1. The figure below displays the location of two point charges \((q_1 = -5q\) and \(q_2 = 2q\)). How far from \(q_2\) and in what direction is there a point in which a third charged particle \((q_3 = +q)\) will be in equilibrium? Express your answer in terms of the distance \(a\) between charges \(q_1\) and \(q_2\).

\[
F = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{r^2}
\]

\[
F_{\text{net}_{q_3}} = F_1 + F_2
\]

\[
0 = F_2 - F_1 = \frac{1}{4\pi\varepsilon_0} q \left( \frac{5q}{(x+a)^2} - \frac{2q}{x^2} \right)
\]

\[
\frac{5}{(x+a)^2} = \frac{2}{x^2} \Rightarrow 3x^2 - 2a^2 - 4xa = 0 \Rightarrow x \approx 1.72a
\]

\[
x = 1.72a
\]
2. The figure below shows a plastic rod with a uniformly distributed charge $-Q$. The rod has been bent in a $120^\circ$ circular arc of radius $r$. Assume a coordinate system as it is shown in the figure, where the axis of symmetry of the rod lies along the $x$ axis and the origin is at the center of curvature $P$ of the rod. In terms of $Q$ and $r$, what is the electric field $E$ (magnitude and direction) due to the rod at point $P$? Hint: use the linear charge density ($\lambda$) in your calculations. Also remember: (arc length = angle $\cdot$ radius).

$$dE = \frac{1}{\varepsilon_0} \frac{dq}{r^2}$$

$$dq = \lambda \cdot ds$$

$$dE = \frac{1}{\varepsilon_0} \frac{\lambda \cdot ds}{r^2}$$

$$dE_x = dE \cos \theta = \frac{1}{\varepsilon_0} \sin \theta \frac{\lambda}{r^2}$$

$$E = \int_{-60^\circ}^{60^\circ} \frac{1}{\varepsilon_0} \sin \theta d\theta = \frac{\lambda}{r^2} \frac{\sin 60^\circ - \sin (-60^\circ)}{2}$$

$$E = \frac{\lambda}{\varepsilon_0 r^2} (1.73) = \left( \frac{Q}{(2 \pi r)} \right) \frac{(1.73)}{(\varepsilon_0 r^2)} = \frac{0.82 Q}{\varepsilon_0 r^2}$$

$$d = \frac{Q}{2 \pi r}$$

$$s = (2 \pi r) \cdot r$$

$$E = \frac{0.82 Q}{(\varepsilon_0 r^2)} \hat{r}$$

Direction: $+x$ axis
3. A conducting sphere of radius 10 cm has an unknown charge. If the electric field 18 cm from the center of the sphere has a magnitude of 4000 N/C and is directed radially inward, what is the net charge on the sphere?

\[ E = 4 \times 10^3 \text{ N/C inward} \]

\[ E = \frac{Q_{\text{end}}}{\varepsilon_0} = \oint E_\perp \cdot dA = \oint E \cdot dA \]

\[ Q_{\text{end}} = \oint \frac{d\Phi}{q + \varepsilon_0 r^2} \cdot (4\pi r^2) = E_\perp \cdot A \]

\[ E_\perp = \frac{Q_{\text{end}}}{\varepsilon_0 \cdot A} = \frac{Q_{\text{end}}}{\varepsilon_0 (4\pi r^2)} = 4 \times 10^3 \text{ N/C} \]

\[ Q_{\text{end}} = \varepsilon_0 \cdot A \cdot E_\perp = \left(8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}\right) \cdot (4\pi) \cdot (0.18 \text{ m})^2 \cdot (4 \times 10^3 \text{ N/C}) \]

\[ Q_{\text{end}} = 14.4 \mu C \quad \text{(and } \Phi \text{, since the } E \text{ is inward)} \]

\[ Q = -14.4 \mu C \]
4. In the rectangle of the figure below, the sides have lengths of 5 cm and 15 cm, $q_1 = -5 \mu C$ and $q_2 = +2 \mu C$. Assume $V = 0$ at infinity. Obtain:

(a) Electric potential at A
(b) Electric potential at B
(c) Work required to move a third charge $q_3 = +3 \mu C$ from B to A through a diagonal of the rectangle.

\[
V_A = \frac{1}{4\pi \varepsilon_0} \left( \frac{q_1}{r_1} + \frac{q_2}{r_2} \right)
\]

\[
V_A = (9 \times 10^9 \frac{Nm^2}{C^2}) \left[ \frac{-5 \times 10^{-6} C}{0.015 m} + \frac{2 \times 10^{-6} C}{0.05 m} \right] = 6 \times 10^4 V
\]

(b) $V_B = (9 \times 10^9 \frac{Nm^2}{C^2}) \left[ \frac{-5 \times 10^{-6} C}{0.015 m} + \frac{2 \times 10^{-6} C}{0.015 m} \right] = -78 \times 10^4 V$

(c) $\Delta W_E = W_{B\to A} = q_3 \cdot \left[ V_B - V_A \right] = (3 \times 10^{-6} C) \left[ -78 \times 10^4 V - 6 \times 10^4 V \right] = -2.52 J$

Work done by electric field

\[
W_{E\to A} = -2.52 J \Rightarrow W_{E\to A} = -W_{\text{applied force}} = +2.52 J
\]
5. In the circuit shown below, \(C_1 = 10 \ \mu\text{F},\ C_2 = 4.3 \ \mu\text{F}\) and \(C_3 = 2.5 \ \mu\text{F}\).

(a) Find the equivalent capacitance for the combination of three capacitors shown.

(b) If the potential difference applied to the input terminals is \(V = 11\ \text{V}\), what is the charge on \(C_1\)?

\[
(a) \ C_{eq} = 2.13 \ \mu\text{F} \hspace{2cm} (b) \ q_1 = 16.4 \ \mu\text{C}
\]