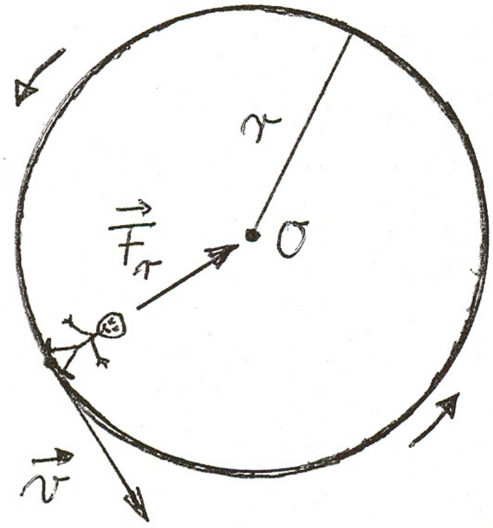
SOLUTIONS TO PROBLEMS of Chapter 6

(The values given in the book have been used for the solutions)

Problem 1 (problem 7 of book chapter 6):



uniform circular motion

$d = 2r = 120\ m$

$a_r = 3.00\ m/s^2$ (centripetal acceleration)

Standing on the inner surface of the rim, and moving with it, a person will feel a normal force $\vec{F}_r$ (= centripetal or radial force) exerted by the rim onto the person. This inward force causes the centripetal acceleration $a_r = 3.00\ \frac{m}{s^2}$:

$$\vec{F}_r = m \cdot \vec{a}_r \ .$$

Since $a_r = \frac{v^2}{r}$, so

$$v = \sqrt{a_r \cdot r} = \sqrt{(3.00\ m/s^2) \cdot (60.0\ m)} = 13.4\ \frac{m}{s} \ .$$

We find the rate of rotation of the wheel from the period of rotation, T:

$$v = \frac{2\pi r}{T} \ ,$$

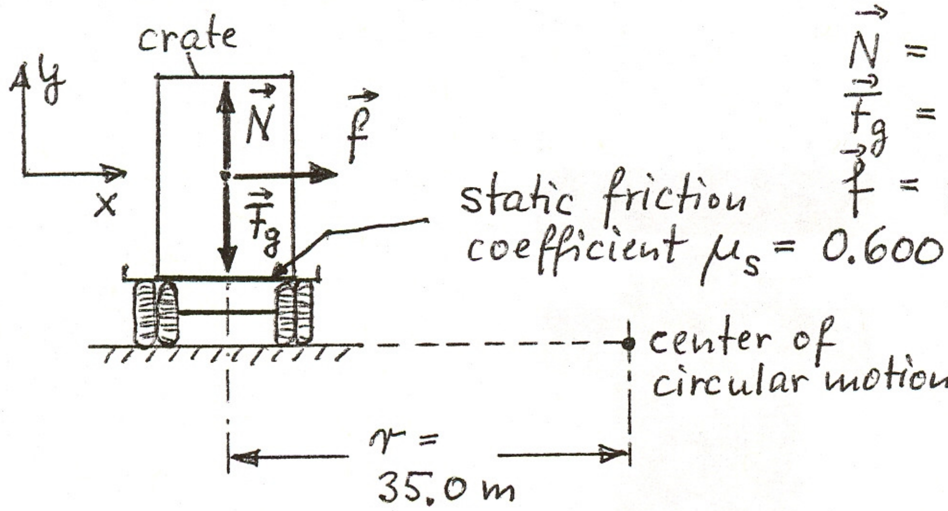
$$T = \frac{2\pi r}{v} = \frac{2\pi\ (60.0\ m)}{13.4\ m/s} = 28.1\ s \ ,$$

so the rate of rotation (= frequency) is

$$\underline{\underline{f}} = \frac{1}{T} = \frac{1}{28.1\ s} = \frac{1}{(28.1\ s)} \cdot \left(\frac{60\ s}{1\ min}\right) = \underline{\underline{2.14\ \frac{rev}{min}}}$$

Problem 2 (problem 9 of book chapter 6)

$\vec{N}$ = normal force
$\vec{F}_g = m \cdot \vec{g}$ (weight)
$\vec{f}$ = frictional force



along y-direction (vertical): $0 = \vec{N} - \vec{F}_g$, $\vec{N} = \vec{F}_g = m \cdot \vec{g}$
(since $a_y = 0$), $\underline{N = m \cdot g}$.

along x-direction (horizontal):

the force causing the centripetal acceleration is the frictional force $\vec{f}$. From Newton's 2. law, and since $a_c = \frac{v^2}{r}$, we get:

$$f = m \cdot a_c = m \cdot \frac{v^2}{r} .$$

Since the frictional force is $f \leq \mu_s \cdot N$,

i.e. $$m \cdot \frac{v^2}{r} \leq \mu_s \cdot m \cdot g$$

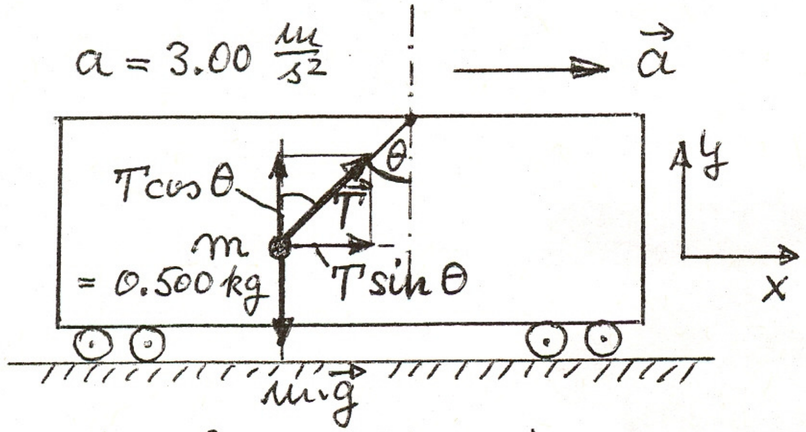
$$\rightarrow \quad v \leq \sqrt{\mu_s r g} = \sqrt{0.600\,(35.0\,m)(9.80\,\tfrac{m}{s^2})}$$

$$= 14.3\ \tfrac{m}{s}$$

$$\underline{\underline{v \leq 14.3\ \tfrac{m}{s}}}$$

Problem 3 (problem 21 of book chapter 6)

$a = 3.00 \frac{m}{s^2}$



view from inertial observer

The only forces acting on m are the force of gravity $m\cdot\vec{g}$ and the force of tension $\vec{T}$ in the string upward at angle θ with the vertical.

(a) Apply Newton's 2. law:

x-direction (horizontal): $\Sigma F_x = T\cdot\sin\theta = m\cdot a$ (1)

y-direction (vertical): $\Sigma F_y = T\cdot\cos\theta - m\cdot g$

or $T\cdot\cos\theta = m\cdot g$ (2)

Dividing eq.(1) by eq.(2) gives:

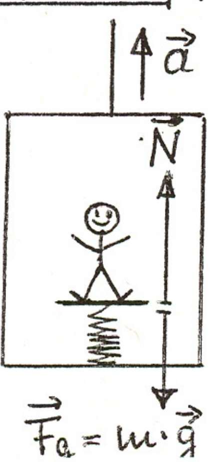
$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{a}{g} = \frac{3.00\,\frac{m}{s^2}}{9.80\,m/s^2} = 0.306$$

$$\rightarrow \theta = \tan^{-1}(0.306) = 17.0^\circ$$

(b) from eq.(1):

$$T = \frac{m\cdot a}{\sin\theta} = \frac{(0.500\,kg)\cdot(3.00\,\frac{m}{s^2})}{\sin(17.0^\circ)} = 5.13\,N$$

Problem 4 (problem 23 of book chapter 6)



Newton's 2. law: $m\cdot\vec{a} = \vec{N} - m\cdot\vec{g}$

$N_{max} = F_{max} = m\cdot g + m\cdot a = 591\,N$

$N_{min} = F_{min} = m\cdot g - m\cdot a = 391\,N$

(a) Adding: $2\cdot m\cdot g = 2\cdot F_g = 982\,N$, $\rightarrow F_g = 491\,N$

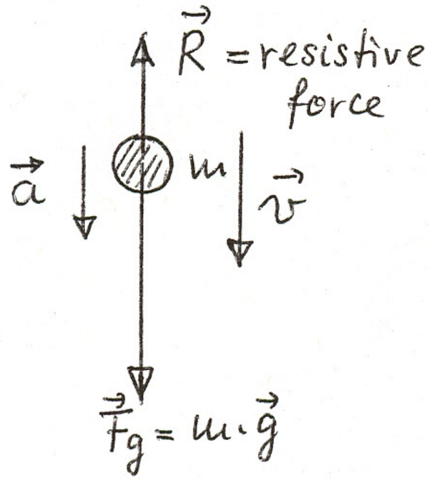
(b) Since $F_g = m\cdot g$, $m = \frac{F_g}{g} = \frac{491\,N}{9.80\,m/s^2} = 50.1\,kg$

(c) Subtracting:

$2m\cdot a = 200\,N$, $\rightarrow a = 2.00\,m/s^2$.

Problem 5 (problem 31 in book chapter 6)

Here: $\vec{R} = -b \cdot \vec{v}$ eq. (6.2).



(a) $m = 3.00\,g$,

terminal velocity $v_T = 2.00\,\frac{cm}{s}$.

At terminal velocity,

$$R = b \cdot v_T = m \cdot g \;, \quad \left(a = 0\,\frac{m}{s^2}\right)$$

$$\rightarrow \underline{\underline{b}} = \frac{m \cdot g}{v_T} = \frac{(3.00 \cdot 10^{-3}\,kg)(9.80\,\frac{m}{s^2})}{(2.00 \cdot 10^{-2}\,\frac{m}{s})}$$

$$= \underline{\underline{1.47\ N \cdot s/m}}$$

(b) The equation describing the time evolution of the velocity is

$$v(t) = v_T \cdot \underbrace{\left(1 - e^{-\frac{b}{m} \cdot t}\right)}_{= 0.632} \qquad \text{eq. (6.5)}$$

If $v = 0.632\, v_T$,

then $e^{-\frac{b}{m} \cdot t} = 0.368$,

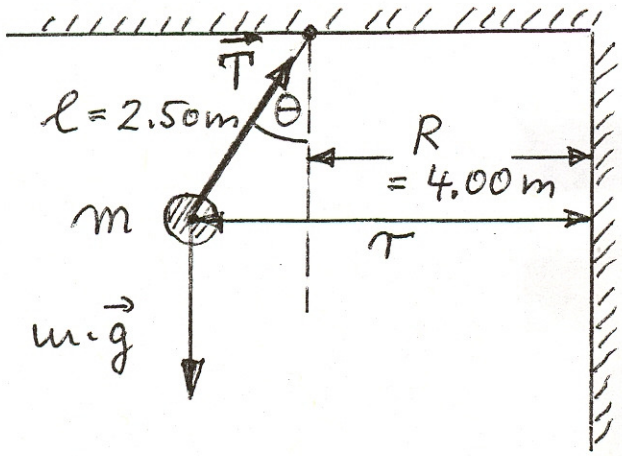
or at time $t = -\left(\frac{m}{b}\right) \cdot \ln(0.368)$,

$$\underline{\underline{t = 2.04 \cdot 10^{-3}\,s}}$$

(c) at terminal velocity,

$$\underline{\underline{R}} = v_T \cdot b = m \cdot g = \underline{\underline{2.94 \cdot 10^{-2}\,N}}$$

Problem 6 (problem 53 of book chapter 6)



$\theta = 28°$

$m = 10.0\ kg$ (seat)

(a) Newton's 2. law:

vertical direction: $m \cdot a_y = 0 = T \cdot \cos\theta - m \cdot g$

$$\rightarrow \quad \underline{T \cdot \cos\theta = m \cdot g} \qquad (1)$$

horizontal direction:

$$m \cdot a_r = m \cdot \frac{v^2}{r} = T \cdot \sin\theta$$

$$\underline{T \cdot \sin\theta = m \cdot \frac{v^2}{r}} \qquad (2)$$

Radius $r = (2.50 \cdot \sin\theta + 4.00)\ m$

$$= (2.50 \cdot \sin 28° + 4.00)\ m = \underline{5.17\ m}$$

Divide eq. (2) by (1): $\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{\left(\frac{v^2}{r}\right)}{g}$

$$\rightarrow v^2 = r \cdot g \cdot \tan\theta$$

$$= (5.17) \cdot (9.80) \cdot (\tan 28°)\ \frac{m^2}{s^2}$$

$$\underline{\underline{v = 5.19\ \frac{m}{s}}}$$

(b) $m_{child} = 40.0\ kg$. $m_{tot} = m_{child} + m_{seat} = 50.0\ kg$

from (1):

$$\underline{\underline{T}} = \frac{m_{tot} \cdot g}{\cos\theta} = \frac{(50.0\ kg) \cdot (9.80\ \frac{m}{s^2})}{\cos(28.0°)} = \underline{\underline{555\ N}}$$