1. An object moves along the x-axis according to the equation: $x(t) = (2t^3 - t + 5)$ m, where t is in seconds. Determine: *(20 points)*

   (a) the average speed between t = 1 s and t = 3 s
   (b) the instantaneous speed at t = 2 s
   (c) the average acceleration between t = 1 s and t = 3 s
   (d) the instantaneous acceleration at t = 2 s

$x(t) = 2t^3 - t + 5$

$v(t) = \frac{dx(t)}{dt} = 6t^2 - 1$

(a) $v_{avg} = \frac{x(3) - x(1)}{3s - 1s} = \frac{(56-6)m}{2s} = \boxed{25\ m/s}$

$x(3) = 54 - 3 + 5 = 56\ m$

$x(1) = 2 - 1 + 5 = 6$

(b) $v(2) = 6 * (2)^2 - 1 = \boxed{23 m/s}$

(c) $a_{avg} = \frac{v(3) - v(1)}{3 - 1} = \frac{53 m/s - 5 m/s}{2s} = \boxed{24 m/s^2}$

$v(3) = 6 * 3^2 - 1 = 53 m/s$

$v(1) = 6 * 1^2 - 1 = 5 m/s$

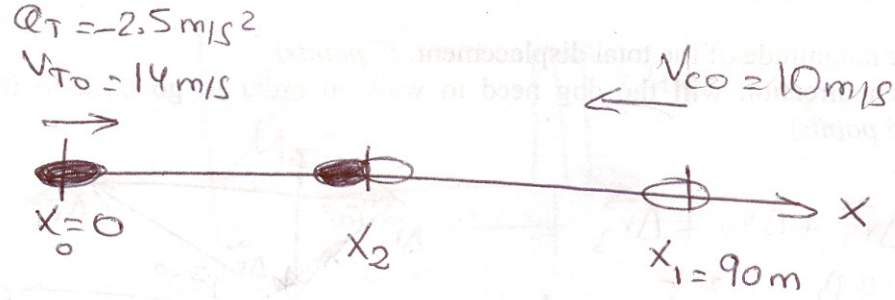
(d) $a(2) = 12 * 2 = \boxed{24 m/s^2}$

$a = \frac{dv}{dt} = 12 \cdot t$

(a) 25 m/s	(b) 23 m/s	(c) $24 m/s^2$	(d) $24 m/s^2$

2. A truck is travelling with an initial constant speed of 14 m/s along a one-way street until the truck driver sees a car from a drunk driver travelling towards him in the same lane with a constant speed of 10 m/s (contrary direction to the velocity of the truck). The truck driver immediately pushes his breaks and decelerates at a rate of -2.5 m/s^2. The distance between the truck and the car when the truck driver first saw the car was 90 m. Unfortunately, the truck driver could not avoid the collision.

(a) How far from the initial position of the truck (when he saw the car) will they collide? *(7 points)*
(b) When will they collide? *(7 points)*
(c) What will be the speed of the truck directly before the collision? *(6 points)*



TRUCK: $x_2 - x_0 = V_{To} \cdot t + \frac{1}{2} a_T t^2$ (1)

CAR: $x_2 - x_1 = V_{Co} \cdot t$ (2)

(1) $x_2 = 14t - 1.25t^2$

(2) $x_2 = 90 - 10t$

$14t - 1.25t^2 = 90 - 10t$

$-1.25t^2 + 24t - 90 = 0$

$t_1 = 5.11\ s$ shortest time

~~$t_2 = 14.09\ s$~~

(b) $x_2 = 90m - \left(10 \frac{m}{s}\right) \cdot (5.11\ s) = 38.9\ m$

(c) $a_T = \frac{V_{fT} - V_{To}}{t} \Rightarrow V_{fT} = a_T \cdot t + V_{To} = \left(-2.5\ m/s^2\right) \cdot (5.11\ s) + 14\ m/s = 1.22\ m/s$

(a) 38.9 m	(b) 5.11 s	(c) 1.22 m/s

3. A dog undergoes three consecutive displacements:

$\Delta r_1 = 5$ m, 30° North of East

$\Delta r_2 = 2$ m, West

$\Delta r_3 = 7$ m, 20° South of East

Consider North to be the positive direction of the y-axis and East that of the x-axis.

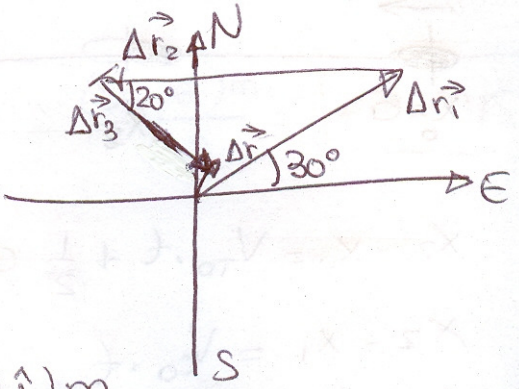
(a) Find the x and y components of the resultant displacement vector of the dog. *(10 points)*
(b) Find the magnitude of the total displacement. *(5 points)*
(c) In which direction will the dog need to walk in order to go back to its original position? *(5 points)*

$\vec{\Delta r_T} = \vec{\Delta r_1} + \vec{\Delta r_2} + \vec{\Delta r_3}$

$\vec{\Delta r_1} = (5\cos 30^\circ \hat{i} + 5\sin 30^\circ \hat{j})$ m (4.3, 2.5)

$\vec{\Delta r_2} = -2\hat{i}$ m

$\vec{\Delta r_3} = (7\cos 20^\circ \hat{i} - 7\sin 20^\circ \hat{j})$ m (6.58, −2.39)



$\vec{\Delta r_T} = (5\cos 30^\circ - 2 + 7\cos 20^\circ)\hat{i} + (5\sin 30^\circ - 7\sin 20^\circ)\hat{j}$

$\vec{\Delta r_T} = (8.9\hat{i} + 0.1\hat{j})$ m

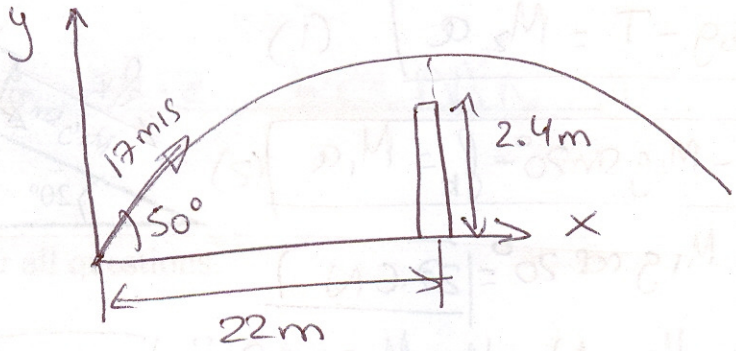
$|\vec{\Delta r_T}| = \sqrt{8.9^2 + 0.1^2} \Rightarrow$ **8.9 m**

$\tan\theta = \frac{0.1}{8.9} \Rightarrow \theta = 0.6^\circ$

(a) $(8.9\hat{i} + 0.1\hat{j})$ m	(b) 8.9	(c) 0.6° South West

4. A soccer player kicks a football from the ground from a point located 22 m (horizontally) away from the goal. The initial speed of the ball is 17 m/s and it is launched at an angle of 50° with respect to ground level. If the goal is 2.4 meters high and there is no goalkeeper: (Neglect air friction).

   (a) What will be the time of flight of the ball from its launching position until it reaches the goal? *(10 points)*
   (b) Will the soccer player score a goal? What will be the height of the ball when it reaches the goal? *(10 points)*



a)

$$x - x_0 = v_{ox} \cdot t = v_0 \cos\theta\, t \Rightarrow 22\,m = 17 \cos 50^\circ\, t \Rightarrow t = 2.01\,s$$

$$y - y_0 = v_{oy} t - \frac{1}{2} g t^2$$

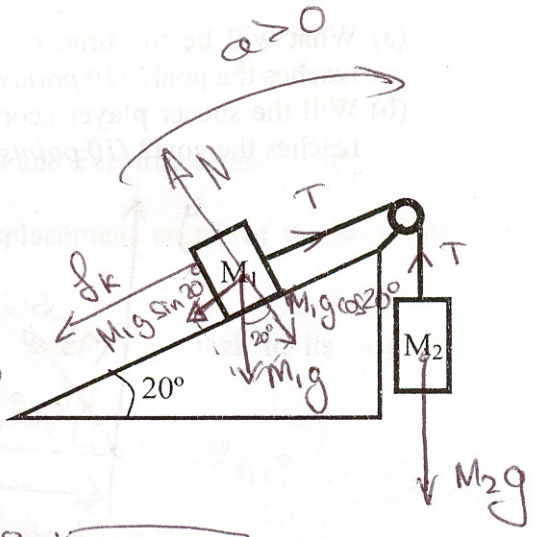
$$y = v_0 \sin\theta\, t - \frac{1}{2} g t^2 \Rightarrow y = 17 \sin 50\, t - 4.9 t^2 = 6.38\,m > 2.4\,m$$

No goal!

(a)	2.01 s	(b)	6.38 m	NO

5. The two blocks in the diagram below are connected by a massless cord to a massless pulley. The masses of the blocks are: M_1 = 3 kg, M_2=7 Kg. The coefficient of kinetic friction between M_1 and the incline plane is 0.1.

(a) Draw the force diagram. *(2 points)*
(b) Obtain the acceleration of the system. *(9 points)*
(c) Calculate the tension force on the cord. *(9 points)*



$M_2 \Rightarrow \boxed{M_2 g - T = M_2 a}$ (1)

$M_1 \Rightarrow \boxed{T - M_1 g \sin 20° - f_k = M_1 a}$ (2)

$N = M_1 g \cos 20° = \boxed{27.6\,N}$

$f_k = \mu_k \cdot N = \mu_k \cdot M_1 g \cos 20° = \boxed{2.76\,N}$

(1) $M_2 g - T = M_2 a$

(2) $T - M_1 g \sin 20° - \mu_k M_1 g \cos 20° = M_1 a$ }

(1)=(2) $\begin{cases} T = M_2 g - M_2 a \\ T = M_1 g \sin 20° + \mu_k M_1 g \cos 20° + M_1 a \end{cases}$

$M_2 g - M_2 a = M_1 g \sin 20° + \mu_k M_1 g \cos 20° + M_1 a$

$(M_1 + M_2) a = M_2 g - M_1 g \sin 20° - \mu_k M_1 g \cos 20°$

$$a = \frac{M_2 g - M_1 g \sin 20° - \mu_k M_1 g \cos 20°}{(M_1 + M_2)} = \frac{55.78\,kg\,m/s^2}{10\,kg} = \boxed{5.58\,m/s^2}$$

$T = M_2 (g - a) = \boxed{29.54\,N}$

(b)	5.58 m/s²	(c)	29.54 N