Chapter 6 – Force and Motion II

I. Drag forces and terminal speed.

II. Uniform circular motion.

III. Non-Uniform circular motion.

I. Drag force and terminal speed

-Fluid: anything that can flow. Example: gas, liquid.

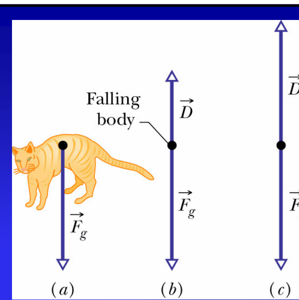
-Drag force: $\vec{D}$

- Appears when there is a relative velocity between a fluid and a body.
- Opposes the relative motion of a body in a fluid.
- Points in the direction in which the fluid flows.

Assumptions:

* Fluid = air.
* Body is blunt (baseball).
* Fast relative motion → turbulent air.

$$D = \frac{1}{2} C \rho A v^2 \qquad (6.3)$$



C = drag coefficient (0.4-1).

ρ = air density (mass/volume).

A= effective body's cross sectional area → area perpendicular to $\vec{v}$

-Terminal speed: v_t

- Reached when the acceleration of an object that experiences a vertical movement through the air becomes zero → F_g=D

$$D - F_g = ma \rightarrow if\ a = 0 \rightarrow \frac{1}{2} C \rho A v^2 - F_g = 0$$

$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \qquad (6.4)$$

II. Uniform circular motion

-Centripetal acceleration:

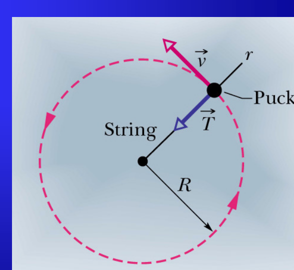
$$a = \frac{v^2}{r} \qquad (6.5)$$

v, a = ctes, but direction changes during motion.

A centripetal force accelerates a body by changing the direction of the body's velocity without changing its speed.

-Centripetal force:

$$F = m\frac{v^2}{R} \qquad (6.6)$$



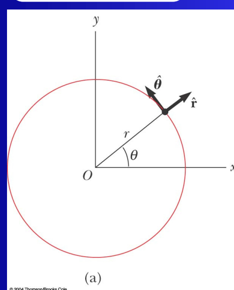
$\vec{a}$, $\vec{F}$ are directed toward the center of curvature of the particle's path.

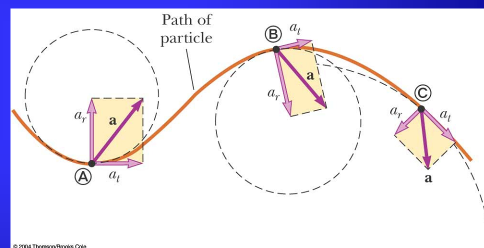
III. Non-Uniform circular motion

- A particle moves with varying speed in a circular path.
- The acceleration has two components: - Radial → $a_r = v^2/R$

  - Tangential → $a_t = dv/dt$
- a_t causes the change in the speed of the particle.

$$a = \sqrt{a_r^2 + a_t^2}$$



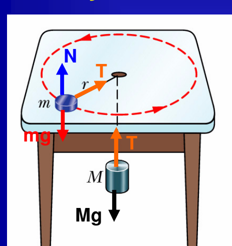


$$\vec{a} = \vec{a}_t + \vec{a}_r = \frac{d|\vec{v}|}{dt}\hat{\theta} - \frac{v^2}{r}\hat{r}$$

$$\sum \vec{F} = \sum \vec{F}_r + \sum \vec{F}_t$$

- In uniform circular motion, v = constant → $a_t = 0$ → $a = a_r$

49. A puck of mass m slides on a frictionless table while attached to a hanging cylinder of mass M by a cord through a hole in the table. What speed keeps the cylinder at rest?



$$For\, M \rightarrow T = Mg \rightarrow a_c = 0$$

$$For\ m \rightarrow T = m\frac{v^2}{r} \rightarrow Mg = m\frac{v^2}{r} \rightarrow v = \sqrt{\frac{Mgr}{m}}$$

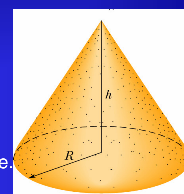
33E. Calculate the drag force on a missile 53cm in diameter cruising with a speed of 250m/s at low altitude, where the density of air is 1.2kg/m³. Assume C=0.75

$$D = \frac{1}{2}C\rho A v^2 = 0.5\cdot 0.75\cdot(1.2kg/m^3)\cdot\pi\cdot(0.53m/2)^2(250m/s)^2 = 6.2kN$$

32. The terminal speed of a ski diver is 160 km/h in the spread eagle position and 310 km/h in the nose-dive position. Assuming that the diver's drag coefficient C does not change from one point to another, find the ratio of the effective cross sectional area A in the slower position to that of the faster position.

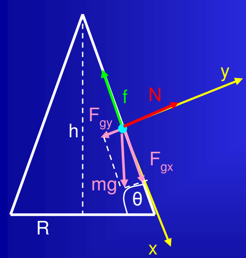
$$v_t = \sqrt{\frac{2F_g}{C\rho A}} \rightarrow \frac{160km/h}{310km/h} = \frac{\sqrt{\frac{2F_g}{C\rho A_E}}}{\sqrt{\frac{2F_g}{C\rho A_D}}} = \frac{\sqrt{A_D}}{\sqrt{A_E}} \rightarrow \frac{A_E}{A_D} = 3.7$$

11P. A worker wishes to pile a cone of sand onto a circular area in his yard. The radius of the circle is R, and no sand is to spill into the surrounding area. If μ_s is the static coefficient of friction between each layer of sand along the slope and the sand beneath it (along which it might slip), show that the greatest volume of sand that can be stored in this manner is $\pi \mu_s R^3/3$. (The volume of a cone is Ah/3, where A is the base area and h is the cone's height).



- To pile the most sand without extending the radius, sand is added to make the height "h" as great as possible.
- Eventually, the sides become so steep that sand at the surface begins to slip.
- Goal: find the greatest height (greatest slope) for which the sand does not slide.

Cross section of sand's cone



Static friction → grain does not move

$$N = F_{gy} = mg\cos\theta$$
$$f = F_{gx} = mg\sin\theta$$

If grain does not slide

$$F_{gx} = mg\sin\theta \le f_{s,\max} = \mu_s N = \mu_s mg\cos\theta \rightarrow \mu_s \ge \tan\theta$$

The surface of the cone has the greatest slope and the height of the cone is maximum if :

$$\mu_s = \tan\theta = \frac{h}{R} \rightarrow h = R\mu_s$$

$$V_{cone} = \frac{A\cdot h}{3} = \frac{\pi R^2 (R\mu_s)}{3} = \frac{\pi \mu_s R^3}{3}$$

21. Block B weighs 711N. The coefficient of static friction between the block and the table is 0.25; assume that the cord between B and the knot is horizontal. Find the maximum weight of block A for which the system will be stationary.

$$System\ \ stationary \ \rightarrow f_{s,\max} = \mu_s N$$

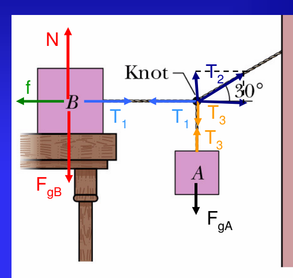
$$Block\ \ B \rightarrow N = m_B g$$

$$T_1 - f_{s,\max} = 0 \rightarrow T_1 = 0.25 \cdot 711\,N = 177.75\,N$$

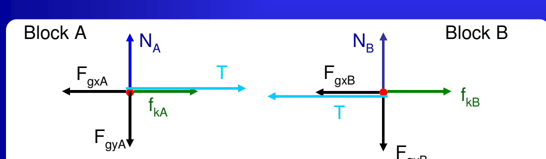
$$Knot \ \rightarrow T_1 = T_{2x} = T_2\cos 30^\circ \rightarrow T_2 = \frac{177.75\,N}{\cos 30^\circ} = 205.25\,N$$

$$T_{2y} = T_2 \sin 30^\circ = T_3$$

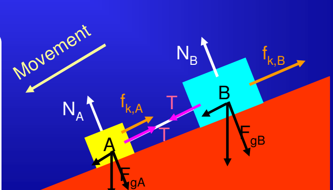
$$Block\ \ A \rightarrow T_3 = m_A g = T_2 \sin 30^\circ = 0.5 \cdot 205.25\,N = 102.62\,N$$



23P. Two blocks of weights 3.6N and 7.2N, are connected by a massless string and slide down a 30º inclined plane. The coefficient of kinetic friction between the lighter block and the plane is 0.10; that between the heavier block and the plane is 0.20. Assuming that the lighter block leads, find (a) the magnitude of the acceleration of the blocks and (b) the tension in the string. (c) Describe the motion if, instead, the heavier block leads.



Light block A leads



Light block A leads

$$Block \quad A \rightarrow N_A = F_{gyA} = m_A g \cos 30^\circ = 3.12\,N$$
$$f_{kA} = \mu_{kA} N_A = (0.1)(3.12\,N) = 0.312\,N$$
$$F_{gxA} - f_{kA} - T = m_A a \rightarrow (3.6N)\sin 30^\circ - 0.312\,N - T = 0.37\,a \rightarrow \boxed{1.49 - T = 0.37\,a}$$

$$a = 3.49\,m/s^2$$
$$T = 0.2\,N$$

$$Block \quad B \rightarrow N_B = F_{gyB} = m_B g \cos 30^\circ = 6.23\,N$$
$$f_{kB} = \mu_{kB} N_B = (0.2)(6.23\,N) = 1.25\,N$$
$$F_{gxB} + T - f_{kB} = m_B a \rightarrow (7.2N)\sin 30^\circ + T - 1.25\,N = 0.73\,a \rightarrow \boxed{2.35 + T = 0.73\,a}$$

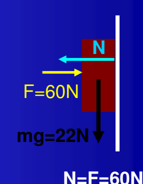
$$T = \left(\frac{W_A W_B}{W_A + W_B}\right)(\mu_{kB} - \mu_{kA})\cos\theta = 0.2\,N$$

Heavy block B leads

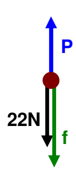
Reversing the blocks is equivalent to switching the labels. This would give $T \sim (\mu_{kA} - \mu_{kB}) < 0$ impossible!!!

The above set of equations is not valid in this circumstance → $a_A \neq a_B$ →The blocks move independently from each other.

74. A block weighing 22N is held against a vertical wall by a horizontal force F of magnitude 60N. The coefficient of static friction between the wall and the block is 0.55 and the coefficient of kinetic friction between them is 0.38. A second P acting parallel to the wall is applied to the block. For the following magnitudes and directions of P, determine whether the block moves, the direction of motion, and the magnitude and direction of the frictional force acting on the block: (a) 34N up (b) 12N up, (c) 48N up, (d) 62N up, (e) 10N down, (f) 18N down.



(a) P=34N, up

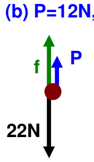


Without P, the block is at rest →

$$f_{s,\max} = \mu_s N = 0.55(60\,N) = 33\,N$$
$$f_k = \mu_k N = 0.38(60\,N) = 22.8\,N$$

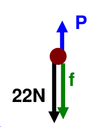
$$P - mg - f = ma$$
$$If \ we \ assume \ f = f_s \rightarrow a = 0$$
$$34\,N - 22\,N = f \rightarrow f = 12\,N \ down$$
$$f < f_{s,\max} = 33\,N \rightarrow Block \ does \ not \ move$$

(b) P=12N, up



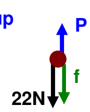
$$P + f - mg = ma = 0$$
$$f = 22\,N - 12\,N = 10\,N \ up$$
$$f < f_{s,\max} = 33\,N \rightarrow Not \ moving$$

(c) P=48N, up



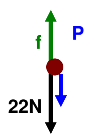
$$P - f - mg = ma = 0$$
$$f = 48\,N - 22\,N = 26\,N \ down$$
$$f < f_{s,\max} = 33\,N \rightarrow Not \ moving$$

(d) P=62N, up



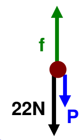
$$P - f - mg = 0 \ (*) \rightarrow f = 62\,N - 22\,N = 40\,N \ up$$
$$f > f_{s,\max} = 33\,N \rightarrow Block \ moves \ up \rightarrow Assumption \ (*) \ wrong$$
$$\rightarrow P - f - mg = ma \ with \ f = f_k = 22.8\,N \ down$$

(e) P=10N, down



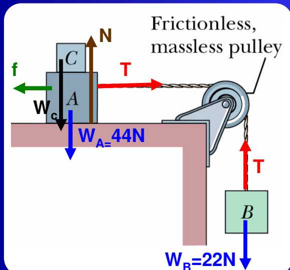
$$f - P - mg = ma = 0$$
$$f = 22\,N + 12\,N = 32\,N \;\; up$$
$$f < f_{s,\max} = 33\,N \rightarrow \; Not \;\; moving$$

(f) P=18N, down



$$f - P - mg = ma = 0$$
$$f = 18\,N + 22\,N = 40\,N \;\; up$$
$$f > f_{s,\max} = 33\,N \rightarrow moves$$
$$f = f_k = 22.8\,N \;\; up$$

28. Blocks A and B have weights of 44N and 22N, respectively. (a) Determine the minimum weight of block C to keep A from sliding if μ_s between A and the table is 0.2. (b) Block C suddenly is lifted of A. What is the acceleration of block A if μ_k between A and the table is 0.15?



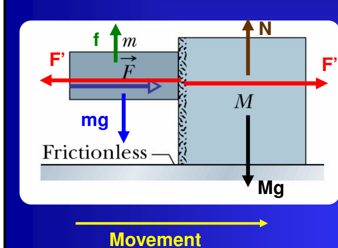
(a)

$$f = f_{s,\max} = \mu_s N$$
$$Block \;\; A \rightarrow a = 0 \rightarrow T - f_{s,\max} = 0 \rightarrow T = \mu_s N \qquad (1)$$
$$Block \;\; B \rightarrow -T + m_B g = 0 \rightarrow T = 22\,N \qquad (2)$$
$$(1)+(2) \;\; N = \frac{T}{\mu_s} = \frac{22\,N}{0.2} = 110\,N$$
$$Blocks \;\; A, B \rightarrow N = W_A + W_C \rightarrow W_C = 110\,N - 44\,N = 66\,N$$

(b) $C \;\; disappears \;\; \rightarrow N = m_A g = 44\,N$

$$T - \mu_k N = m_A a$$
$$m_B g - T = m_B a \qquad T - 6.6 = 4.5a \qquad \rightarrow a = 2.3 m/s^2$$
$$22 - T = 2.2a \qquad \rightarrow T \approx 17\,N$$

29. The two blocks (with m=16kg and m=88kg) shown in the figure below are not attached. The coefficient of static friction between the blocks is: μ_s=0.38 but the surface beneath the larger block is frictionless What is the minimum value of the horizontal force F required to keep the smaller block from slipping down the larger block?

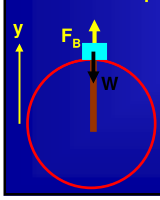


F_{min} required to keep m from sliding down?

Treat both blocks as a single system sliding across a frictionless floor

$$F = m_{total}\, a \rightarrow a = \frac{F}{m+M}$$
$$Small \;\; block \;\; \rightarrow F - F' = ma = m\left(\frac{F}{m+M}\right) \qquad (1)$$
$$f_s - mg = 0 \rightarrow \mu_s F' - mg = 0 \qquad (2)$$
$$(1)+(2) \qquad \mu_s M\left(\frac{F}{m+M}\right) = mg \rightarrow F = \frac{mg}{\mu_s}\left(\frac{m+M}{M}\right) = 488\,N$$

44. An amusement park ride consists of a car moving in a vertical circle on the end of a rigid boom of negligible mass. The combined weigh of the car and riders is 5kN, and the radius of the circle is 10m. What are the magnitude and the direction of the force of the boom on the car at the top of the circle if the car's speed is (a) 5m/s (b) 12m/s?



The force of the boom on the car is capable of pointing any direction

$$F_B - W = m\left(-\frac{v^2}{R}\right) \rightarrow F_B = W\left(1 - \frac{v^2}{Rg}\right)$$

$$(a) \;\; v = 5m/s \rightarrow F_B = 3.7\,N \;\; up \qquad (b) \;\; v = 12\,m/s \rightarrow F_B = -2.3 \;\; down$$