Please answer all questions.

#1

#2

#3

#4

Total: 

Show all work and enter answers in boxes, if provided.
1. Two particles of masses $m_1 = 3 \text{ kg}$ and $m_2 = 0.5 \text{ kg}$ slide in opposite directions over a frictionless surface with constant velocities $v_1 = 1.5 \text{ m/s}$ and $v_2 = -5 \text{ m/s}$, respectively. The particles collide and bounce as it is indicated in the figure below. If the magnitude of the velocity of particle 1 after the collision is 0.4 m/s, determine:

(a) Velocity (magnitude and direction) of particle 2 after colliding with particle 1. 
(10 points)

(b) Kinetic energy lost during the collision. (10 points)
2. (a) The angular speed of a point on the rim of a rotating wheel is given by: \( \omega = bt - ct^2 \), where \( t \) is in seconds, \( b = 2 \text{ rad/s}^2 \) and \( c = 5 \text{ rad/s}^3 \). Find an expression for the angular acceleration as a function of time. Assume that \( \omega = 0 \text{ rad/s} \) at \( t = 0 \text{ s} \). (5 points)

(b) The figure below shows a rigid structure consisting of a circular hoop of radius \( R \) and mass “\( m \)” and a square made of four thin bars, each of length \( R \) and mass “\( m \)”. The rigid structure rotates at a constant speed about a perpendicular axis at the location shown. Assuming \( R = 0.2 \text{ m} \) and \( m = 3 \text{ kg} \). Calculate the structure’s rotational inertia about the common axis of rotation. The moment of inertia of the hoop about its COM is \( I_{\text{hoop}} = mR^2 \). The moment of inertia of a thin rod about an axis through its COM perpendicular to its length is \( I_{\text{rod}} = 1/12 m L^2 \). (15 points)
3. Two blocks, $m_1 = 0.2 \text{ kg}$ and $m_2 = 7 \text{ kg}$ are connected by a massless string through a pulley of mass $M$. The rotational inertia of the pulley is $I = \frac{1}{2} M R^2$, with $M = 0.3 \text{ kg}$ and a radius $R = 0.2 \text{ m}$. Block $m_2$ is pulled rightward by applying a force of 100 N.

(a) Write down, using symbols, the equations of motion of the masses and the pulley. (5 points)
(b) Obtain the acceleration of the masses (the same for both masses). (7.5 points)
(c) Calculate the forces of tension in the two sides of the pulley. (7.5 points)

(b) $a = \ldots$

(c) $T_1 = \ldots$

$T_2 = \ldots$
4. A uniform circular platform of mass \( M = 40 \) kg and radius \( R = 3 \) m rotates in a horizontal plane about a vertical axis through its center at 50 rad/s. The moment of inertia of the platform is \( \frac{1}{2} MR^2 \). A 8 kg wad of wet putty drops at negligible speed \((v_0=0)\) onto the platform hitting it at a point 2.5 m from the platform’s center and then sticking to it.

(a) What is the angular velocity of the platform-putty system immediately after the impact? (10 points)

(b) If the platform’s rotational axis was located 0.3 m away from its geometrical center, what will be the angular velocity of the system after the impact? (7.5 points)

\[
\begin{array}{c}
\text{R} = 3 \text{ m} \\
d = 2.5 \text{ m}
\end{array}
\]
\[ \ddot{v} = v_0 + \ddot{a}t \]
\[ \ddot{r} = r_0 + \ddot{v}_0 t + 0.5\ddot{a}t^2 \]
\[ v^2 = v_0^2 + 2a(x - x_0) \]
\[ \ddot{p} = m \ddot{v} \]
\[ \dddot{F} = m \dddot{a} \]
\[ F_s = -kx \]
\[ a_c = \frac{v^2}{r} \]
\[ E_{mec} = K + U \]
\[ W = \int F(x)dx \]
\[ W_{net} = \Delta K \]
\[ W_{net} = -\Delta U \]
\[ F = -\frac{dU(x)}{dx} \]
\[ \Delta E = \Delta K + \Delta U \]
\[ \Delta E = \Delta K + \Delta U + \Delta E_{th} \]
\[ \Delta E_{th} = f_k d \]
\[ K = \frac{1}{2}mv^2 \]
\[ s = \theta \cdot r \]
\[ \omega = \omega_0 + \alpha \cdot t \]
\[ a_i = \alpha \cdot r \]
\[ v = \omega \cdot r \]
\[ F_{ext} = \frac{d\ddot{p}}{dt} \]
\[ \dddot{F}_{ext} = M \dddot{a}_{COM} \]
\[ I = I_{COM} + Mr^2 \]
\[ a_r = \frac{v^2}{r} = r\omega^2 \]
\[ I_{disk} (COM) = \frac{1}{2}mR^2 = I_{cylinder} (COM) \]
\[ I_{ring} (COM) = mR^2 \]
\[ I_{rod} (COM) = \frac{1}{12}ML^2 \]
\[ I_{sphere} (COM) = \frac{2}{5}mR^2 \]
\[ I_m = md^2 \]
\[ Rv_{rel} = Ma \]
\[ v_f - v_i = v_{rel} \ln \frac{M_i}{M_f} \]
\[ L = I \cdot \omega \]
\[ \dddot{L} = \dddot{r} \times \dddot{p} \]
\[ \dddot{r}_{ext} = \frac{dL}{dt} \]
MINTERM 3

1. \( m_1 = 3 \text{ kg} \)
   \( m_2 = 0.5 \text{ kg} \)

2. \( V_{1x} = 1.5 \text{ m/s} \)
   \( V_{1y} = 0.4 \text{ m/s} \)

3. \( V_{2x} = 0.5 \text{ m/s} \)

4. \( V_{2y} \?)

\[ V_{1f} = V_{1f} \cos 35^\circ \]
\[ V_{1fy} = V_{1f} \sin 35^\circ \]

a) \( P \) conservation
\[ P_c = P_f \]

\[ x - \text{Axis:} \quad m_1 V_{1x} - m_2 V_{2x} = m_1 V_{1f} + m_2 V_{2f} \]
\[ (2) \quad (3 \text{ kg}) (1.5 \text{ m/s}) - (0.5 \text{ kg}) (0.5 \text{ m/s}) = (3 \text{ kg}) (V_{1f} \cos 35^\circ) + (0.5 \text{ kg}) V_{2f} \]
\[ \Rightarrow V_{2f} = 2.03 \text{ m/s} \]
\[ = V_{2f} \cos \theta \]

\[ y - \text{Axis:} \quad 0 = m_1 V_{1y} - m_2 V_{2y} \quad \Rightarrow 0 = (3 \text{ kg}) (0.4 \text{ m/s}) (\sin 35^\circ) - (0.5) V_{2fy} \]
\[ (2) \quad V_{2fy} = 1.376 \text{ m/s} \]

\[ V_{2f} = \sqrt{V_{2fx}^2 + V_{2fy}^2} = 2.453 \text{ m/s} \]

\[ \tan \theta = \frac{V_{2fy}}{V_{2fx}} \quad \Rightarrow \theta = 34.1^\circ \]

b) \( \Delta K = K_f - K_i = \frac{1}{2} m_1 V_{1f}^2 + \frac{1}{2} m_2 V_{2f}^2 - \frac{1}{2} m_1 V_{1i}^2 - \frac{1}{2} m_2 V_{2i}^2 \]
\[ \Delta K = \frac{1}{2} (3 \text{ kg}) (0.4 \text{ m/s})^2 + \frac{1}{2} (0.5 \text{ kg}) (2.453 \text{ m/s})^2 - \frac{1}{2} (3 \text{ kg}) (1.5 \text{ m/s})^2 - \frac{1}{2} (0.5 \text{ kg}) (0.5 \text{ m/s})^2 \]
\[ \Delta K = -7.9 \text{ J} \]
2. (a) \( \omega = b t - c t^2 = 2 t - 5 t^2 \)
\[ \alpha = \frac{d\omega}{dt} = b - 2ct = 2 - 10t \]

(b) \( R = 0.2 \text{m} \)
\( m = 3 \text{kg} \)
\( L = R \)

\[ I_{\text{System}} = I_{\text{hoop}} + I_{1p} + I_{2p} + I_{3p} + I_{4p} \]

\[ I_{\text{hoop}} = I_{\text{com}} + m \cdot d_h^2 = mR^2 + m \cdot R^2 = 2mR^2 \]

\[ I_{1p} = I_{1\text{com}} + m \cdot d_1^2 = \frac{1}{12} mL^2 + m \cdot R^2 = \frac{1}{12} mR^2 + mR^2 \]

\[ I_{4p} = \frac{1}{12} mR^2 \]

\[ I_3 = I_3 = I_{\text{com}} + m \cdot d_2^2 = \frac{1}{12} mR^2 + m \cdot \frac{R^2}{2} \]

\[ d_2^2 = (\frac{R}{2})^2 + (\frac{R}{2})^2 - \frac{2}{4} = \frac{R^2}{2} \Rightarrow d_2 = \frac{R}{\sqrt{2}} \]

\[ I_{\text{TOTAL}} = 2mR^2 + \frac{1}{12} mR^2 + mR^2 + \frac{1}{12} mR^2 + \frac{2}{12} mR^2 + mR^2 + mR^2 \]

\[ I_{\text{TOTAL}} = 4mR^2 + \frac{1}{3} mR^2 = \frac{13}{3} mR^2 = 0.52 \text{kg m}^2 \]
\[ T_1 - m_1 g = m_1 a \]  \hspace{1cm} \text{(1)}

\[ F - T_2 = m_2 a \]  \hspace{1cm} \text{(2)}

\[ T_2 - T_1 = \frac{1}{2} M a \]  \hspace{1cm} \text{(3)}

\[ \frac{d T_1}{d t} = F \times F \]  \hspace{1cm} \Rightarrow -RT_2 + RT_1 = T \cdot \alpha = T \cdot \left( -\frac{a}{R} \right) 

\[ R(T_1 - T_2) = -\frac{1}{2} M R^2 \left( \frac{a}{R} \right) \]

\[ -m_1 g + F = (m_1 + m_2 + 0.5 M) a \Rightarrow a = 13.34 \text{ m/s}^2 \]

\[ T_1 = m_1 (a + g) = 4.63 \text{ N} \]

\[ T_2 = F - m_2 a = 6.62 \text{ N} \]
(a) \( \omega_p \)?

\[
\omega_i = I \omega_i = \left( \frac{1}{2} MR^2 \right) \omega_i = \frac{1}{2} \left[ 40 \, \text{kg} \times (3 \, \text{m})^2 \times 50 \, \text{rad/s} \right] = 9000 \, \text{kg} \cdot \text{m}^2 / \text{s}
\]

\[
L_f = L_f \text{ platform} + L_f \text{ pulley} = \frac{1}{2} M \omega_p \left( \frac{1}{2} MR^2 + m \cdot d_1^2 \right)
\]

\[
L_f = \omega_p \left( 0.5 \times 40 \, \text{kg} \times 3^2 \, \text{m}^2 + 8 \, \text{kg} \times 2.5^2 \, \text{m}^2 \right) = \omega_p \left( 230 \right)
\]

\[
\omega_i \omega_f \approx 9000 = 230 \omega_f \Rightarrow \omega_f = \frac{9000}{230} \approx 39.13 \, \text{rad/s}
\]

(b) New relationship with \( d_2 \)

\[
L_i = I \omega_i = \left( \frac{1}{2} MR^2 + M \cdot d_2^2 \right) \omega_i
\]

\[
L_i = \left( 0.5 \times 40 \, \text{kg} \times 9 \, \text{m}^2 + 40 \, \text{kg} \times 0.3^2 \, \text{m}^2 \right) \times 50 \, \text{rad/s}
\]

\[
L_i = 9180 \, \text{kg} \cdot \text{m}^2 / \text{s}
\]

\[
L_f = \frac{1}{2} M \omega_p \left( \frac{1}{2} MR^2 + M \omega_p \left( d_2^2 + d_3^2 \right) \right) \omega_p
\]

\[
L_f = \left( 0.5 \times 40 \, \text{kg} \times 9 \, \text{m}^2 + 40 \, \text{kg} \times 0.3^2 \, \text{m}^2 + 8 \, \text{kg} \times 2.2^2 \, \text{m}^2 \right) \omega_p
\]

\[
L_f = 22232 \, \text{kg} \cdot \text{m} / \text{s}
\]

\[
L_i = L_f \Rightarrow 9180 = 22232 \Rightarrow \omega_p = \frac{9180}{22232} \approx 41.29 \, \text{rad/s}
\]