Chapter 12 – Equilibrium and Elasticity

I. Equilibrium
- Definition
- Requirements
- Static equilibrium

II. Center of gravity

III. Elasticity
- Tension and compression
- Shearing
- Hydraulic stress

I. Equilibrium

- Definition: An object is in equilibrium if:

- The linear momentum of its center of mass is constant.
- Its angular momentum about its center of mass is constant.

Example: block resting on a table, hockey puck sliding across a frictionless surface with constant velocity, the rotating blades of a ceiling fan, the wheel of a bike traveling across a straight path at constant speed.

- Static equilibrium:

$$\vec{P} = 0, \qquad \vec{L} = 0$$

Objects that are not moving either in TRANSLATION or ROTATION

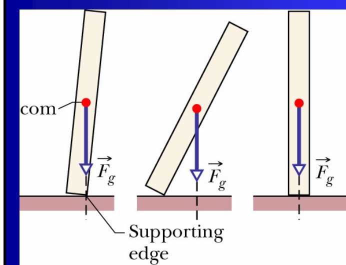
Example: block resting on a table.

Stable static equilibrium:

If a body returns to a state of static equilibrium after having been displaced from it by a force → marble at the bottom of a spherical bowl.

Unstable static equilibrium:

A small force can displace the body and end the equilibrium.



(1) Torque about supporting edge by F_g is 0 because line of action of F_g passes through rotation axis → domino in equilibrium.

(2) Slight force ends equilibrium → line of action of F_g moves to one side of supporting edge → torque due to F_g increases domino rotation.

(3) Not as unstable as (1) → in order to topple it, one needs to rotate it beyond balance position in (1).

- Requirements of equilibrium:

$$\vec{P} = cte \rightarrow \vec{F}_{net} = \frac{d\vec{P}}{dt} = 0$$

Balance of forces → translational equilibrium

$$\vec{L} = cte, \qquad \vec{\tau}_{net} = \frac{d\vec{L}}{dt} = 0$$

Balance of torques → rotational equilibrium

- Vector sum of all external forces that act on body must be zero.
- Vector sum of all external torques that act on the body, measured about any possible point must be zero.

Balance of forces → $F_{net,x} = F_{net,y} = F_{net,z} = 0$

Balance of torques → $\tau_{net,x} = \tau_{net,y} = \tau_{net,z} = 0$

II. Center of gravity

Gravitational force on extended body → vector sum of the gravitational forces acting on the individual body's elements (atoms) .

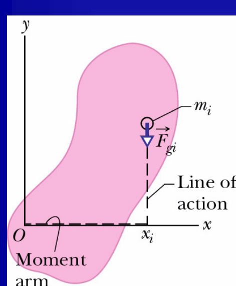
cog = Body's point where the gravitational force "effectively" acts.

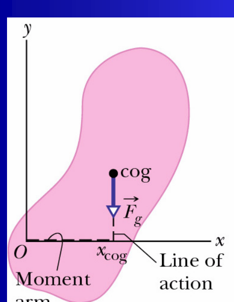
- **This course initial assumption:** The center of gravity is at the center of mass.

If $\vec{g}$ is the same for all elements of a body, then the body's Center Of Gravity (COG) is coincident with the body's Center Of Mass (COM).

Assumption valid for every day objects → "g" varies only slightly along Earth's surface and decreases in magnitude slightly with altitude.

Proof:





Each force F_{gi} produces a torque τ_i on the element of mass about the origin O, with moment arm x_i.

$$\tau = r_{\perp} F \rightarrow \tau_i = x_i F_{gi} \rightarrow \tau_{net} = \sum_i \tau_i = \sum_i x_i F_{gi}$$

$$\tau = x_{cog} F_g = x_{cog} \sum_i F_{gi} = \tau_{net}$$

$$x_{cog} \sum_i F_{gi} = \sum_i x_i F_{gi} \rightarrow x_{cog} \sum_i m_i g_i = \sum_i x_i m_i g_i \rightarrow x_{cog} \sum_i m_i = \sum_i x_i m_i$$

$$\rightarrow x_{cog} = \frac{1}{M} \sum_i x_i m_i = x_{com}$$

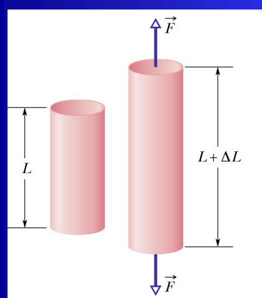
III. Elasticity

Branch of physics that describes how real bodies deform when forces are applied to them.

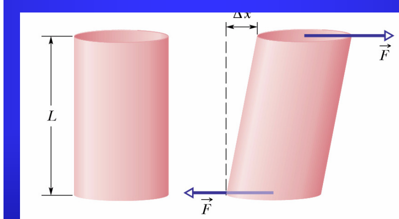
Real rigid bodies are elastic → we can slightly change their dimensions by pulling, pushing, twisting or compressing them.

Stress: Deforming force per unit area.

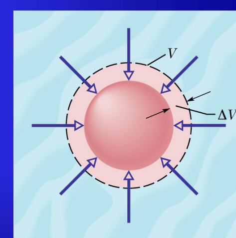
Strain: Unit deformation



Tensile stress: associated with stretching



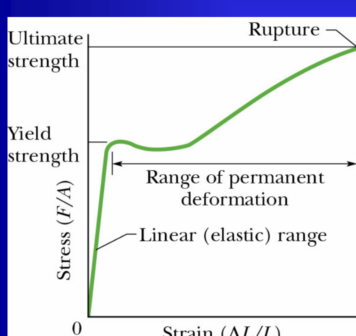
Shearing stress



Hydraulic stress

Elastic modulus: describes the elastic behavior (deformations) of objects as they respond to forces that act on them.

Stress = Elasticity Modulus x Strain



(1) Stress = cte x Strain → Recovers original dimensions when stress removed.

(2) Stress > yield strength S_y → specimen becomes permanently deformed.

(3) Stress > ultimate strength S_u → specimen breaks.

Tension and compression:

$$Stress = \frac{F}{A}$$

(F= force applied perpendicular to the area A of the object)

$Strain = \frac{\Delta L}{L}$ (fractional change in length of the specimen)

Stress = (Young's modulus) x Strain $\frac{F}{A} = E\frac{\Delta L}{L}$

Units of Young modulus: F/m^2

Shearing: $Stress = \frac{F}{A}$ (F= force in the plane of the area A) $\frac{F}{A} = G\frac{\Delta x}{L}$

$Strain = \frac{\Delta x}{L}$ (fractional change in length of the specimen)

Stress = (Shear modulus) x Strain

Hydraulic stress: $Stress = Fluid\ \ pressure = p = \frac{F}{A}$ $p = B\frac{\Delta V}{V}$

Hydraulic Stress = (Bulk modulus) x Hydraulic compression

$Strain = \frac{\Delta V}{V}$