Homework 3

PHZ 3151

Due Friday, February 20, 2009

1. The gravitational potential Φ (potential energy per unit mass) at a distance r away from the center of the Earth is given by,

$$\Phi(r) = -\frac{GM}{r}$$

where $r = (x^2 + y^2 + z^2)^{\frac{1}{2}}$, and $G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2}$, and the mass of the Earth is $M = 5.97 \times 10^{24} kg$.

a) Using differentials, *estimate* the energy needed to move a $10kg$ mass from a point with coordinates $(4000km, 4000km, 3000km)$ to a point with coordinates given by $(4020km, 4050km, 3010km)$. The center of the Earth corresponds to $(0, 0, 0)$.

b) Determine *exactly* the energy required to move a $10kg$ mass between the two points given in part a). Compare the exact answer to the estimate determined in part a) and comment on the difference.

c) Take the Earth as a sphere of radius $R = 6371km$. Determine how the gravitational potential depends on distance z above the surface when $\frac{z}{R} \ll 1$. By making a Taylor's series expansion of the potential near the surface, show that $\Phi(z) \approx -\frac{GM}{R} + gz$. Show that $g = 9.81 \frac{m}{s^2}$.

2. Simulate asteroids near the 2/1 Kirkwood gap. Use the parameters found in Table 4.4 for the initial conditions of the asteroids and also Jupiter. Plot for each of these 3 starting conditions the position of the asteroid relative to the Sun for about 10 orbits of Jupiter. Your picture should look similar to Fig 4.15. Use a time step of 0.005 years.

Next, run the code for a very long time (e.g. 100 orbits of Jupiter) and determine if the trajectories vary dramatically from what you found for 10 orbits of Jupiter. Given enough time, the asteroids are believed to rapidly change to highly elliptical orbits which can become chaotic.