Problem 5 (20 points)

Several years ago, at 8 AM the eye of hurricane Floyd passed over Grand Bahama Island heading due west at a speed of 30.0 km/h. Four hours later, the course of hurricane Floyd shifted to Northwest towards the Florida coast and its speed increased to 40.0 km/h. Floyd continued on this course at this speed for two hours before turning due north again.

A. How far from Grand Bahama was hurricane Floyd 6 hours after it passes over the island?

The hurricane makes two constant velocity motions:
- \( \Delta \vec{r}_1 \) from going West at 30.0 km/hr for 4 hrs &
- \( \Delta \vec{r}_2 \) from going NW at 40.0 km/hr for 2 hrs

The hurricane’s displacement \( \Delta \vec{r} = \Delta \vec{r}_1 + \Delta \vec{r}_2 \)

\( \Delta \vec{r} = |\Delta \vec{r}_1|(-\cos \phi \hat{i} + \sin \phi \hat{j}) + |\Delta \vec{r}_2|(-\cos \alpha \hat{i} + \sin \alpha \hat{j}) \)

where \( \phi = 0^\circ \) is the angle between \( \Delta \vec{r}_1 \) & the \(-x\) axis, \( \alpha = 45^\circ \) is the angle between \( \Delta \vec{r}_2 \) and the \(-x\) axis,

- \( |\Delta \vec{r}_1| = v_1 \Delta t_1 = (30.0 \text{ km/hr})(4 \text{ hrs}) = 120 \text{ km} \)
- \( |\Delta \vec{r}_2| = v_2 \Delta t_2 = (40.0 \text{ km/hr})(2 \text{ hrs}) = 80 \text{ km} \)

\( \Delta \vec{r} = (-|\Delta \vec{r}_1| \cos \phi \hat{i} + 0 \hat{j}) + (-|\Delta \vec{r}_2| \cos \alpha \hat{i} + |\Delta \vec{r}_2| \sin \alpha \hat{j}) = (-|\Delta \vec{r}_1| \cos \phi - |\Delta \vec{r}_2| \cos \alpha) \hat{i} + |\Delta \vec{r}_2| \sin \alpha \hat{j} \)

\( |\Delta \vec{r}| = \sqrt{(-120 \text{ km})^2 + (80 \text{ km} \sin 45^\circ)^2} = 185 \text{ km} (185.4 \text{ km}) \)

B. What was Floyd’s average speed during this time?

\[ \text{Average Speed} = \frac{\text{distance}}{\text{time}} = \frac{(d_1 + d_2)}{(\Delta t_1 + \Delta t_2)} = \frac{(120 \text{ km} + 80 \text{ km})}{(4 \text{ hrs} + 2 \text{ hrs})} \]

\[ \text{Average Speed} = 200 \text{ km} / 6 \text{ hrs} = 33.3 \text{ km} / \text{hr} \]

C. What was Floyd’s average velocity during this time?

\[ \langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t} \]

\[ |\langle \vec{v} \rangle| = \frac{|\Delta \vec{r}|}{\Delta t} = \frac{|\Delta \vec{r}|}{(\Delta t_1 + \Delta t_2)} = \frac{(185.4 \text{ km})}{(4 \text{ hrs} + 2 \text{ hrs})} = 30.9 \text{ km} / \text{hr} \]

direction: \( \theta = \arctan \left( \frac{\Delta \vec{r}_y}{\Delta \vec{r}_x} \right) = \arctan \left( \frac{56.56 \text{ km}}{176.6 \text{ km}} \right) \approx 17.8^\circ \)

Alternatively, \( \langle \vec{v} \rangle = \frac{\Delta \vec{r}}{\Delta t} = (-176.6 \text{ km} \hat{i} + 56.57 \text{ km} \hat{j}) / (4 \text{ hrs} + 2 \text{ hrs}) = -29.4 \text{ km} \hat{i} + 9.43 \text{ km} \hat{j} \)

D. Sketch a vector representing hurricane Floyd’s average acceleration during this time.

Since \( \Delta \vec{v} \) is proportional to the average acceleration, \( \Delta \vec{v} \) vector points in the direction of the acceleration. Recall that

\[ \vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{(\vec{v}_2 - \vec{v}_1)}{\Delta t} \]