Chapter 6 – Kinetic energy and work

I. Kinetic energy.

II. Work.

III. Work - Kinetic energy theorem.

IV. Work done by a constant force

- Gravitational force

V. Work done by a variable force.

- Spring force.
- General.
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VI. Power

Energy: scalar quantity associated with a state (or condition) of one or more objects.

I. Kinetic energy

Energy associated with the state of motion of an object.

$$K = \frac{1}{2}mv^2 \qquad (7.1)$$

Units: 1 Joule = 1J = 1 kgm^2/s^2 = N m

II. Work

Energy transferred "to" or "from" an object by means of a force acting on the object.

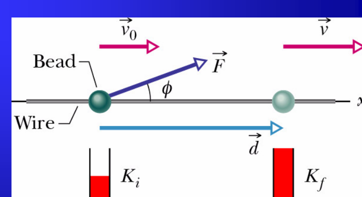
To → +W
From → -W

- Constant force: $F_x = ma_x$

$$v^2 = v_0^2 + 2a_x d \rightarrow a_x = \frac{v^2 - v_0^2}{2d}$$

$$F_x = ma_x = \frac{1}{2}m(v^2 - v_0^2)\frac{1}{d} \rightarrow ma_x d = \frac{1}{2}m(v^2 - v_0^2)$$

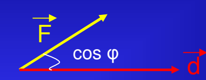
$$\rightarrow \frac{1}{2}m(v^2 - v_0^2) = K_f - K_i = F_x d \rightarrow \boxed{W = F_x d}$$



Work done by the force = Energy transfer due to the force.

- *To calculate the work done on an object by a force during a displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.*

$$W = F_x d = F\cos\varphi \cdot d = \vec{F}\cdot\vec{d} \qquad (7.3)$$



- Assumptions: 1) F=cte, 2) Object particle-like.

Units: 1 Joule = 1J = 1 kgm^2/s^2

$$\varphi < 90^\circ \rightarrow +W$$
$$180^\circ > \varphi > 90^\circ \rightarrow -W$$
$$\varphi = 90^\circ \rightarrow 0$$

A force does +W when it has a vector component in the same direction as the displacement, and –W when it has a vector component in the opposite direction. W=0 when it has no such vector component.

Net work done by several forces = Sum of works done by individual forces.

Calculation: 1) $W_{net} = W_1 + W_2 + W_3 + \ldots$

2) $\vec{F}_{net} \rightarrow W_{net} = \vec{F}_{net} \cdot \vec{d}$

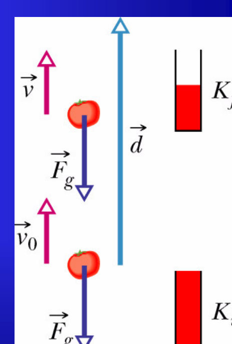
II. Work-Kinetic Energy Theorem

$$\Delta K = K_f - K_i = W \qquad (7.4)$$

Change in the kinetic energy of the particle = Net work done on the particle

III. Work done by a constant force

- Gravitational force: $W = \vec{F}\cdot\vec{d} = mgd\cos\varphi \qquad (7.5)$

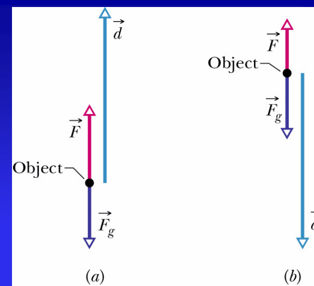


Rising object: W= mgd cos180º = -mgd → F_g transfers mgd energy from the object's kinetic energy.

Falling object: W= mgd cos 0º = +mgd → F_g transfers mgd energy to the object's kinetic energy.

- External applied force + Gravitational force:

$$\Delta K = K_f - K_i = W_a + W_g \qquad (7.6)$$



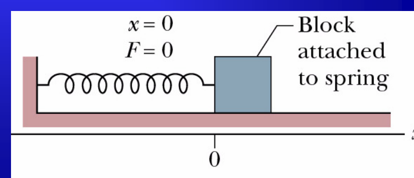
Object stationary before and after the lift: $W_a + W_g = 0$

The applied force transfers the same amount of energy to the object as the gravitational force transfers from the object.

IV. Work done by a variable force

- Spring force: $$\vec{F} = -k\vec{d} \qquad (7.7)$$

Hooke's law

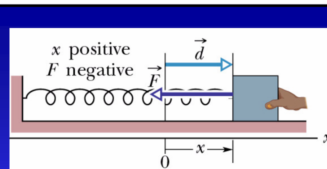


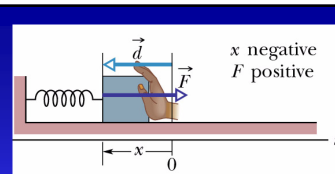
k = spring constant → measures spring's stiffness.

Units: N/m

Hooke's law

$$1D \rightarrow F_x = -kx$$

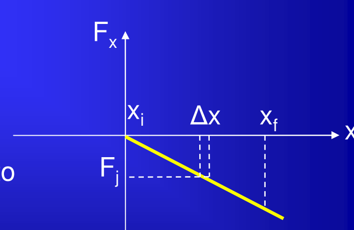




Work done by a spring force:

- Assumptions:
  - Spring is massless → $m_{spring} << m_{block}$
  - Ideal spring → obeys Hooke's law exactly.
  - Contact between the block and floor is frictionless.
  - Block is particle-like.

- Calculation:



1) The block displacement must be divided into many segments of infinitesimal width, Δx.

2) $F(x) \approx$ cte within each short Δx segment.

$$W_s = \sum F_j \Delta x \Rightarrow \ \Delta x \to 0 \ \ \Rightarrow W_s = \int_{x_i}^{x_f} F\ dx = \int_{x_i}^{x_f} (-kx)\ dx$$

$$W_S = -k\int_{x_i}^{x_f} x\ dx = \left(-\frac{1}{2}k\right)\left[x^2\right]_{x_i}^{x_f} = \left(-\frac{1}{2}k\right)(x_f^2 - x_i^2)$$

$$W_s = \frac{1}{2}k\ x_i^2 - \frac{1}{2}k\ x_f^2$$

$W_s=0$ → If Block ends up at $x_f=x_i$.

$$W_s = -\frac{1}{2}k\ x_f^2 \qquad if\ x_i = 0$$

Work done by an applied force + spring force:

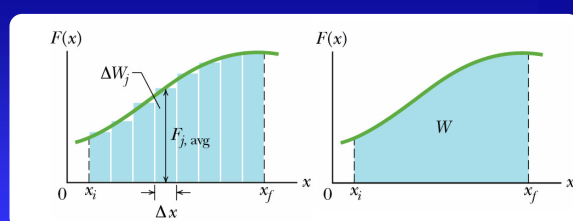
$$\Delta K = K_f - K_i = W_a + W_s$$

Block stationary before and after the displacement: $\Delta K=0$ → $W_a = -W_s$

→The work done by the applied force displacing the block is the negative of the work done by the spring force.

Work done by a general variable force:

1D-Analysis



$$\Delta W_j = F_{j,avg} \Delta x$$

$$W = \sum \Delta W_j = \sum F_{j,avg} \Delta x$$

$$better\ approximation \Rightarrow more\ \Delta x,\ \ \Delta x \to 0$$

$$\Rightarrow W = \lim_{\Delta x \to 0} F_{j,avg} \Delta x \Rightarrow \boxed{W = \int_{x_i}^{x_f} F(x)dx} \qquad (7.10)$$

Geometrically: Work is the area between the curve F(x) and the x-axis.

3D-Analysis

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \;; \qquad F_x = F(x),\; F_y = F(y),\; F_z = F(z)$$

$$d\vec{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$dW = \vec{F}\cdot d\vec{r} = F_x dx + F_y dy + F_z dz \Rightarrow \boxed{W = \int_{r_i}^{r_f} dW = \int_{x_i}^{x_f} F_x dx + \int_{y_i}^{y_f} F_y dy + \int_{z_i}^{z_f} F_z dz}$$

Work-Kinetic Energy Theorem - Variable force

$$W = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} ma\; dx$$

$$ma\; dx = m\boxed{\frac{dv}{dt}}dx \quad \rightarrow\; m\frac{dv}{dx}v\; dx = mvdv$$

$$\left(\frac{dv}{dt} = \frac{dv}{dx}\frac{dx}{dt} = \frac{dv}{dx}v\right)$$

$$W = \int_{v_i}^{v_f} mv\; dv = m\int_{v_i}^{v_f} v\; dv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = K_f - K_i = \Delta K$$

V. Power

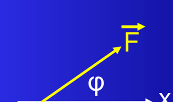
Time rate at which the applied force does work.

- Average power: amount of work done in an amount of time Δt by a force.

$$P_{avg} = \frac{W}{\Delta t} \qquad (7.12)$$

- Instantaneous power: instantaneous time rate of doing work.

$$P = \frac{dW}{dt} \qquad (7.13)$$

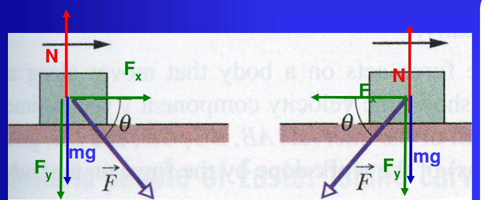


$$P = \frac{dW}{dt} = \frac{F\cos\varphi\; dx}{dt} = F\cos\varphi\left(\frac{dx}{dt}\right) = Fv\cos\varphi = \vec{F}\cdot\vec{v} \qquad (7.14)$$

Units: 1 Watt= 1 W = 1J/s

1 kilowatt-hour = 1 kW·h = 3.60 x 10^6 J = 3.6 MJ

54. In the figure (a) below a 2N force is applied to a 4kg block at a downward angle θ as the block moves rightward through 1m across a frictionless floor. Find an expression for the speed v_f at the end of that distance if the block's initial velocity is: (a) 0 and (b) 1m/s to the right. (c) The situation in (b) is similar in that the block is initially moving at 1m/s to the right, but now the 2N force is directed downward to the left. Find an expression for the speed of the block at the end of the 1m distance.



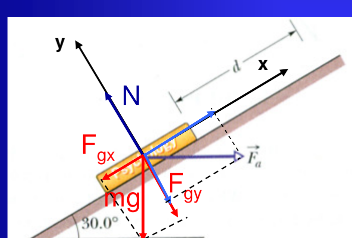
$$W = \vec{F} \cdot \vec{d} = (F\cos\theta)d$$
$$W = \Delta K = 0.5m(v_f^2 - v_0^2)$$

$$(a)\ \ v_0 = 0 \rightarrow \Delta K = 0.5mv_f^2$$
$$(2N)\cos\theta = 0.5(4kg)v_f^2$$
$$\rightarrow v_f = \sqrt{\cos\theta}\ \ m/s$$

$$(b)\ \ v_0 = 1m/s \rightarrow \Delta K = 0.5mv_f^2 - 0.5\cdot(4kg)\cdot(1m/s)^2$$
$$(2N)\cos\theta = 0.5(4kg)v_f^2 - 2J$$
$$\rightarrow v_f = \sqrt{1+\cos\theta}\ \ m/s$$

$$(c)\ \ v_0 = 1m/s \rightarrow \Delta K = 0.5mv_f^2 - 2J$$
$$-(2N)\cos\theta = 0.5(4kg)v_f^2 - 2J$$
$$\rightarrow v_f = \sqrt{1-\cos\theta}\ \ m/s$$

18. In the figure below a horizontal force F_a of magnitude 20N is applied to a 3kg psychology book, as the book slides a distance of d=0.5m up a frictionless ramp. (a) During the displacement, what is the net work done on the book by F_a, the gravitational force on the book and the normal force on the book? (b) If the book has zero kinetic energy at the start of the displacement, what is the speed at the end of the displacement?

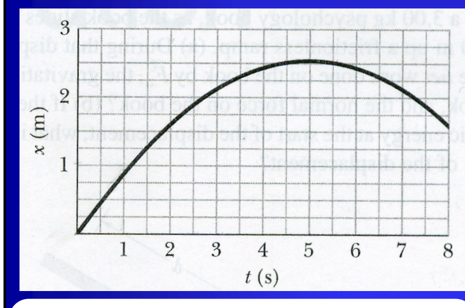


$$\vec{N} \perp \vec{d} \rightarrow W = 0$$
$$Only\ F_{gx},\ F_{ax}\ do\ work$$

$$(a)\ \ W = W_{Fa\,x} - W_{Fg\,x} \qquad or \qquad W_{net} = \vec{F}_{net} \cdot \vec{d}$$
$$F_{net} = Fa_x - Fg_x = 20\cos 30^\circ - mg\sin 30^\circ$$
$$W_{net} = (17.32N - 14.7N)0.5m = 1.31J$$

$$(b)\ \ K_0 = 0 \rightarrow W = \Delta K = K_f$$
$$W = 1.31J = 0.5mv_f^2 \rightarrow v_f = 0.93m/s$$

55. A 2kg lunchbox is sent sliding over a frictionless surface, in the positive direction of an x axis along the surface. Beginning at t=0, a steady wind pushes on the lunchbox in the negative direction of x, Fig. below. Estimate the kinetic energy of the lunchbox at (a) t=1s, (b) t=5s. (c) How much work does the force from the wind do on the lunch box from t=1s to t=5s?



$$Motion \rightarrow concave\ downward\ parabola$$

$$x = \frac{1}{10}t(10-t)$$

$$v = \frac{dx}{dt} = 1 - \frac{2}{10}t$$

$$a = \frac{dv}{dt} = -\frac{2}{10} = -0.2m/s^2$$

$$F = cte = ma = (2kg)(-0.2m/s^2) = -0.4N$$

$$W = F \cdot x = (-0.4N)(t - 0.1t^2)$$

$$(a)\ \ t = 1s \rightarrow v_f = 0.8m/s$$

$$K_f = 0.5(2kg)(0.8m/s)^2 = 0.64J$$

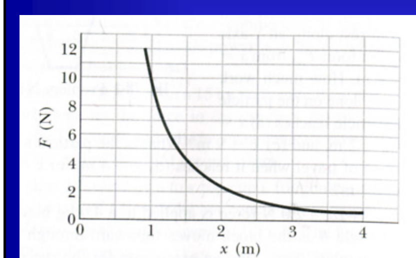
$$(b)\ \ t = 5s \rightarrow v_f = 0$$

$$K_f = 0J$$

$$(c)\ \ W = \Delta K = K_f(5s) - K_f(1s)$$

$$W = 0 - 0.64 = -0.64J$$

74. (a) Find the work done on the particle by the force represented in the graph below as the particle moves from x=1 to x=3m. (b) The curve is given by F=a/x^2, with a=9Nm^2. Calculate the work using integration

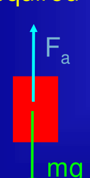


$$(a)\ \ W = Area\ under\ curve$$

$$W = (11.5 squares)(0.5m)(1N) = 5.75J$$

$$(b)\ \ W = \int_1^3 \frac{9}{x^2}dx = -9\left[\frac{1}{x}\right]_1^3 = -9(\frac{1}{3} - 1) = 6J$$

73. An elevator has a mass of 4500kg and can carry a maximum load of 1800kg. If the cab is moving upward at full load at 3.8m/s, what power is required of the force moving the cab to maintain that speed?



$$m_{total} = 4500kg + 1800kg = 6300kg$$

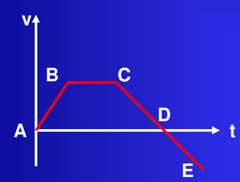
$$\vec{F}_a + m\vec{g} = \vec{F}_{net} = \vec{0} \rightarrow F_a - F_g = 0 \rightarrow$$

$$F_a = mg = (6300kg)(9.8m/s^2) = 61.74kN$$

$$P = \vec{F} \cdot \vec{v} = (61.74kN)(3.8m/s)$$

$$P = 234.61kW$$

A single force acts on a body that moves along an x-axis. The figure below shows the velocity component versus time for the body. For each of the intervals AB, BC, CD, and DE, give the sign (plus or minus) of the work done by the force, or state that the work is zero.



$$W = \Delta K = K_f - K_0 = \frac{1}{2}m\left(v_f^2 - v_0^2\right)$$

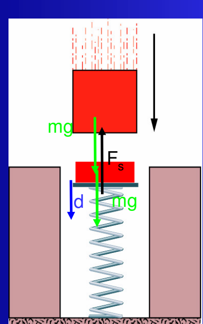
$$AB \rightarrow v_B > v_A \rightarrow W > 0$$

$$BC \rightarrow v_C = v_B \rightarrow W = 0$$

$$CD \rightarrow v_D < v_C \rightarrow W < 0$$

$$DE \rightarrow v_E < 0,\ \ v_D = 0 \rightarrow W > 0$$

50. A 250g block is dropped onto a relaxed vertical spring that has a spring constant of k=2.5N/cm. The block becomes attached to the spring and compresses the spring 12 cm before momentarily stopping. While the spring is being compressed, what work is done on the block by (a) the gravitational force on it and (b) the spring force? (c) What is the speed of the block just before it hits the spring? (Friction negligible) (d) If the speed at impact is doubled, what is the maximum compression of the spring?



$$(a)\ \ W_{Fg} = \vec{F}_g \vec{d} = mgd = (0.25kg)(9.8m/s^2)(0.12m) = 0.29J$$

$$(b)\ \ W_s = -\frac{1}{2}kd^2 = -0.5\cdot(250N/m)(0.12m)^2 = -1.8J$$

$$(c)\ \ W_{net} = \Delta K = 0.5mv_f^2 - 0.5mv_i^2$$

$$v_f = 0 \rightarrow K_f = 0 \rightarrow \Delta K = -K_i = -0.5mv_i^2 = W_{Fg} + W_s$$

$$0.29J - 1.8J = -0.5\cdot(0.25kg)v_i^2$$

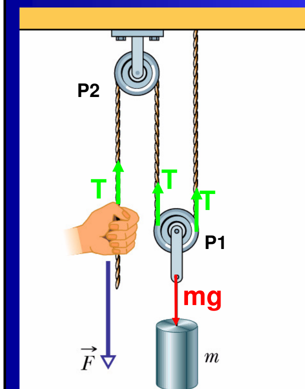
$$\rightarrow v_i = 3.47m/s$$

$$(d)\ \ If\ \ v_i' = 6.95m/s \rightarrow Maximum\ \ spring\ \ compression?\ v_f = 0$$

$$W_{net} = mgd' - 0.5kd'^2 = \Delta K = -0.5mv_i'^2$$

$$d' = 0.23m$$

62. In the figure below, a cord runs around two massless, frictionless pulleys; a canister with mass m=20kg hangs from one pulley; and you exert a force F on the free end of the cord. (a) What must be the magnitude of F if you are to lift the canister at a constant speed? (b) To lift the canister by 2cm, how far must you pull the free end of the cord? During that lift, what is the work done on the canister by (c) your force (via the cord) and (d) the gravitational force on the canister?



$$(a)\ \ Pulley\ 1:\ \ v = cte \rightarrow F_{net} = 0 \rightarrow 2T - mg = 0 \rightarrow T = 98N$$

$$Hand - cord:\ \ T - F = 0 \rightarrow F = \frac{mg}{2} = 98N$$

(b) To rise "m" 0.02m, two segments of the cord must be shorten by that amount. Thus, the amount of the string pulled down at the left end is: 0.04m

$$(c)\ \ W_F = F \cdot d = (98N)(0.04m) = 3.92J$$

$$(d)\ \ W_{Fg} = -mgd = (-0.02m)(20kg)(9.8m/s^2) = -3.92J$$

$W_F + W_{Fg} = 0$ There is no change in kinetic energy.