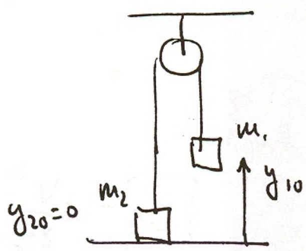
Chapter 8

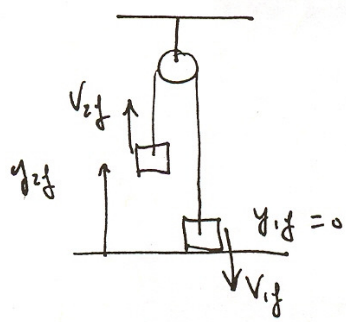
Problem 7:

a) There are only conservative forces, therefore $\Delta E = 0$

$$E_i = \underbrace{\cancel{m_2 g y_{20}}}_{0} + m_1 g y_{10} + \underbrace{\cancel{\tfrac{1}{2} m_1 v_{10}^2}}_{0} + \underbrace{\cancel{\tfrac{1}{2} m_2 v_{20}^2}}_{0} = m_1 g \underbrace{y_{10}}_{y} = m_1 g y$$



$$E_f = \underbrace{\cancel{m_1 g y_{1f}}}_{0} + m_2 g \underbrace{y_{2f}}_{y} + \tfrac{1}{2} m_1 \underbrace{v_{1f}}_{v}{}^2 + \tfrac{1}{2} m_2 \underbrace{v_{2f}}_{v}{}^2 = m_2 g y + \tfrac{1}{2}(m_1 + m_2) v^2$$



links:

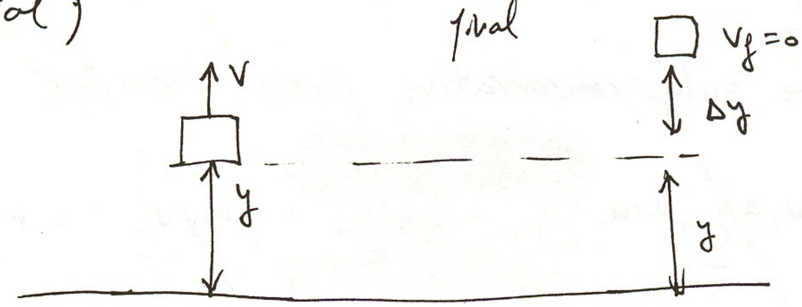
a) $v_{2f} = v_{1f} \equiv v$

b) $y_{2f} = y_{10} \equiv y$

$$E_i = E_f \Rightarrow m_1 g y = m_2 g y + \tfrac{1}{2}(m_1 + m_2) v^2$$

$$\boxed{v = \sqrt{\frac{2(m_1 - m_2) g y}{m_1 + m_2}} = 4.43 \text{ m/s}}$$

b) initial)



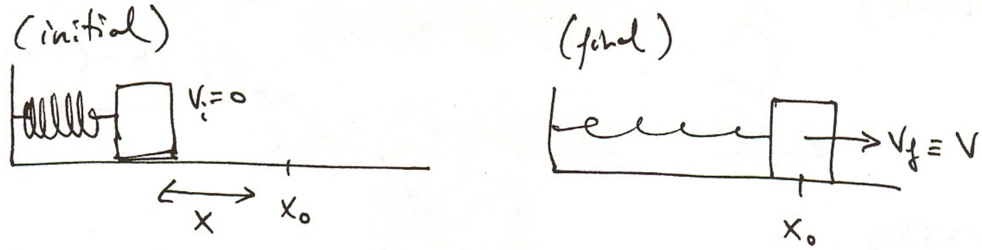
$\Delta E = 0 \qquad E_i = E_f$

$$\left.\begin{aligned} E_i &= m_2 g y + \frac{1}{2} m_2 v^2 \\ E_f &= m_2 g (y + \Delta y) + \cancel{\frac{1}{2} m_2 \underbrace{v_f^2}_{0}} \end{aligned}\right\} \boxed{\Delta y} = \frac{v^2}{2g} = \boxed{1m}$$

Total maximum height $\quad y + \Delta y = 4m + 1m = 5m$

Problem 14:

a) $\mu_K = 0$ (frictionless surface), only conservative forces $\Rightarrow \Delta E = 0$

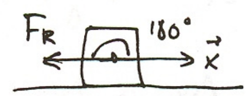


$E_i = \frac{1}{2} k X^2$ (potential spring) $\qquad E_f = \frac{1}{2} m V^2$ (kinetic)

$E_i = E_f \rightarrow \frac{1}{2} k X^2 = \frac{1}{2} m V^2 \qquad \boxed{V = \sqrt{\frac{k}{m}}\, X = 0.791 \frac{m}{s}}$

b) $\mu_K = 0.35$ (friction), non-conservative forces $\Rightarrow \Delta E = W_{NC}$

$E_f - E_i = W_{NC}$



$\frac{1}{2} m V^2 - \frac{1}{2} k X^2 = \vec{F}_K \cdot \vec{X} = F_K X \underbrace{\cos 180°}_{-1} = -F_K X$

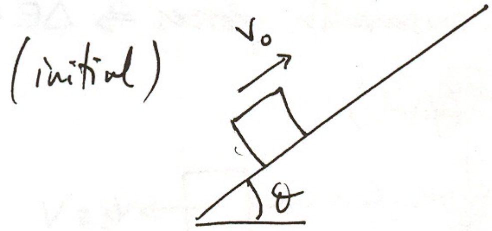
Therefore:

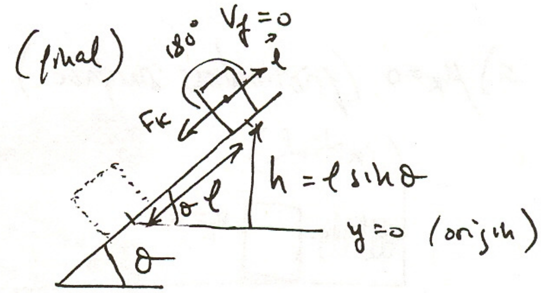
$\frac{1}{2} m V^2 = \frac{1}{2} k X^2 - F_K X \qquad F_K = \mu_K m g$

$$\boxed{V = \sqrt{\frac{k}{m} X^2 - \frac{2 F_K}{m} X} = \sqrt{\frac{k}{m} X^2 - 2 \mu_K g X} = 0.531 \frac{m}{s}}$$

$V_{\text{with friction}} < V_{\text{without friction}}$

Problem 21:





$m = 5\,kg$

$\ell = 3\,m$

$v_0 = 8\,m/s$

$\theta = 30°$

a) ΔK?

b) ΔU?

c) F_K?

d) μ_K?

a) $\Delta K = K_f - K_i = 0 - \frac{1}{2} m v_0^2 = -160\,J$

b) $\Delta U = U_f - U_i = mgh - 0 = mg\ell\sin\theta = 73.5\,J$

Note that $\Delta K \neq -\Delta U$, therefore $\Delta E = \Delta K + \Delta U \neq 0$

Energy is not conserved, therefore, a non conservative force is responsible to transfer the lost mechanical energy in other type of energy:

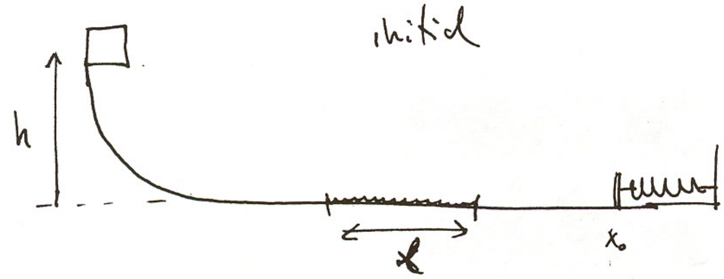
c) $$\Delta E = W_{NC} = \vec{F_K} \cdot \vec{\ell} = F_K \,\ell \underbrace{\cos 180°}_{-1} = -F_K\,\ell$$

$$\Delta E = \Delta K + \Delta U = -F_K\,\ell \qquad \boxed{F_K = -\frac{\Delta K + \Delta U}{\ell}} = -\frac{(-160\,J + 73.5\,J)}{3\,m} = \boxed{28.8\,N}$$

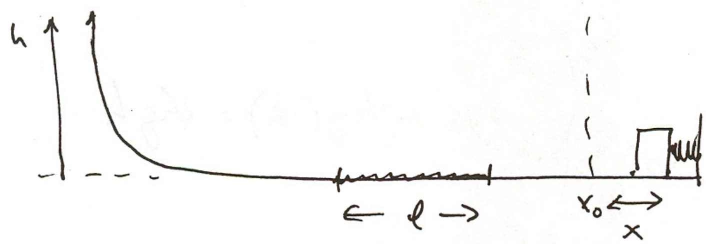
d) $$F_K = \mu_K N = \mu_K \underbrace{mg\cos\theta}_{\text{component of the weight perpendicular to the surface}} \rightarrow \boxed{\mu_K = \frac{F_K}{mg\cos\theta} = 0.674}$$

Problem 55 :

initial



$$E_i = mgh$$



$$E_f = \frac{1}{2} k x^2$$

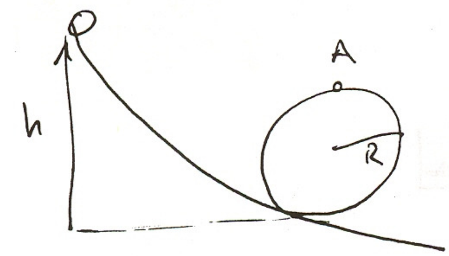
In the presence of non-conservative forces (friction), the change in mechanical energy is equal to the work done by the non-conservative forces:

$$\Delta E = E_f - E_i = W_{NC}$$

$$\frac{1}{2} k x^2 - \frac{1}{2} m v^2 = F_k \cdot \ell \underbrace{\cos 180^\circ}_{-1} = -F_k \ell = -\mu_k \underbrace{mg}_{F_k} \ell$$

$$\boxed{\mu_k = \frac{\frac{1}{2} m v^2 - \frac{1}{2} k x^2}{mg\ell} = 0.328}$$

Problem 3:



a) No friction ; $\Delta E = 0$

$$E_i = mgh$$

$$E_f = \frac{1}{2} m V_A^2 + mg(2R)$$

$$\frac{1}{2}\cancel{m}V_A^2 + \cancel{m}g(2R) = \cancel{m}gh$$

$$V_A = \sqrt{2gh - 4gR} = \sqrt{3gR}$$

$$h = 3.50R = \frac{7}{2}R$$

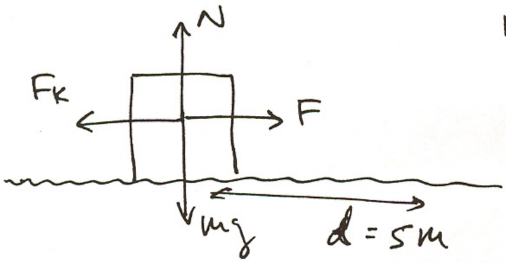
b)

$$\Sigma F_y = m a_c = m\frac{V_A^2}{R}$$

$$\Sigma F_y = N + mg$$

$$\boxed{N = m\frac{V_A^2}{R} - mg = 0.098\ \text{N}}$$

Problem 13:



$m = 40 kg$
$F = 130 N$
$\mu_k = 0.3$

a) $\boxed{W_F = \vec{F} \cdot \vec{d} = F d \cos 0° = F d = \boxed{650 J}}$

b) $\boxed{\Delta E_{int}} = -\Delta E = -W_{NC} = -\left(+F_k d \cos 180°\right) = -F_k d = \boxed{588 J}$

c) $W_N = N \cdot d \underbrace{\cos 90°}_{0} = 0$

d) $W_g = mg \cdot d \cos \underbrace{90°}_{0} = 0$

e) $\Delta K = W_T = W_{F_k} + W_F + \cancel{W_N}^{0} + \cancel{W_g}^{0} = 650 J - 588 J = 62 J$

f) $v_f = \sqrt{\frac{2K_f}{m}} = 1.76 \, m/s$

P8.11 (a) For a 5-m cord the spring constant is described by $F = kx$, $mg = k(1.5\text{ m})$. For a longer cord of length L the stretch distance is longer so the spring constant is smaller in inverse proportion:

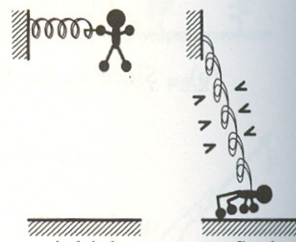


FIG. P8.11(a)

$$k = \frac{5\text{ m}}{L}\frac{mg}{1.5\text{ m}} = 3.33\,mg/L$$

$$\left(K+U_g+U_s\right)_i = \left(K+U_g+U_s\right)_f$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}kx_f^2$$

$$mg\left(y_i - y_f\right) = \frac{1}{2}kx_f^2 = \frac{1}{2}3.33\frac{mg}{L}x_f^2$$

here $y_i - y_f = 55\text{ m} = L + x_f$

$$55.0\text{ m}L = \frac{1}{2}3.33(55.0\text{ m} - L)^2$$

$$55.0\text{ m}L = 5.04\times10^3\text{ m}^2 - 183\text{ m}L + 1.67L^2$$

$$0 = 1.67L^2 - 238L + 5.04\times10^3 = 0$$

$$L = \frac{238 \pm \sqrt{238^2 - 4(1.67)\left(5.04\times10^3\right)}}{2(1.67)} = \frac{238\pm152}{3.33} = \boxed{25.8\text{ m}}$$

only the value of L less than 55 m is physical.

(b) $k = 3.33\dfrac{mg}{25.8\text{ m}}$ $\qquad x_{\max} = x_f = 55.0\text{ m} - 25.8\text{ m} = 29.2\text{ m}$

$\sum F = ma$ $\qquad +kx_{\max} - mg = ma$

$$3.33\frac{mg}{25.8\text{ m}}29.2\text{ m} - mg = ma$$

$$a = 2.77g = \boxed{27.1\text{ m/s}^2}$$