Chapter 8 – Potential energy and conservation of energy

I. Potential energy → Energy of configuration

II. Work and potential energy

III. Conservative / Non-conservative forces

IV. Determining potential energy values:

- Gravitational potential energy
- Elastic potential energy

I. V. Conservation of mechanical energy

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VIII. Power

I. Potential energy

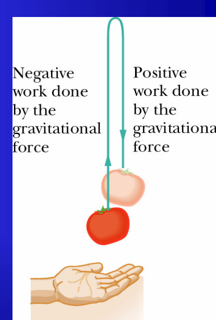
Energy associated with the arrangement of a system of objects that exert forces on one another.

Units: J

Examples:

- Gravitational potential energy: associated with the state of separation between objects which can attract one another via the gravitational force.
- Elastic potential energy: associated with the state of compression/extension of an elastic object.

II. Work and potential energy

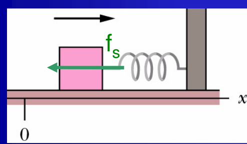


If tomato rises → gravitational force transfers energy "from" tomato's kinetic energy "to" the gravitational potential energy of the tomato-Earth system.

If tomato falls down → gravitational force transfers energy "from" the gravitational potential energy "to" the tomato's kinetic energy.

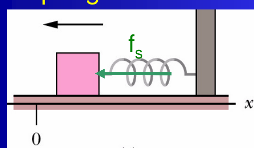
$\Delta U = -W$ Also valid for elastic potential energy

Spring compression



Spring force does –W on block → energy transfer from kinetic energy of the block to potential elastic energy of the spring.

Spring extension



Spring force does +W on block → energy transfer from potential energy of the spring to kinetic energy of the block.

General:

- System of two or more objects.

- A force acts between a particle in the system and the rest of the system.

- When system configuration changes → force does work on the object (W_1) transferring energy between KE of the object and some other form of energy of the system.

- When the configuration change is reversed → force reverses the energy transfer, doing W_2.

III. Conservative / Nonconservative forces

- If $W_1=W_2$ always → conservative force.

Examples: Gravitational force and spring force → associated potential energies.

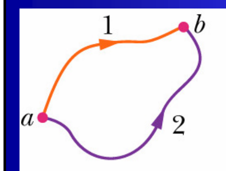
- If $W_1 \neq W_2$ → nonconservative force.

Examples: Drag force, frictional force → KE transferred into thermal energy. Non-reversible process.

- Thermal energy: Energy associated with the random movement of atoms and molecules. This is not a potential energy.

- Conservative force: The net work it does on a particle moving around every closed path, from an initial point and then back to that point is zero.

- The net work it does on a particle moving between two points does not depend on the particle's path.

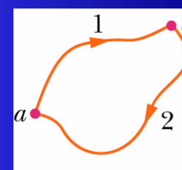


Conservative force → $W_{ab,1} = W_{ab,2}$

Proof:

$W_{ab,1} + W_{ba,2} = 0$ → $W_{ab,1} = -W_{ba,2}$

$W_{ab,2} = -W_{ba,2}$ → $W_{ab,2} = W_{ab,1}$



IV. Determining potential energy values

$$W = \int_{x_i}^{x_f} F(x)dx = -\Delta U$$

Force F is conservative

Gravitational potential energy:

$$\Delta U = -\int_{y_i}^{y_f} (-mg)dy = mg[y]_{y_i}^{y_f} = mg(y_f - y_i) = mg\Delta y$$

Change in the gravitational potential energy of the particle-Earth system.

$$U_i = 0, \quad y_i = 0 \rightarrow U(y) = mgy$$

Reference configuration

The gravitational potential energy associated with particle-Earth system depends only on particle's vertical position "y" relative to the reference position y=0, not on the horizontal position.

Elastic potential energy:

$$\Delta U = -\int_{x_i}^{x_f} (-kx)dx = \frac{k}{2}[x^2]_{x_i}^{x_f} = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

Change in the elastic potential energy of the spring-block system.

Reference configuration → when the spring is at its relaxed length and the block is at $x_i=0$.

$$U_i = 0, \quad x_i = 0 \rightarrow U(x) = \frac{1}{2}kx^2$$

Remember! Potential energy is always associated with a system.

V. Conservation of mechanical energy

Mechanical energy of a system: Sum of its potential (U) and kinetic (K) energies.

$E_{mec} = U + K$

Assumptions: - Only conservative forces cause energy transfer within the system.

- The system is isolated from its environment → No external force from an object outside the system causes energy changes inside the system.

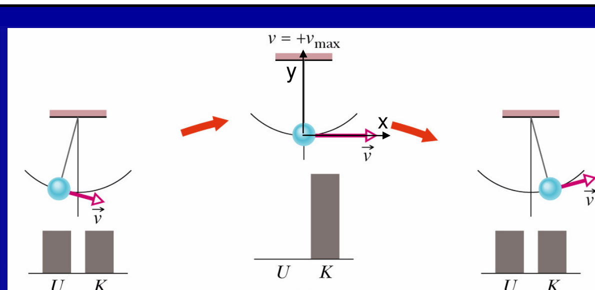
$$W = \Delta K \qquad W = -\Delta U$$

$$\Delta K + \Delta U = 0 \rightarrow (K_2 - K_1) + (U_2 - U_1) = 0 \rightarrow K_2 + U_2 = K_1 + U_1$$

$\Delta E_{mec} = \Delta K + \Delta U = 0$

- In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy of the system cannot change.

- When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant without considering the intermediate motion and without finding the work done by the forces involved.



E_{mec} = constant

$$\Delta E_{mec} = \Delta K + \Delta U = 0$$

$$K_2 + U_2 = K_1 + U_1$$

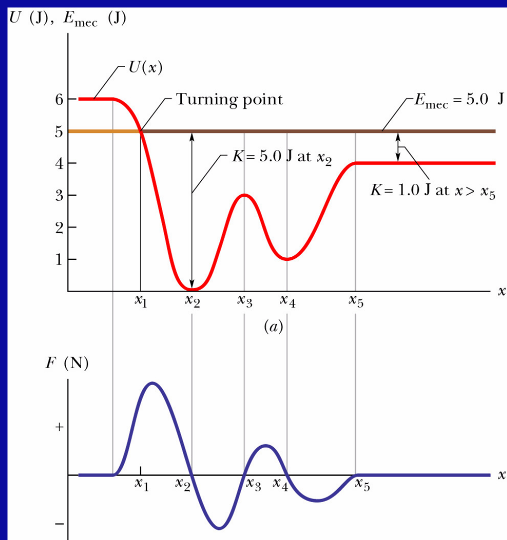
Potential energy curves

Finding the force analytically:

$$\Delta U(x) = -W = -F(x)\Delta x \rightarrow F(x) = -\frac{dU(x)}{dx} \quad (1D\ motion)$$

- The force is the negative of the slope of the curve U(x) versus x.

- The particle's kinetic energy is: $K(x) = E_{mec} - U(x)$



Turning point: a point x at which the particle reverses its motion (K=0).

K always ≥0 ($K=0.5mv^2 \geq 0$)

Examples:

$x= x_1$ → E_{mec}= 5J=5J+K → K=0

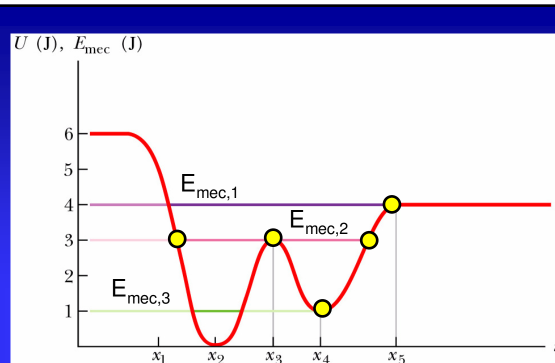
$x<x_1$ → E_{mec}= 5J= >5J+K→ K<0 → impossible

Equilibrium points: where the slope of the U(x) curve is zero → F(x)=0
ΔU = -F(x) dx → ΔU/dx = -F(x)

ΔU(x)/dx = -F(x) → Slope

↓

Equilibrium points



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Example: $x \geq x_5$ → $E_{mec,1}$= 4J=4J+K → K=0 and also F=0 →x_5 **neutral equilibrium**

$x_2>x>x_1$, $x_5>x>x_4$ → $E_{mec,2}$= 3J= 3J+K → K=0 → **Turning points**

x_3 → K=0, F=0 → particle stationary → **Unstable equilibrium**

x_4 → $E_{mec,3}$=1J=1J+K → K=0, F=0, it cannot move to $x>x_4$ or $x<x_4$, since then K<0 → **Stable equilibrium**

Review: Potential energy

$W = -\Delta U$

- The zero is arbitrary → Only potential energy differences have physical meaning.

- The potential energy is a scalar function of the position.

- The force (1D) is given by: $F = -dU/dx$

P1. The force between two atoms in a diatomic molecule can be represented by the following potential energy function:

$$U(x) = U_0\left[\left(\frac{a}{x}\right)^{12} - 2\left(\frac{a}{x}\right)^{6}\right]$$

where U_0 and *a* are constants.

i) Calculate the force F_x

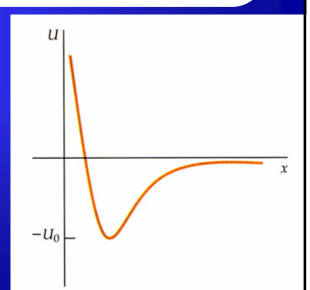
$$F(x) = -\frac{dU(x)}{dx} = -U_0\left[12\left(\frac{-a}{x^2}\right)\left(\frac{a}{x}\right)^{11} - 2\left(\frac{-a}{x^2}\right)6\left(\frac{a}{x}\right)^{5}\right] =$$

$$-U_0\left[-12a^{12}x^{-13} + 12a^6x^{-7}\right] = \frac{12U_0}{a}\left[\left(\frac{a}{x}\right)^{13} - \left(\frac{a}{x}\right)^{7}\right]$$

ii) Minimum value of U(x).

$$U(x)_{\min} \ \ if\ \ \frac{dU(x)}{dx} = -F(x) = 0 \rightarrow \frac{-12U_0}{a}\left[\left(\frac{a}{x}\right)^{13} - \left(\frac{a}{x}\right)^{7}\right] = 0$$

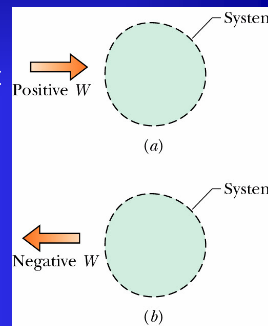
$$\rightarrow x = a \qquad U(a) = U_0[1-2] = -U_0$$



U_0 is approx. the energy necessary to dissociate the two atoms.

VI. Work done on a system by an external force

Work is energy transfer "to" or "from" a system by means of an external force acting on that system.



When more than one force acts on a system their net work is the energy transferred to or from the system.

No Friction: **$W = \Delta E_{mec} = \Delta K + \Delta U$ → Ext. force**

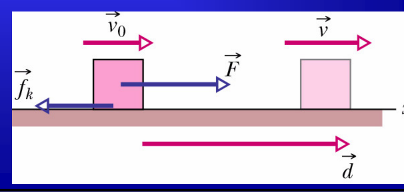
Remember! **$\Delta E_{mec} = \Delta K + \Delta U = 0$** only when:

- System isolated.
- No ext. forces act on a system.
- All internal forces are conservative.

Friction:

$$F - f_k = ma$$

$$v^2 = v_0^2 + 2ad \rightarrow a = 0.5(v^2 - v_0^2)/d$$



$$F - f_k = \frac{m}{2d}(v^2 - v_0^2) \rightarrow Fd - f_k d = \frac{1}{2}m(v^2 - v_0^2) \rightarrow Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_k d$$

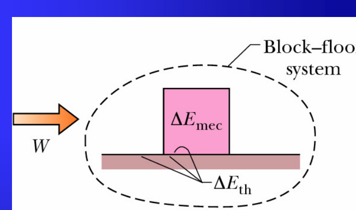
$$W = Fd = \Delta K + f_k d$$

General: $W = Fd = \Delta E_{mec} + f_k d$

Example: Block sliding up a ramp.

Thermal energy: $\Delta E_{th} = f_k d$

Friction due to cold welding between two surfaces. As the block slides over the floor, the sliding causes tearing and reforming of the welds between the block and the floor, which makes the block-floor warmer.



Work done on a system by an external force, friction involved

$$W = Fd = \Delta E_{mec} + \Delta E_{th}$$

VI. Conservation of energy

Total energy of a system = E mechanical + E thermal + E internal

- The total energy of a system can only change by amounts of energy transferred "from" or "to" the system.

$W = \Delta E = \Delta E_{mec} + \Delta E_{th} + \Delta E_{\text{int}}$ → Experimental law

-The total energy of an isolated system cannot change. (There cannot be energy transfers to or from it).

Isolated system: $\Delta E_{mec} + \Delta E_{th} + \Delta E_{\text{int}} = 0$

In an isolated system we can relate the total energy at one instant to the total energy at another instant without considering the energies at intermediate states.

Example: Trolley pole jumper

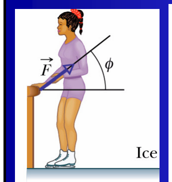
1) Run → Internal energy (muscles) gets transferred into kinetic energy.

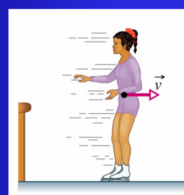
2) Jump/Ascent → Kinetic energy transferred to potential elastic energy (trolley pole deformation) and to gravitational potential energy

3) Descent→ Gravitational potential energy gets transferred into kinetic energy.

VII. External forces and internal energy changes

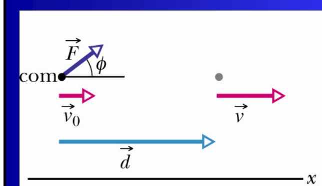
Example: skater pushes herself away from a railing. There is a force F on her from the railing that increases her kinetic energy.





i) One part of an object (skater's arm) does not move like the rest of body.

ii) Internal energy transfer (from one part of the system to another) via the external force F. Biochemical energy from muscles transferred to kinetic energy of the body.



$$W_{F,ext} = \Delta K = F(\cos\varphi)d$$

$$Non-isolated\ system \rightarrow \Delta K + \Delta U = W_{F,ext} = Fd\cos\varphi$$

$$\Delta E_{mec} = Fd\cos\varphi$$

Change in system's mechanical energy by an external force

Proof:

$$v^2 = v_0^2 + 2a_x d \quad (\cdot 0.5M)$$

$$\frac{1}{2}Mv^2 - \frac{1}{2}Mv_0^2 = Ma_x d$$

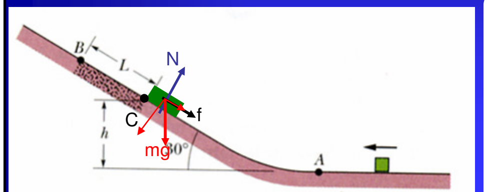
$$\Delta K = (F\cos\varphi)d$$

VIII. Power

Average power: $P_{avg} = \dfrac{\Delta E}{\Delta t}$

Instantaneous power: $P = \dfrac{dE}{dt}$

61. In the figure below, a block slides along a path that is without friction until the block reaches the section of length L=0.75m, which begins at height h=2m. In that section, the coefficient of kinetic friction is 0.4. The block passes through point A with a speed of 8m/s. Does it reach point B (where the section of friction ends)? If so, what is the speed there and if not, what greatest height above point A does it reach?



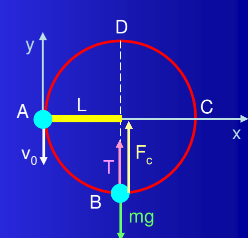
$$N = mg\cos 30^\circ = 8.5m$$
$$f_k = \mu_k N = (0.4)(8.5m) = 3.4m$$
$$A - C \rightarrow Only\ conservative\ forces\ \Delta E_{mec} = 0$$
$$\rightarrow K_A + U_A = K_C + U_C$$
$$\frac{1}{2}mv_A^2 = \frac{1}{2}mv_c^2 + mgh_c \rightarrow v_c = 5m/s$$

The kinetic energy in C turns into thermal and potential energy → Block stops.

$$K_c = 0.5mv_c^2 = 12.4m$$
$$K_c = mgy + f_k d \rightarrow 12.4m = mg(d\sin 30^\circ) + 3.4md \rightarrow d = 1.49\ meters$$
$$d > L = 0.75m \rightarrow Block\ reaches\ B$$

$$Isolated\ system \rightarrow \Delta E = 0 = \Delta E_{mec} + \Delta U + \Delta E_{th} \rightarrow K_C + U_C = K_B + U_B + f_k L$$
$$12.4m = 0.5mv_B^2 + mg(y_B - y_c) + \mu_k mgL\cos 30^\circ = 0.5mv_B^2 + mgL\sin 30^\circ + \mu_k mgL\cos 30^\circ$$
$$12.4m = 0.5mv_B^2 + 3.67m + 2.5m \rightarrow v_B = 3.5m/s$$

129. A massless rigid rod of length L has a ball of mass m attached to one end. The other end is pivoted in such a way that the ball will move in a vertical circle. First, assume that there is no friction at the pivot. The system is launched downward from the horizontal position A with initial speed v_0. The ball just barely reaches point D and then stops. (a) Derive an expression for v_0 in terms of L, m and g. (b) What is the tension in the rod when the ball passes through B? (c) A little girl is placed on the pivot to increase the friction there. Then the ball just barely reaches C when launched from A with the same speed as before. What is the decrease in mechanical energy during this motion? (d) What is the decrease in mechanical energy by the time the ball finally comes to rest at B after several oscillations?



$$(a)\ \Delta E_{mec} = 0 \rightarrow K_f + U_f = K_i + U_i$$
$$K_D = 0;\ U_A = 0$$
$$mgL = \frac{1}{2}mv_0^2 \rightarrow v_0 = \sqrt{2gL}$$

$$(b)\ F_{cent} = ma_c = T - mg$$
$$m\frac{v_B^2}{L} = T - mg \rightarrow T = m\left(\frac{1}{L}v_B^2 + g\right)$$
$$U_A + K_A = U_B + K_B$$
$$\frac{1}{2}mv_0^2 = -mgL + \frac{1}{2}mv_B^2 \rightarrow$$
$$\frac{1}{2}2gL + gL = \frac{1}{2}v_B^2 \rightarrow v_B = 2\sqrt{gL} \qquad T = 5mg$$

$$(c)\ v_c = 0$$
$$W = \Delta E = \Delta E_{mec} + \Delta E_{th}$$
$$\Delta E_{th} = f_k d$$

The difference in heights or in gravitational potential energies between the positions C (reached by the ball when there is friction) and D during the frictionless movement Is going to be the loss of mechanical energy which goes into thermal energy.

$$(c)\ \Delta E_{th} = -mgL$$

(d) The difference in height between B and D is 2L. The total loss of mechanical energy (which all goes into thermal energy) is:

$$\Delta E_{mec} = -2mgL$$

101. A 3kg sloth hangs 3m above the ground. (a) What is the gravitational potential energy of the sloth-Earth system if we take the reference point y=0 to be at the ground? If the sloth drops to the ground and air drag on it is assumed to be negligible, what are (b) the kinetic energy and (c) the speed of the sloth just before it reaches the ground?

$(a)\ \Delta E_{mec} = 0 \rightarrow K_f + U_f = K_i + U_i$

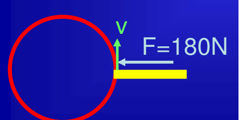
$U_f(ground) = 0;\ K_i = 0$

$U_i = mgh = (3.2kg)(9.8m/s^2)(3m) = 94.1J$

$(b)\ K_f = 94.1J$

$(c)\ K_f = \frac{1}{2}mv_f^2 \rightarrow v_f = \sqrt{\frac{2K_f}{m}} = 7.67m/s$

130. A metal tool is sharpen by being held against the rim of a wheel on a grinding machine by a force of 180N. The frictional forces between the rim and the tool grind small pieces of the tool. The wheel has a radius of 20cm and rotates at 2.5 rev/s. The coefficient of kinetic friction between the wheel and the tool is 0.32. At what rate is energy being transferred from the motor driving the wheel and the tool to the kinetic energy of the material thrown from the tool?



$v = 2.5\left(\frac{rev}{s}\right)\left(\frac{2\pi(0.2m)}{1rev}\right) = 3.14m/s$

$f_k = \mu_k N = \mu_k F = (0.32)(180N) = 57.6N$

Power dissipated by friction = Power supplied motor

$P = \vec{f} \cdot \vec{v} = (-57.6N)(3.14m/s) = -181W$

$P_{motor} = 181W$

82. A block with a kinetic energy of 30J is about to collide with a spring at its relaxed length. As the block compresses the spring, a frictional force between the block and floor acts on the block. The figure below gives the kinetic energy of the block (K(x)) and the potential energy of the spring (U(x)) as a function of the position x of the block, as the spring is compressed. What is the increase in thermal energy of the block and the floor when (a) the block reaches position 0.1 m and (b) the spring reaches its maximum compression?

$Isolated\ system \rightarrow \Delta E = 0 \rightarrow 0 = \Delta E_{mec} + \Delta E_{th}$

$\Delta E_{th} = -\Delta E_{mec}$

$(a)\ x = 0.1m \qquad Graph:\ K_f = 20J,\ U_f = 3J$

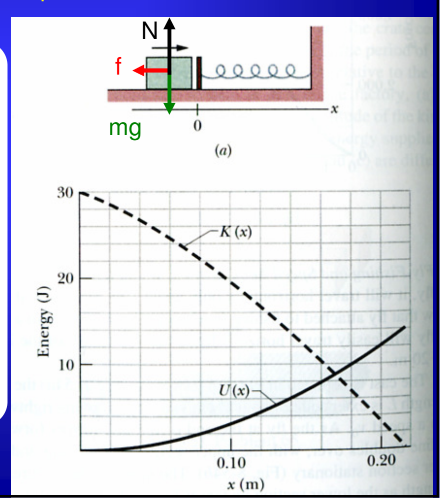
$E_{mec,i} = K_i = 30J \qquad E_{mec,f} = K_f + U_f = 23J$

$\Delta E_{mec} = 23J - 30J = -7J \rightarrow \Delta E_{th} = 7J$

$(b)\ x_{max} \rightarrow v = 0 \rightarrow K = 0 \rightarrow x = 0.21m$

$E_{mec,i} = K_i = 30J \qquad E_{mec,f} = U_f = 14J$

$\Delta E_{mec} = 14J - 30J = -16J \rightarrow \Delta E_{th} = 16J$



B1. A 2kg block is pushed against a spring with spring constant k=500 N/m compressing it 20 cm. After the block is released, it travels along a frictionless horizontal surface and a 45º incline plane. What is the maximum height reached by this block?