:4$
## Results (Porco et al.)

<table>
<thead>
<tr>
<th>Moon</th>
<th>(\frac{a}{R_p})</th>
<th>(\frac{C}{A})</th>
<th>(\frac{A}{A_{\text{Hill}}})</th>
<th>(\frac{\rho}{\rho_{\text{crit}}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pan</td>
<td>2.21</td>
<td>0.60</td>
<td>0.91</td>
<td>0.92±0.32</td>
</tr>
<tr>
<td>Daphnis</td>
<td>2.26</td>
<td>0.69</td>
<td>0.93</td>
<td>0.8±0.5</td>
</tr>
<tr>
<td>Atlas</td>
<td>2.28</td>
<td>0.43</td>
<td>0.97</td>
<td>1.12±0.24</td>
</tr>
<tr>
<td>Prometheus</td>
<td>2.31</td>
<td>0.46</td>
<td>1.05</td>
<td>1.18±0.17</td>
</tr>
<tr>
<td>Pandora</td>
<td>2.35</td>
<td>0.62</td>
<td>0.85</td>
<td>1.32±0.23</td>
</tr>
<tr>
<td>Epimetheus</td>
<td>2.51</td>
<td>0.92</td>
<td>0.57</td>
<td>2.25±0.42</td>
</tr>
<tr>
<td>Janus</td>
<td>2.51</td>
<td>0.78</td>
<td>0.62</td>
<td>2.03±0.21</td>
</tr>
</tbody>
</table>
Conclusions (1)

- Moons have accreted low density mantle onto solid core.
- Pan and Daphnis are more massive than they need to be to have opened the gaps in which they reside. Assuming cores opened gaps, these would be ~1.8-3 km for Daphnis and ~8-12 km for Pan.
- Accretion continued after gap opened. Requires thicker ring at that time, meaning happened very early in history of A ring.
- Additional accretion occurred later, providing “terminal accretionary ornaments” (Charnoz et al.)
Conclusions (2)

• Ridges may be remnants of accretion of material from a disk as the moon opened a gap, analogous to final stages of planet formation.

• Moons have accreted, and thus there may be recycling of fragmented material, helping solve the ring age problem.