Chapter 12 – Equilibrium and Elasticity

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   - Static equilibrium

II. Center of gravity

III. Elasticity
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I. Equilibrium

   - **Definition:** An object is in equilibrium if:
     - The linear momentum of its center of mass is constant.
     - Its angular momentum about its center of mass is constant.

   **Example:** block resting on a table, hockey puck sliding across a frictionless surface with constant velocity, the rotating blades of a ceiling fan, the wheel of a bike traveling across a straight path at constant speed.

   - **Static equilibrium:** \( \bar{P} = 0, \quad \bar{L} = 0 \)
     Objects that are not moving either in TRANSLATION or ROTATION

   **Example:** block resting on a table.
**Stable static equilibrium:**

If a body returns to a state of static equilibrium after having been displaced from it by a force → marble at the bottom of a spherical bowl.

**Unstable static equilibrium:**

A small force can displace the body and end the equilibrium.

1. Torque about supporting edge by \( F_g \) is 0 because line of action of \( F_g \) passes through rotation axis → domino in equilibrium.

2. Slight force ends equilibrium → line of action of \( F_g \) moves to one side of supporting edge → torque due to \( F_g \) increases domino rotation.

3. Not as unstable as (1) → in order to topple it, one needs to rotate it beyond balance position in (1).

**Requirements of equilibrium:**

- Balance of forces → translational equilibrium

\[
\vec{P} = cte \rightarrow \vec{F}_{net} = \frac{d\vec{P}}{dt} = 0
\]

- Balance of torques → rotational equilibrium

\[
\vec{L} = cte, \quad \tau_{net} = \frac{d\vec{L}}{dt} = 0
\]
II. Center of gravity

Gravitational force on extended body $\rightarrow$ vector sum of the gravitational forces acting on the individual body's elements (atoms).

**cog** = Body’s point where the gravitational force “effectively” acts.

This course initial assumption: The center of gravity is at the center of mass.

*If g is the same for all elements of a body, then the body’s Center Of Gravity (COG) is coincident with the body’s Center Of Mass (COM).*

Assumption valid for every day objects $\rightarrow$ “g” varies only slightly along Earth’s surface and decreases in magnitude slightly with altitude.

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Proof:

Each force $F_{gi}$ produces a torque $\tau_i$ on the element of mass about the origin O, with moment arm $x_i$.

$$\tau = r_i F \rightarrow \tau_i = x_i F_{gi} \rightarrow \tau_{net} = \sum_i \tau_i = \sum_i x_i F_{gi}$$

$$\tau = x_{cog} F_g = x_{cog} \sum_i F_{gi} = \tau_{net}$$

$$x_{cog} \sum_i F_{gi} = \sum_i x_i F_{gi} \rightarrow x_{cog} \sum_i m_i g_i = \sum_i x_i m_i g_i \rightarrow x_{cog} \sum_i m_i = \sum_i x_i m_i$$

$$\rightarrow x_{cog} = \frac{1}{M} \sum_i x_i m_i = x_{com}$$
III. Elasticity

Branch of physics that describes how real bodies deform when forces are applied to them.

**Real rigid bodies are elastic** → we can slightly change their dimensions by pulling, pushing, twisting or compressing them.

**Stress**: Deforming force per unit area.  
**Strain**: Unit deformation

- **Tensile stress**: associated with stretching
- **Shearing stress**:  
- **Hydraulic stress**:  

**Elastic modulus**: describes the elastic behavior (deformations) of objects as they respond to forces that act on them.

\[
\text{Stress} = \text{Elasticity Modulus} \times \text{Strain}
\]

(1) Stress = cte x Strain → Recovers original dimensions when stress removed.

(2) Stress > yield strength \( S_y \) → specimen becomes permanently deformed.

(3) Stress > ultimate strength \( S_u \) → specimen breaks.

**Tension and compression:**

\[
\text{Stress} = \frac{F}{A}  
\]

(\( F = \text{force applied perpendicular to the area } A \text{ of the object} \))
Strain = $\frac{\Delta L}{L}$  (fractional change in length of the specimen)

Stress = (Young’s modulus) x Strain  \[ \frac{F}{A} = E \frac{\Delta L}{L} \]

Units of Young modulus: F/m$^2$

Shearing:

\[ \text{Stress} = \frac{F}{A} \]  (F = force in the plane of the area A)

\[ \text{Strain} = \frac{\Delta x}{L} \]  (fractional change in length of the specimen)

\[ \text{Stress} = (\text{Shear modulus}) \times \text{Strain} \]

Hydraulic stress:

\[ \text{Stress} = \text{Fluid pressure} = p = \frac{F}{A} \]

\[ p = B \frac{\Delta V}{V} \]

Hydraulic Stress = (Bulk modulus) x Hydraulic compression

\[ \text{Strain} = \frac{\Delta V}{V} \]