Please answer all questions.

#1

#2

#3

#4

#5

Total: ________

Show all work and enter answers in boxes, if provided.
1. An object moves along the x-axis according to the equation:
\[ x(t) = (2 t^2 + t - 1) \text{ m}, \text{ where } t \text{ is in seconds.} \]
Determine: \hspace{1cm} (20 points)

(a) the average speed between \( t = 1 \) s and \( t = 4 \) s,
(b) the instantaneous speed at \( t = 3 \) s
(c) the average acceleration between \( t = 1 \) s and \( t = 4 \) s,
(d) the instantaneous acceleration at \( t = 3 \) s

\[
\begin{array}{c|c|c|c}
(a) v_{avg} = & \text{m/s} & (b) v = & \text{m/s} \\
(c) a_{avg} = & \text{m/s}^2 & (d) v_{avg} = & \text{m/s}^2 \\
\end{array}
\]
2. A car (initially at rest) starts moving with a constant acceleration of 2 m/s\(^2\) until it reaches a speed of 8 m/s. He then keeps that speed constant for some time. If the total distance traveled is 250 m, how much time does it take for the total trip of the car? (20 points)

\[ t = \text{s} \]
3. A particle undergoes three consecutive displacements: (20 points)

\[ \vec{\Delta r}_1 = (10 \hat{i} - 2 \hat{j}) \text{ m} \]
\[ \vec{\Delta r}_2 = (13 \hat{i} + 11 \hat{j}) \text{ m} \]
\[ \vec{\Delta r}_3 = (-9 \hat{i} + 5 \hat{j}) \text{ m} \]

(a) Find the components of the resultant displacement.
(b) Find the magnitude and the direction (angle) of the resultant displacement.
4. A football player kicks a ball from a point at ground level located 30 m horizontally away from the goal, and he tries to hit the crossbar which is 4 m high. When kicked, the ball leaves the ground at an angle of 47° to the horizontal. What initial speed must the football have in order to hit the crossbar? (Neglect air friction). (20 points)

\[ V_0 = \text{m/s} \]
5. The object in the figure (m = 2 kg) is being pulled up by the external force F and moves with a constant acceleration of 3 m/s². The coefficient of kinetic friction between the object and the incline is 0.2, and the angle θ of the incline is 28°. (20 points)

(a) Calculate the magnitude of the force F.
(b) Calculate the magnitude of the normal force and indicate its direction in the figure below.

\[
\begin{array}{c}
\text{(a) } F = \quad \text{N} \\
\text{(b) } N = \quad \text{N}
\end{array}
\]
\[ \ddot{v} = \ddot{v}_0 + \ddot{a}t \]
\[ \ddot{r} = \ddot{r}_0 + \ddot{v}_0 t + 0.5\ddot{a}t^2 \]
\[ v^2 = v_0^2 + 2a(x - x_0) \]
\[ \vec{F} = m \cdot \vec{\ddot{a}} \]
\[ F_s = -kx \]
\[ a_c = \frac{v^2}{r} \]
\[ E_{mec} = K + U \]
\[ W_{net} = \Delta K \]
\[ W_{net} = -\Delta U \]
\[ \Delta E = \Delta K + \Delta U \]
\[ \Delta E = \Delta K + \Delta U + \Delta E_{th} \]
\[ \Delta E_{th} = f_i d \]
\[ U(y) = mgy \]
\[ U(x) = \frac{1}{2}kx^2 \]
\[ K = \frac{1}{2}mv^2 \]
\[ \vec{F}_{ext} = \frac{d \vec{p}}{dt} \]
\[ \vec{F}_{ext} = M \vec{\ddot{a}}_{COM} \]
\[ \ddot{\vec{R}}_{COM} = \frac{1}{M_{tot}} \sum_i m_i \ddot{r}_i \]
\[ \ddot{\vec{R}}_{COM} = \frac{1}{M_{tot}} \int \ddot{\vec{r}} \, dm \]
\[ \tau = I \alpha = r \omega^2 = r F \perp \]
\[ s = \theta \cdot r \]
\[ \omega = \omega_0 + \alpha \cdot t \]
\[ a_i = \alpha \cdot r \]
\[ K_{rot} = \frac{1}{2} I \omega^2 \]
\[ K_{tot} = \frac{1}{2}mv^2 + \frac{1}{2} I \omega^2 \]
\[ I = \sum_i m_i r_i^2 \]
\[ I = \int r^2 \, dm \]
Midterm 1

7. \( x(t) = 2t^2 + t - 1 \)

8. \( \overline{v} = \frac{x(4) - x(1)}{4 - 1} = \frac{35m - 2m}{3s} = \frac{33m}{3s} = 11 \text{ m/s} \)

\( x(4) = 32 + 4 - 1 = 35 \text{ m} \)

\( x(1) = 2 + 1 - 1 = 2 \text{ m} \)

(b) \( v = \frac{dx}{dt} = 4t + 1 \)

\( v(3) = 12 + 1 = 13 \text{ m/s} \)

(c) \( a_{avg} = \frac{v(4) - v(1)}{4 - 1} = \frac{17 - 5 \text{ m/s}}{3s} = 4 \text{ m/s}^2 \)

\( v(4) = 4\times 4 + 1 = 17 \)

\( v(1) = 5 \)

(d) \( a = \frac{dv}{dt} = \frac{dx^2}{dt^2} = 4 \text{ m/s}^2 \)
\[ a_1 = 2 \text{ m/s}^2 \]

\[ V_1 = ct \]

\[ V_0 = 0 \text{ m/s} \]

\[ x_0 = 0 \]

\[ V_1 = 8 \text{ m/s} \]

\[ x_2 = 250 \text{ m} \]

\[ t_1 \]

\[ x_1 \]

\[ t_2 \]

\[ \int_0^t (t_1 + t_2) = t_2 \]

(4) \[ x_1 - x_0 = \frac{1}{2} a_1 t_1^2 \]

\[ x_1 = \frac{1}{2} \left( \frac{2 \text{ m}}{\text{s}^2} \right) \cdot t_1^2 \]

(2) \[ x_2 - x_1 = V_1 \cdot t_2 \]

\[ 250 - x_1 = 8 \cdot t_2 \]

(2) \[ 250 - x_1 = 8 \cdot t_2 \]

(2) \[ 250 - \frac{x_1}{2} = 8 \cdot t_2 \]

(3) \[ a_1 = \frac{V_1 - V_0}{t_1} \]

\[ V_1 = a_1 \cdot t_1 = \left( \frac{2 \text{ m}}{\text{s}^2} \right) \cdot 8 \text{ m/s} \]

\[ \Rightarrow t_1 = \frac{V_1}{a_1} = \frac{8 \text{ m/s}}{2 \text{ m/s}^2} = 4 \text{ s} \]

(4) \[ x_1 = t_1^2 = 16 \text{ m} \]

(2) \[ t_2 = \frac{250 - x_1}{8} = \frac{250 - 16 \text{ m}^2}{8 \text{ m/s}^2} = 29.25 \text{ s} \]

\[ b_1 = 29.25 \text{ s} + 4 \text{ s} = 33.25 \text{ s} \]
\[ \begin{align*}
\vec{A}_1 &= 10 \hat{e}_x - 2 \hat{e}_y \\
\vec{A}_2 &= 13 \hat{e}_x + 11 \hat{e}_y \\
\vec{A}_3 &= -9 \hat{e}_x + 5 \hat{e}_y \\
\{ \end{align*} \]

(a) \[ \vec{A}_T = 14 \hat{e}_x + 14 \hat{e}_y \]

(b) \[ |\vec{A}_T| = \sqrt{14^2 + 14^2} = 19.80 \text{ m} \]

(c) \[ \tan \theta = \frac{14}{14} \rightarrow \theta = 45^\circ \]
\[
\begin{align*}
X - X_0 &= (V_0 \cos 47^\circ) \cdot t \\
Y - Y_0 &= (V_0 \sin 47^\circ) t - \frac{1}{2} gt^2
\end{align*}
\]

\[
\begin{align*}
30 \text{ m} &= 0.682 \cdot V_0 \cdot t \\
4 \text{ m} &= V_0 (0.731) \cdot t - 4.9 t^2
\end{align*}
\]

\[4 = (\frac{44}{t}) \cdot (0.731) - 4.9 t^2 \Rightarrow 4 = 32.16 - 4.9 t^2 \]

\[t = 2.397 \text{ s}\]
5. $m = 2\, kg$

$a = 3\, m/s^2$

$\mu_k = 0.2$

a) $F$?

b) $N$?

$f_k = \mu_k \cdot N = \mu_k \cdot mg \cos 28^\circ = (0.2) (2\, kg) (9.8\, m/s^2) \cos 28^\circ = 3.46\, N$

(a) $F - mg \sin 28^\circ - f_k = ma$

$F - 3.46\, N - 9.20\, N = 2a = 6\, N \Rightarrow F = 18.66\, N$

(b) $N = mg \cos 28^\circ = 17.3\, N$