Spring 2002 Test 2 Solutions

Problem 1 (Short Answer: 15 points)

A sled on ice moves in the ways described in questions 1-7 below. *Friction is so small that it can be ignored.* A person wearing spiked shoes standing on the ice can apply a force to the sled and push it along the ice. Choose the one force (**A** through **G**) which would **keep the sled moving** as described in each statement below. You may use a choice more than once or not at all but choose only one answer for each blank. If you think that none is correct, answer choice **J**.

The two key points to this problem are that (1) the applied force is the net force and must point in the direction of the acceleration, and (2) since the motions all have either constant or zero acceleration, this means only responses B, D, & F apply since only these responses describe constant forces.

Problem 2 (Estimation Problem: 15 points)

You have joined a volunteer fire department. They are looking to buy a new rescue net because the old one broke. (A rescue net is the circular trampoline-like net fireman hold to catch people falling from buildings.) The cost increases dramatically with the strength of the net and your fire department has a very limited budget. You need to buy the cheapest rescue net that meets your department's needs. Assuming that the force exerted by the rescue net is constant and that the tallest building in your region is 5 stories high, what is the maximum force the rescue net would need to withstand? Remember the idea is stop people before they hit the ground.

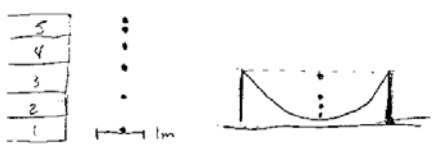
From a student solution:

Height of Building = 5 stories

Force exerted = ?

Assume
1 story = 5 m.
Net is held 1m above ground
Person jumps from top of building
Max. Person mass = 200 kg.

$\Delta x_1 = 24\,m \qquad V_{f1} = V_{i2}$
$\Delta x_2 = 1\,m$



Motion 1   Motion 2

vf = ?

$V_f^2 = v_i^2 + 2a\Delta x$
$V_f^2 = \sqrt{0 + 2 \cdot 9.8 m/s^2 \cdot 24 m}$
$V_f = 21.68\,m/s$

a = ?

$V_f^2 = V_i^2 + 2a\Delta x$
$a = \dfrac{-V_i^2}{2\Delta x}$
$a = \dfrac{-(21.68 m/s)^2}{2 \cdot 1m}$
$a = -235.2\,m/s^2$

F = ?

$F = m \cdot a$
$F = 200kg \cdot 235.2 m/s^2$
$F = 47040\,N$
$F = 47\,kN$

Maximum force net must withstand is 47 kN.

Dr. Saul's comments: *Although missing some details that make it hard to follow and one mistake, this is a good solution. The analysis of the motion is divided into two parts. Motion 1 is for the person falling from the building and Motion 2 is when the person is being stopped by the net. Let's refer to the point where the person has left the building as point 0. The point right before the person hits the net is point 1 and the point where the person comes to a stop is point 2. We also need to define a coordinate system, so let's define down as + (this is consistent with the above solution).*

For motion 1:
$v_0 = 0$ m/s and $a_1 = g$, => $v_1 =$ "v_f" = 21.68 m/s (note that if g is –, so is the displacement. Thus we are not taking the square root of a negative number.)

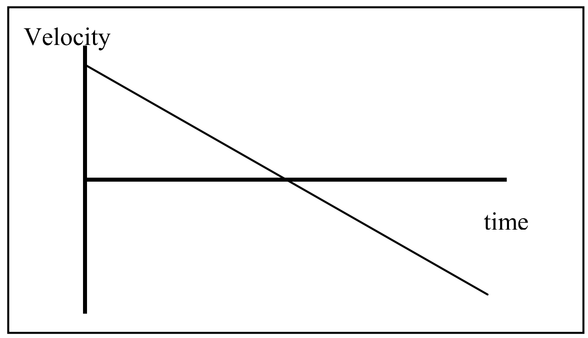
For motion 2:
Initial v = v_1 = 21.68 m/s and final v = v_2 = 0 m/s => a_2 = -235.2 m/s/s

So far, so good but now comes the mistake, using a_2 to find the force. Using Newton's 2^{nd} law, the net force is 47 kNewtons as shown above. However, the question asked to find the average force of the net on the person which = net force (person) – Weight force of the earth on the person,

So F (net => person) = net F (=> person) – W(earth => person)
= 47040 N – (200 kg) * (- 9.8 m/s/s)
= 49 kN

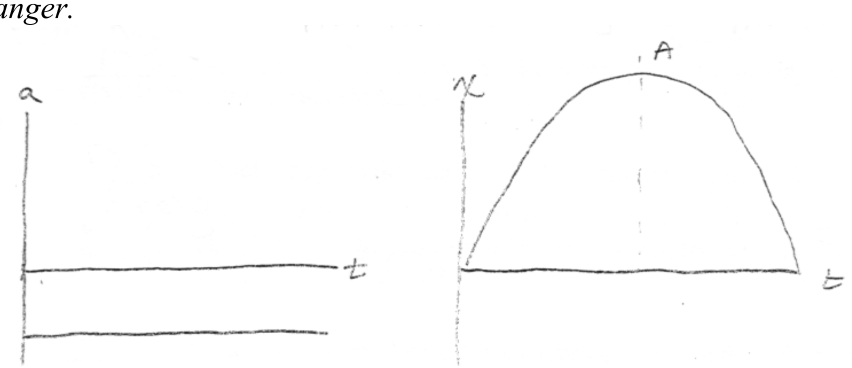
Problem 3 (Essay 10 points)
You may use diagrams and equations but no calculations in your response for this problem.

A cart can move to the right or left along a horizontal track (the positive part of the x axis) as shown in the figure below. Assume that friction is small enough that it can be ignored. A sonic ranger is used (as shown) to measure the position, velocity, and acceleration of the cart. The track is not necessarily flat or horizontal. In addition, the track may be tipped or the cart may be pulled or pushed. For the first run, the sonic ranger displays a graph of the velocity that looks like the graph shown at the right.



- Describe the motion of the cart in words
- Draw graphs showing what the sonic ranger would display for the cart's position and the cart's acceleration.
- USE WHAT YOU'VE LEARNED FROM CLASS SO FAR to explain in words how you came up with your answers

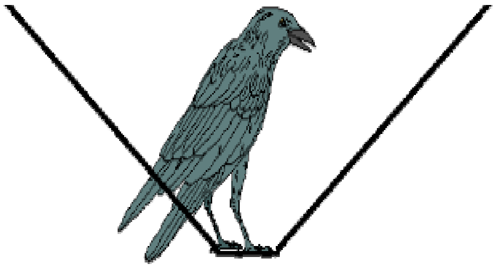
(mark point A as the point where v = 0 m/s.)
From a student solution: *The cart starts with an initial positive velocity which means it is moving away from the sonic ranger. The rate at which it is traveling goes down until it reaches 0 at point A. Then it continues down then having a negative velocity so moving towards the ranger.*



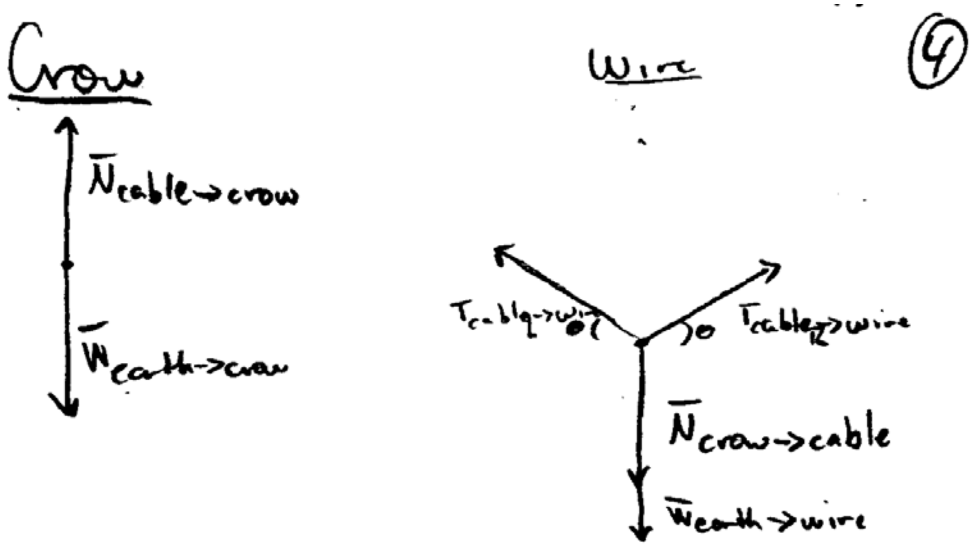
In class we've learned that velocity, acceleration, and position are all intertwined. From the velocity graph given you can find the position graph by the integral or area under the graph of the velocity. We see at point A it [position] *and because the area after A in v vs. t is negative (below t-axis) that the values for x vs. t decrease. We also learned in class that the a vs. t graph is the derivative (or graph of the slope) of the velocity vs. time graph. As we can see from the v vs. t graph, the slope is always the same and always negative, so the a vs. t values correspond giving a constant negative value.*

Dr. Saul's comment: Although choppy, this essay received 10 out of 10 points and is correct in each detail. However, it could be improved by noting in the first paragraph that the slope is constant and therefore the acceleration is constant. In addition, one could note that since velocity is the slope of the x vs. t graph, the slope of the x vs. t graph should decrease in slope throughout.

Problem 4 (15 points)
A crow is sitting on a telephone wire near a field, waiting patiently for the farmer to leave so he can eat the farmer's corn. The crow is standing on a small piece of wire made straight by the force of his feet.



a. Identify all the forces acting on the crow and on the piece of wire under the crow's feet. Give any relationships between these forces that you can and explain where they come from.



From Newton's second law, since neither the crow nor the wire is accelerating (they are both at rest) we know that the net force on both objects is zero and therefore the sum of the x-components and the y-components of the forces exerted on the two objects is also zero. Thus,

- $\vec{N}_{wire \to crow} = -\vec{W}_{earth \to wire}$
- The sum of the vertical forces acting on the wire = 0
- The sum of the horizontal forces acting on the wire = 0,
  $0 = T_{Left\,cable \to wire\;x} + T_{Right\,cable \to wire\;x} = T_{RHS \to wire} \cos\theta_{RHS} - T_{LHS \to wire} \cos\theta_{LHS}$
  $\left|\vec{T}_{RHS \to wire}\right| \cos\theta_{RHS} = \left|\vec{T}_{LHS \to wire}\right| \cos\theta_{LHS}$

From Newton's Third law, we know that $\vec{N}_{crow \to wire} = -\vec{N}_{wire \to crow}$

- Therefore, $\left|\vec{N}_{crow \to wire}\right| = \left|\vec{W}_{earth \to crow}\right| = m_{crow} g$

b. The angles that the left piece of wire and the right piece of wire make with the flat piece on the bottom are the same and equal to θ. If the crow has a mass of 0.5 kg, and θ = 150º, find the tension in the wire.

acceleration = 0

$\sin\theta = \frac{T_{1y}}{T_1}$

$T_{1y} = T_1 \sin\theta$

$\cos\theta = \frac{T_{1x}}{T_1}$

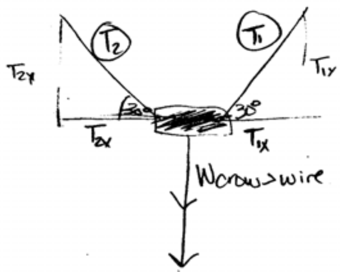
$T_{1x} = T_1 \cos\theta$

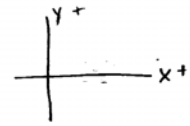
$\sin\theta = \frac{T_{2y}}{T_2}$

$T_{2y} = \sin\theta \cdot T_2$

$\cos\theta = \frac{T_{2x}}{T_2}$

$T_{2x} = T_2 \cos\theta$





crow = 0.5 kg

θ = 30°

$T_1 = T_2$, θ = same for both wires.

$a_x = \frac{\Sigma F_{netx}}{m} = T_{1x} - T_{2x} = T_1\cos\theta - T_2\cos\theta \equiv T_1\cos\theta = T_2\cos\theta,\ T_1 = T_2$

$a_y = \frac{\Sigma F_{nety}}{m} = \frac{T_{1y} + T_{2y} - W_{crow\to wire}}{m} = \frac{T_1\sin\theta + T_2\sin\theta - mg}{m} = 0$

$m a_y = T_1\sin\theta + T_2\sin\theta - mg$

$m a_y + mg = T_1\sin\theta + T_1\sin\theta$

$m(a_y + g) = 2T_1\sin\theta$

$T_1 = \frac{m(a_y+g)}{2\sin\theta} = \frac{(0.5kg)(0 - 9.8m/s^2)}{2\sin(30°)} = -4.9\ N$

Comments: This is a pretty good solution starting from the component versions of Newton's 2nd Law and working toward the symbol solution before plugging in numbers and calculating the final answer. However, there are some minor errors.

- In the x-component equation, $T_1 \cos\theta - T_2 \cos\theta$ should have been set = 0 since $a = a_x = 0$.
- In the x-component equation shown above, it appears that the ≡ is saying that $T_1 \cos\theta - T_2 \cos\theta$ is defined to be $T_1 \cos\theta$.
- There is no weight force exerted by the crow on the wire (the gravitational force exerted by the crow on the wire is very small and can be neglected). However, there is a contact force exerted by the crow on the wire which by Newton's third law is equal in magnitude to the contact force that the wire exerts on the crow which in turn is equal in magnitude to the weight force the earth exerts on the crow since the crow is not accelerating and therefore the net force on the crow is zero. (There is a small weight force exerted on the wire by the earth but we will assume that it is small and can be neglected.)
- Also, if we take the component equations directly from the vector form of Newton's 2nd law $\vec{a} = \frac{\sum \vec{F}_{wire}}{m} = \frac{\vec{T}_1 + \vec{T}_2 + \vec{N}_{crow\to wire}}{m}$, then

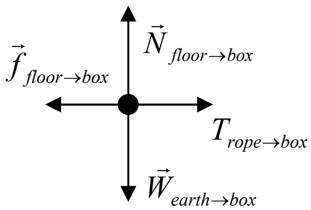
$$a_x = 0 = \frac{T_{1x} + T_{2x}}{m} = \frac{T_1\cos\theta - T_2\cos\theta}{m}$$ => $T_1 = T_2$ (since the angles are the same) and

$$a_y = 0 = \frac{T_{1y} + T_{2y} + F_{y\ crow\to wire}}{m} = \frac{T_1\sin\theta + T_2\sin\theta - mg}{m}$$ => $2T_1 \sin\theta = mg$

$T_1 = T_2 = mg / 2\sin\theta$

Problem 5 (20 points)

a. Need free body diagrams to see this. Since v = constant, $F^{net} = 0$ for both the rope and the crate (Newton's first law of motion). Let's assume that the magnitude of the tension in the rope is constant so that $T_{box\to rope} = T_{rope\to box} = T_{worker\to rope}$



$\vec{F}^{Net} = 0$

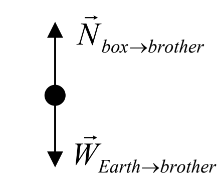
Since the net force = 0, the horizontal forces acting on the box add up to zero. Since there are only two of them, these two forces are equal in magnitude and opposite in direction. Thus, the force that the worker exerts is equal in magnitude and opposite in direction to the friction force exerted by the floor on the box. The magnitude of this friction force = $\mu N_{floor\to box}$. However, since the normal force is one of only two vertical forces acting on the box with the other force being the Weight force exerted by the Earth on the box, the two vertical forces are of equal magnitude. Therefore

$T_{worker\to rope} = f_{floor\to box} = \mu N_{floor\to box} = \mu W_{Earth\to box} = \mu mg$ and

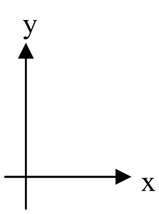
The force exerted by the worker on the rope points to the right.

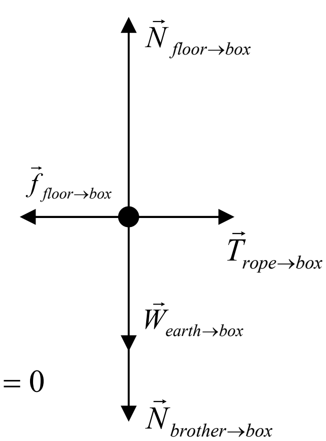
b. Yes, how hard the worker pulls does depend on whether or not her little brother is on top. As we showed above, how hard the worker pulls depends on the strength of the normal force exerted by the floor on the box. As we saw in the two-book problem in class, placing the little brother on top of the box increases the normal force the floor exerts on the box. Looking at the forces acting on the box, the net force on the box is zero since it is stationary and not moving. So the normal force of the floor on the box must be equal in magnitude to the weight force of the Earth on box plus the normal force of the little brother on the box. Since the weight force on the box has not changed, the normal force exerted by the floor on the box is larger than if the brother were removed. (See the free body diagrams in part c). This increases the frictional force exerted by the floor on the box, which means the worker must pull harder to keep the box moving with constant velocity.

c.



$\vec{F}^{Net} = 0$





$\vec{F}^{Net} = 0$

$\boxed{T_{wkr\to rope} = 300\text{ N}}$

Since $\vec{F}^{Net} = 0$, then $F_x^{Net} = F_y^{Net} = 0$

For little brother,

$$0 = F_y^{Net} = \vec{N}_{y\ box\to brother} + \vec{W}_{y\ Earth\to brother}$$

$$\vec{N}_{y\ bx\to br} = -\vec{W}_{y\ E\to br} = -(-m_{br}g) = m_{br}g$$

From Newton's 3rd law of motion

$$\vec{N}_{y\ br\to bx} = -\vec{N}_{y\ bx\to br} = -m_{br}g$$

For the box

$$0 = F_y^{Net} = \vec{N}_{y\ f\to bx} + \vec{N}_{y\ br\to bx} + \vec{W}_{y\ E\to bx}$$

$$\vec{N}_{y\ f\to bx} = -\vec{N}_{y\ br\to bx} - \vec{W}_{y\ E\to bx}$$

$$\vec{N}_{y\ f\to bx} = -(-m_{br}g) - (-m_{bx}g) = (m_{br} + m_{bx})g$$

$$T_{wrkr\to rope} = f_{fl\to bx} = \mu\left|\vec{N}_{y\ fl\to bx}\right| = \mu(m_{br} + m_{bx})g$$

$$T_{wkr\to rope} = (0.4)\ [(30\text{ kg}) + (50\text{ kg})]\ (9.8\text{ m/s}^2)$$