$$
\begin{array}{ll}
v_{l}=30.0 \mathrm{~km} / \mathrm{hr} W & v_{2}=40.0 \mathrm{~km} \mathrm{NW} \\
\Lambda t_{,}=4 \mathrm{hrs} & \Lambda t_{八}=7 \mathrm{hrs}
\end{array}
$$

## Problem 5 (20 points)

make Grand Bahama Island the origin of our coordinate system with East being the $+x$ direction and North being the $+y$ direction (see diagram below). Use constant velocity motion and addition of vectors

Several years ago, at 8 AM the eye of hurricane Floyd passed over Grand Bahama Island heading due west at a speed of $30.0 \mathrm{~km} / \mathrm{h}$. Four hours later, the course of hurricane Floyd shifted to Northwest towards the Florida coast and its speed increased to $40.0 \mathrm{~km} / \mathrm{h}$. Floyd continued on this course at this speed for two hours before turning due north again.
A. How far from Grand Bahama was hurricane Floyd 6 hours after it passes over the island?

## The hurricane makes two constant velocity motions:

$\Delta \vec{r}_{1}$ from going West at $30.0 \mathrm{~km} / \mathrm{hr}$ for $4 \mathrm{hrs} \&$
$\Delta \vec{r}_{2}$ from going $N W$ at $40.0 \mathrm{~km} / \mathrm{hr}$ for 2 hrs
The hurricane's displacement $\Delta \bar{r}=\Delta \vec{r}_{1}+\Delta \vec{r}_{2}$
$\Delta \vec{r}=\left|\Delta \vec{r}_{1}\right|(-\cos \phi \hat{i}+\sin \phi \hat{j})+\left|\Delta \vec{r}_{2}\right|(-\cos \alpha \hat{i}+\sin \alpha \hat{j})$ where

$\phi=0^{\circ}$ is the angle between $\Delta \vec{r}_{1}$ \& the $-x$ axis, $\alpha=45^{\circ}$ is the angle between $\Delta \vec{r}_{2}$ and the $-x$ axis,
$\left|\Delta \vec{r}_{1}\right|=v_{1} \Delta t_{1}=(30.0 \mathrm{~km} / \mathrm{hr})(4 \mathrm{hrs})=120 \mathrm{~km} \quad\left|\Delta \vec{r}_{2}\right|=v_{2} \Delta t_{2}=(40.0 \mathrm{~km} / \mathrm{hr})(2 \mathrm{~h})=80 \mathrm{~km}$
$\Delta \vec{r}=\left(-\left|\Delta \vec{r}_{1}\right| \cos \phi \hat{i}+0 \hat{j}\right)+\left(-\left|\Delta \vec{r}_{2}\right| \cos \alpha \hat{i}+\left|\Delta \vec{r}_{2}\right| \sin \alpha \hat{j}\right)=\left(-\left|\Delta \vec{r}_{1}\right| \cos \phi-\left|\Delta \vec{r}_{2}\right| \cos \alpha\right) \hat{i}+\left|\Delta \vec{r}_{2}\right| \sin \alpha \hat{j}$
$\Delta \vec{r}=\left[-(120 \mathrm{~km}) \cos 0^{\circ}-80 \mathrm{~km} \cos 45^{\circ}\right] \hat{i}+80 \mathrm{~km} \sin 45^{\circ} \hat{j}=-176.6 \mathrm{~km} \hat{i}+56.57 \mathrm{~km} \hat{j}$
$|\Delta \vec{r}|=\sqrt{(-176.6 \mathrm{~km})^{2}+(56.57 \mathrm{~km})^{2}}=185 \mathrm{~km}(185.4 \mathrm{~km})$
B. What was Floyd's average speed during this time?

$$
\begin{aligned}
& \text { Average Speed }=\text { distance } / \text { time }=(d 1+d 2) /(\text { Dt1 }+ \text { Dt2 })=(120 \mathrm{~km}+80 \mathrm{~km}) /(4 \mathrm{hrs}+2 \mathrm{hrs}) \\
& \text { Average Speed }=200 \mathrm{~km} / 6 \mathrm{hrs}=33.3 \mathrm{~km} / \mathrm{hr}
\end{aligned}
$$

C. What was Floyd's average velocity during this time?

Average velocity $\langle\vec{v}\rangle=$ displacement/time $=\Delta \vec{r} / \Delta t$
$|\langle\vec{v}\rangle|=|\Delta \vec{r} / \Delta t|=|\Delta \vec{r}| /\left(\Delta t_{1}+\Delta t_{2}\right)=(185.4 \mathrm{~km}) /(4 \mathrm{hrs}+2 \mathrm{hrs})=30.9 \mathrm{~km} / \mathrm{hr}$
direction: $\vartheta=\arctan \frac{\left|\Delta \vec{r}_{y}\right|}{\left|\Delta \vec{r}_{x}\right|}=\arctan \left(\frac{56.56 \mathrm{~km}}{176.6 \mathrm{~km}}\right)=17.8^{\circ}$
alternatively, $\langle\vec{v}\rangle=\Delta \vec{r} / \Delta t=(-176.6 \mathrm{~km} \hat{i}+56.57 \mathrm{~km} \hat{j}) /(4 \mathrm{hrs}+2 \mathrm{hrs})=-29.4 \mathrm{~km} \hat{i}+9.43 \mathrm{~km} \hat{j}$
D. Sketch a vector representing hurricane Floyd's average acceleration during this time.


Since Delta v is proportional to the average acceleration, Delta $v$ vector points in the direction of the acceleration. Recall that
$\vec{a}=\Delta \vec{v} / \Delta t=\left(\vec{v}_{f}-\vec{v}_{i}\right) / \Delta t$

