Homework 1
PHZ 3151
Due Friday, January 23, 2009

1. Solve the following differential equations:
   a) \( \frac{dN}{dt} = -\frac{N}{\tau} \)
   b) \( \frac{d^2y}{dt^2} = -\omega_0^2 y \)

In a), write your answer in terms of \( N \) at \( t = 0 \), \( N(t = 0) \). For b), write your answer in terms of the \( t = 0 \) position and velocity, \( y(t = 0) = y_0 \) and \( \frac{dy}{dt} \rvert_{t=0} = v_0 \) respectively. Notice for the second-order differential equation, two initial conditions are needed to determine a particular solution.

2. Implement the Fortran radioactive decay program using the Euler method. You may directly copy the code given in Giordano. Choose for the time constant \( \tau = 1.0 \) s.

Run the code and generate a picture like Fig. 1.1. What happens if you choose a time step of \( \Delta t = 0.05 s \)? Does the code compare well with the exact solution when \( \Delta t = 0.1 s \), or \( \Delta t = 0.5 s \)?

3. Consider the following rate equations that describe a second-order chemical reaction \( A + B \leftrightarrow C \)

\[
\begin{align*}
\frac{dn_A}{dt} &= -k_{AB}n_An_B + k_Cn_C \\
\frac{dn_B}{dt} &= -k_{AB}n_An_B + k_Cn_C \\
\frac{dn_C}{dt} &= k_{AB}n_An_B - k_Cn_C
\end{align*}
\]

where \( k_{AB} \) and \( k_C \) are rate constants that typically depend on temperature (and perhaps other conditions as well). The concentrations of the chemical species are \( n_A \), \( n_B \), and \( n_C \). A second-order reaction occurs at a rate dependent on either a second-order concentration (e.g. \( n_A^2 \)) or the product of two first-order concentrations (e.g. \( n_An_B \)).

Write a code using the Euler method to integrate the three equations above. Run the code for a few different starting conditions and plot the results. Examine in particular the cases \( n_A = 1, n_B = 1, \) and \( n_C = 0 \) at \( t = 0 \), and also the case \( n_A = 1, n_B = \frac{1}{2}, \) and \( n_C = \frac{1}{2} \).
Determine the appropriate conservation laws for the above equations, and verify that your code is obeying it. In particular, mass cannot be created or destroyed, so as $A$ and $B$ are consumed, $C$ must be created in predictable amounts. The same considerations apply as $C$ decomposes into $A$ and $B$.

For simplicity, choose $k_{AB} = k_C = 1$, and determine an appropriate time step $\Delta t$ for which the code runs accurately and efficiently.