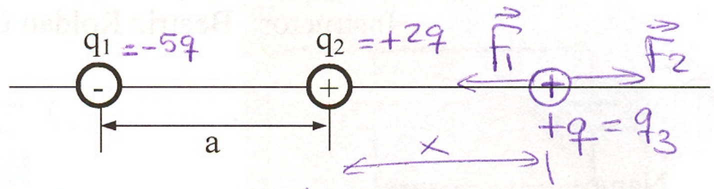
1. The figure below displays the location of two point charges ($q_1 = -5q$ and $q_2 = 2q$). How far from q_2 and in what direction is there a point in which a third charged particle ($q_3 = +q$) will be in equilibrium? Express your answer in terms of the distance a between charges q_1 and q_2.

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 \cdot q_2|}{r^2}$$



$$\vec{F}_{net\, q_3} = \vec{F_1} + \vec{F_2}$$

magnitude

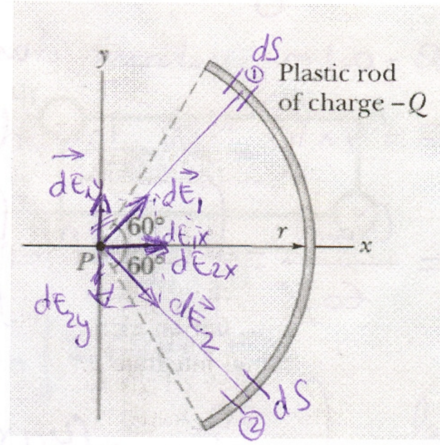
$$0 = F_2 - F_1 = \frac{1}{4\pi\epsilon_0} q\left(\frac{5q}{(x+a)^2} - \frac{2q}{x^2}\right)$$

$$\frac{5}{(x+a)^2} = \frac{2}{x^2} \rightarrow 3x^2 - 2a^2 - 4xa = 0 \Rightarrow \boxed{x \simeq 1.72a}$$

$\boxed{x = 1.72\, a}$

2. The figure below shows a plastic rod with a uniformly distributed charge **–Q**. The rod has been bent in a 120° circular arc of radius **r**. Assume a coordinate system as it is shown in the figure, where the axis of symmetry of the rod lies along the x axis and the origin is at the center of curvature P of the rod. In terms of Q and r, what is the electric field $\vec{E}$ (magnitude and direction) due to the rod at point P? Hint: use the linear charge density (λ) in your calculations. Also remember: (arc length = angle · radius).

$$\lambda = \frac{Q}{\underset{\text{"arc length}}{S}} = \frac{Q}{(\text{Angle})\cdot(\text{Radius})}$$



$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}$$

$$dq = \lambda \cdot ds$$

$$dE = \frac{1}{4\pi\epsilon_0}\cdot\frac{\lambda ds}{r^2}$$

$$S = \theta \cdot r$$

$$S = (120°)\cdot\left(\frac{\pi}{180°}\right)\cdot r$$

$$ds = r\,d\theta$$

$$dE_x = dE\cos\theta = \frac{1}{4\pi\epsilon_0}\cdot\frac{\lambda\cdot(\cancel{r}\cdot d\theta)}{r^{\cancel{2}}}\cos\theta$$

$$E = \int dE_x = \int_{-60°}^{60°}\frac{1}{4\pi\epsilon_0}\cdot\frac{\lambda}{r}\cos\theta\,d\theta = \frac{1}{4\pi\epsilon_0}\frac{\lambda}{r}[\sin\theta]_{-60°}^{60°}$$

$$E = \frac{\lambda}{4\pi\epsilon_0\cdot r}(1.73) = \frac{(Q/(2.1r))(1.73)}{(4\pi\epsilon_0)r} = \boxed{\frac{0.82\,Q}{4\pi\epsilon_0 r^2}\,\hat{i}}$$

$$\lambda = \frac{Q}{2.1r}$$

$$S = (2.1\,\text{rad})\cdot r$$

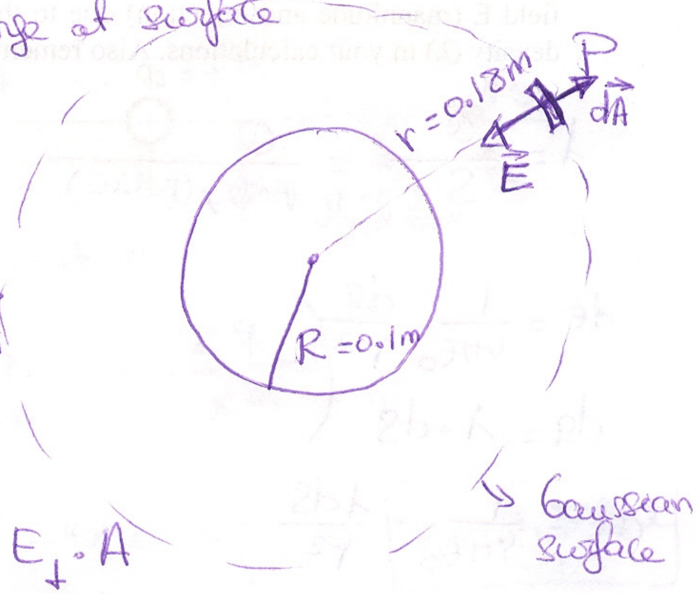
E = $\frac{0.82\,Q}{(4\pi\epsilon_0)r^2}\,\hat{i}$	Direction: $+X$- Axis

3. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 18 cm from the center of the sphere has a magnitude of 4000 N/C and is directed radially inward, what is the net charge on the sphere?

Conducting sphere → all charge at surface

$R = 0.1\,m$, unknown charge

$E = 4\times10^{3}\,N/C$ inward



$$\Phi_E = \frac{Q_{encl}}{\epsilon_0} = \oint E_\perp \cdot dA = \oint \vec{E}\cdot\vec{dA}$$

$$\frac{Q_{encl}}{\epsilon_0} = \int \underbrace{\frac{dq}{4\pi\epsilon_0 r^2}}_{\text{constant}} \cdot (4\pi r^2) = E_\perp \cdot A$$

$$E_\perp = \frac{Q_{encl}}{\epsilon_0 \cdot A} = \frac{Q_{encl}}{\epsilon_0 (4\pi r^2)} = 4\times10^{3}\,N/C$$

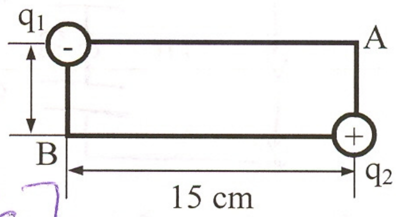
$$Q_{encl} = \epsilon_0 \cdot A \cdot E_\perp = \left(8.85\times10^{-12}\,\frac{C^2}{N m^2}\right)\cdot(4\pi)\cdot(0.18\,m)^2\cdot(4\times10^{3}\,N/C)$$

$Q_{encl} = 14.4\,nC$ (and $\ominus$, since the $\vec{E}$ is inward)

Q = $-14.4\,nC$

4. In the rectangle of the figure below, the sides have lengths of 5 cm and 15 cm, $q_1 = -5\mu C$ and $q_2 = +2\mu C$. Assume V = 0 at infinity. Obtain:

(a) Electric potential at A
(b) Electric potential at B
(c) Work required to move a third charge $q_3 = +3\mu C$ from B to A through a diagonal of the rectangle.



(a) $V_A = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2}\right]$

$V_A = \left(9\times10^{9}\frac{Nm^2}{C^2}\right)\left[\frac{-5\times10^{-6}C}{0.15\,m} + \frac{2\times10^{-6}C}{0.05m}\right] = 6\times10^{4}V$

(b) $V_B = \left(9\times10^{9}\frac{Nm^2}{C^2}\right)\cdot\left[\frac{-5\times10^{-6}C}{0.05m} + \frac{2\times10^{-6}C}{0.15m}\right] = -78\times10^{4}V$

(c) $W_{E\,B\to A} = -\Delta U = U_B - U_A = q_3\cdot[V_B - V_A] = (3\times10^{-6}C)[-78\times10^{4}V - 6\times10^{4}V]$

$W_{E\,B\to A} = -2.52\,J \Rightarrow W_{E\,B\to A} = -W_{\text{applied force}} \Rightarrow \boxed{+2.52\,J} = W_{\text{applied force}}$

work done by electric field

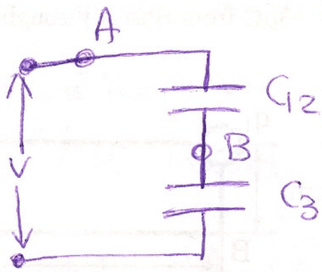
(a) $6\times10^{4}V$	(b) $-78\times10^{4}V$	(c) $W_{appl.} = +2.52J$

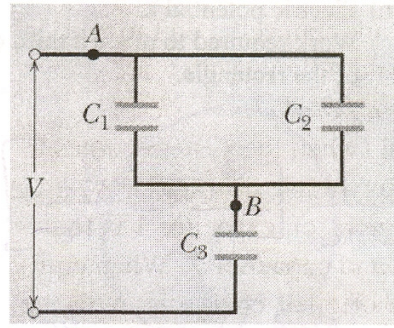
5. In the circuit shown below, $C_1 = 10\ \mu F$, $C_2 = 4.3\ \mu F$ and $C_3 = 2.5\ \mu F$.

(a) Find the equivalent capacitance for the combination of three capacitors shown.

(b) If the potential difference applied to the input terminals is $V = 11$ V, what is the charge on C_1?

(a)





Parallel $\Rightarrow$ $C_{12} = C_1 + C_2 = (10 + 4.3)\mu F = 14.3 \mu F$

Series $\Rightarrow$ $(C_{123\,eq})^{-1} = \dfrac{1}{C_{12}} + \dfrac{1}{C_3}$ $\Rightarrow$ $\dfrac{1}{C_{eq\,123}} = \dfrac{1}{(14.3\mu F)} + \dfrac{1}{(2.5\mu F)}$

$\Rightarrow \boxed{C_{eq_{123}} \simeq 2.13 \mu F}$

(b) $V = 11\,V = \dfrac{Q}{C_{123}}$ $\Rightarrow$ $Q_{TOTAL} = C_{123} \cdot V = (2.13\mu F)\cdot(11V) = 23.43 \mu C$

Series capacitors have the same charge $\Rightarrow$ $Q_{TOTAL} = Q_{12} = Q_3$

$\Rightarrow Q_{12} = 23.4 \mu C$

$V_{12} = \dfrac{Q_{12}}{C_{12}} = \dfrac{23.4\mu C}{14.3 \mu F} = 1.64 V = \underbrace{V_1 = V_2}_{\text{Parallel}}$

$Q_1 = C_1 \cdot V_1 = (10\mu F)\cdot(1.64V) = 16.4 \mu C$

(a) C_{eq} = $2.13\ \mu F$	(b) q_1 = $16.4\ \mu C$