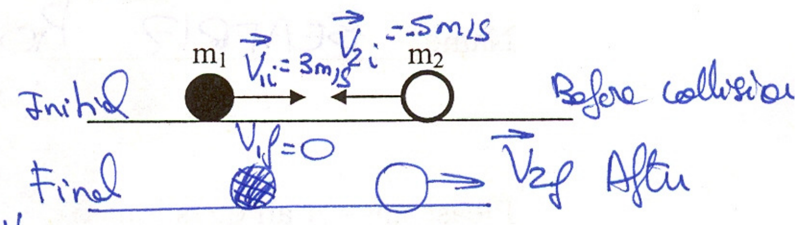
1. Two balls approach each other head-on with initial speeds of magnitude $v_{1i} = 3$ m/s and $v_{2i} = 5$ m/s and collide *elastically.* After the collision, ball one, whose mass is $m_1 = 0.3$ kg, remains at rest, while ball 2 bounces back. Assume one dimensional motion.

   (a) What is the mass of ball 2 (m_2)? *(12.5 points)*
   (b) What is the velocity of m_2 after the collision? *(12.5 points)*

$$\vec{P}_i = \vec{P}_f$$



$$(1)\quad m_1 V_{1i} - m_2 V_{2i} = \cancel{m_1 V_{1f}} + m_2 V_{2f}$$

$$(2)\quad \frac{1}{2} m_1 V_{1i}^2 + \frac{1}{2} m_2 V_{2i}^2 = 0 + \frac{1}{2} m_2 V_{2f}^2$$

From (1) $\Rightarrow$ $(0.3\,\text{kg})(3\,\text{m/s}) - m_2 (5\,\text{m/s}) = m_2 V_{2f}$

$$\rightarrow \boxed{m_2 = \frac{0.9}{V_{2f} + 5}} \quad (1^*)$$

From (2) $\Rightarrow$ $\cancel{\tfrac{1}{2}} (0.3\,\text{kg})\left(9\,\frac{\text{m}^2}{\text{s}^2}\right) + \cancel{\tfrac{1}{2}} m_2 (5\,\text{m/s})^2 = \cancel{\tfrac{1}{2}} m_2 V_{2f}^2$

$$2.7 + 25 m_2 = m_2 V_{2f}^2 \Rightarrow \boxed{m_2 = \frac{2.7}{V_{2f}^2 - 25}} \quad (2^*)$$

$(1^*) = (2^*)$ $\Rightarrow$

$$\frac{0.9}{V_{2f} + 5} = \frac{2.7}{V_{2f}^2 - 25}$$

$$\Rightarrow \quad 0.9 V_{2f}^2 - 2.7 V_{2f} - 36 = 0 \rightarrow \boxed{V_{2f} = 8\,\text{m/s}}$$

From (1^*) $\Rightarrow$ $m_2 = \dfrac{0.9}{8 + 5} \simeq \boxed{0.07\,\text{kg}}$

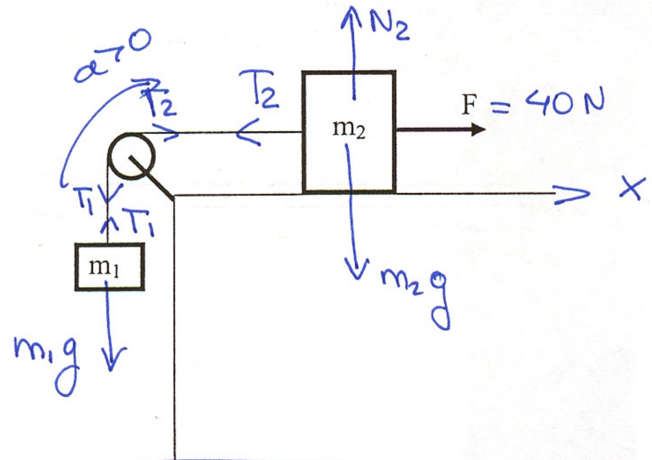
(a) m_2 = 0.07 kg	(b) v_{2f} = 8 m/s

2. Two blocks, m_1= 1 kg and m_2= 4 kg are connected by a massless string through a pulley of mass M. The rotational inertia of the pulley is I = ½ MR^2, with M = 0.3 kg and a radius R= 0.2 m. Block m_2 is pulled by a horizontal force of 40 N. m_2 lies on a frictionless surface.

a) Using symbols, write down the equations of motion of the masses and the pulley. (7 points)

b) Obtain the linear acceleration of the masses. *(9 points)*

c) Calculate the forces of tension in the two sides of the pulley. *(9 points)*



$I = \frac{1}{2} M R^2$

$M = 0.3\,\text{Kg}, \; R = 0.2\,\text{m}$

$m_1 \Rightarrow \boxed{T_1 - m_1 g = m_1 a} \quad (1)$

$m_2 \Rightarrow \boxed{F - T_2 = m_2 a} \quad (2)$

$M \Rightarrow \left\{ \begin{array}{l} \tau_{TOTAL} = I \cdot \alpha = \frac{1}{2} M R^2 \cdot \alpha = \frac{1}{2} M R^2 \left(-\frac{a}{R}\right) \\ \tau_{TOTAL} = \tau_{T_1} + \tau_{T_2} = RT_1 - RT_2 \end{array} \right\} \Rightarrow -\frac{1}{2} M \cancel{R}^2 \frac{a}{\cancel{R}} = \cancel{R}(T_1 - T_2)$

$\alpha = -\frac{a}{R}$

$\Downarrow$

$(3) \; \boxed{\frac{1}{2} M a = (T_2 - T_1)}$

$(1)+(2) \Rightarrow \cancel{T_1} - \cancel{T_2} - m_1 g + F = (m_1 + m_2) a$

$(3) \Rightarrow \cancel{T_2} - \cancel{T_1} = \frac{1}{2} M a$

$+$

$F - m_1 g = (m_1 + m_2 + \frac{1}{2} M) \cdot a \Rightarrow a = \frac{F - m_1 g}{m_1 + m_2 + \frac{1}{2} M} = \boxed{5.86 \frac{m}{s^2}}$

From (1) $\Rightarrow T_1 = m_1 (a + g) = \boxed{15.66\,N}$; From (2) $T_2 = F - m_2 a = \boxed{16.56\,N}$

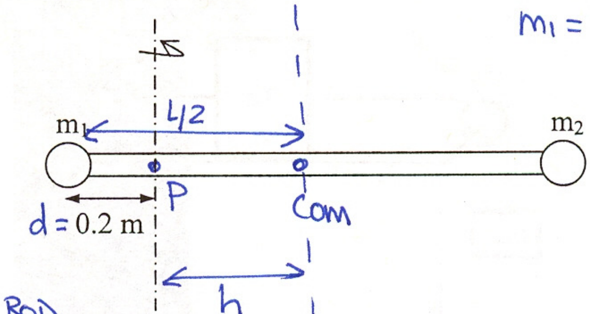
(b) 5.86 m/s^2	(c) T_1 = 15.66 N;	T_2 = 16.56 N

3. The system of objects displayed below is rotating in the horizontal plane with respect to a perpendicular axis located 0.2 m away from mass 1. The system is made of two small particles of equal mass $m_1 = m_2 = 0.2$ kg and of one rod of length L = 3 m and mass M = 0.5 kg. The system is rotating with an angular speed of 8 rad/s. [I_{rod} (COM) = $ML^2/12$]

(a) Calculate the total moment of inertia of the system with respect to the rotational axis shown in the figure below *(12.5 points)*

(b) Calculate the rotational kinetic energy *(12.5 points)*

$m_1 = m_2 = m$



(a) $I_{TOTAL} = I_{m_1} + I_{m_2} + I_{ROD_P}$

$I_{m_1} = m_1 \cdot d^2$

$I_{m_2} = m_2 \cdot (L-d)^2$

$I_{ROD} = I_{COM} + h^2 \cdot M = \frac{1}{12} M L^2 + M\left(\frac{L}{2} - d\right)^2$

$I_{TOTAL} = m_1 d^2 + m_2 (L-d)^2 + \frac{1}{12} M L^2 + M\left(\frac{L}{2} - d\right)^2$

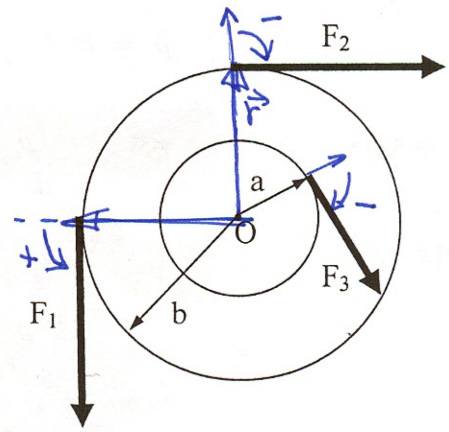
$I_{TOTAL} = \boxed{2.796\ Kg\,m^2}$

(b) $KE_{ROTATION} = \frac{1}{2} I \omega^2 = (0.5)(2.796\ Kg\,m^2) \cdot (8\ rad/s)^2$

$= \boxed{89.47\ \left(\frac{Kg\,m^2}{s}\right) \text{" J}}$

(a) I = 2.796 Kgm^2	(b) 89.47 J

4. Find the net torque on the wheel in the figure below about the axle through O, taking a = 1 m and b = 3 m. The magnitudes of the forces acting on the wheel are: F_1 = 30 N, F_2 = 15 N, F_3 = 18 N *(25 points)*



$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\vec{\tau}_{TOTAL} = \vec{\tau}_{F_1} + \vec{\tau}_{F_2} + \vec{\tau}_{F_3}$$

$$\left.\begin{aligned} \tau_{F_1} &= +F_1 . b \\ \tau_{F_2} &= -F_2 . b \\ \tau_{F_3} &= -F_3 . a \end{aligned}\right\} \quad \tau_{TOTAL} = (F_1 - F_2).b - F_3 . a$$

$$= \boxed{27 \text{ Nm}}$$

Torque =	$\boxed{27 \text{ Nm}}$