Chapter 31 – <u>Alternating Current</u>

- Phasors and Alternating Currents
- Resistance and Reactance
- Magnetic-Field Energy
- The L-R-C Series Circuit
- Power in Alternating-Current Circuits
- Resonance in Alternating-Current Circuits
- Transformers

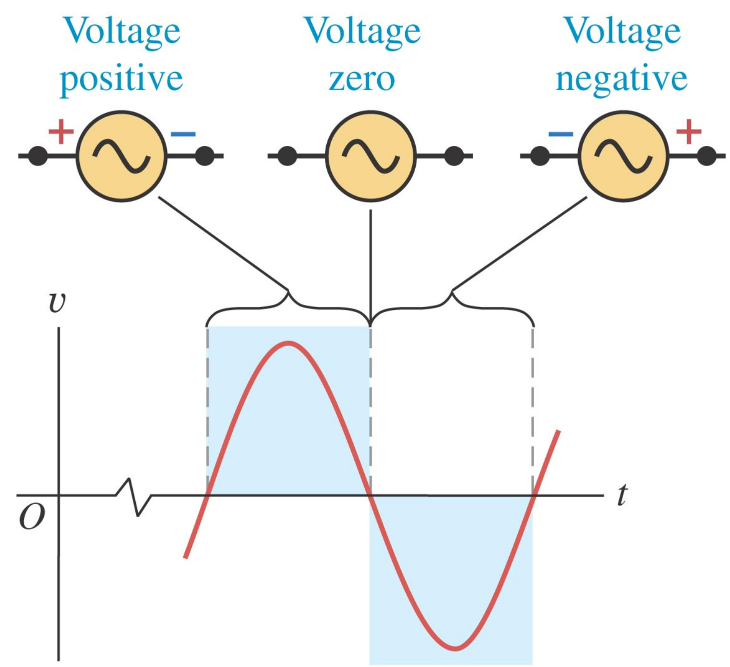
1. Phasors and Alternating Currents

Ex. source of ac: coil of wire rotating with constant ω in a magnetic field → sinusoidal alternating emf.

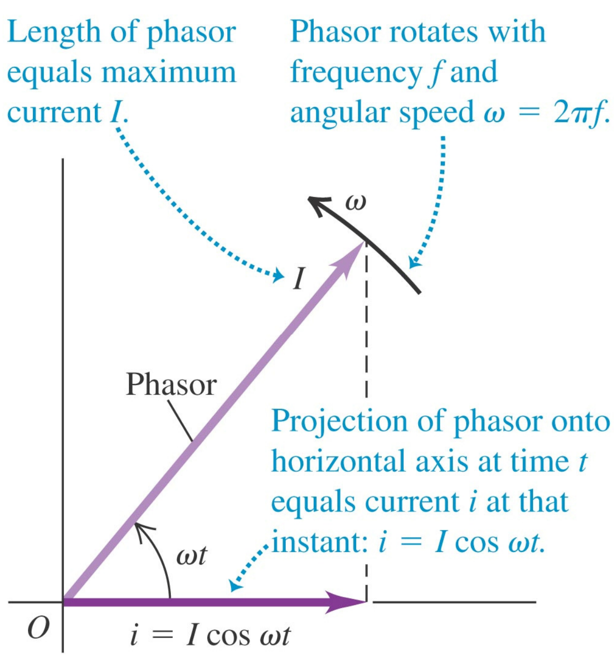
$$\boxed{v = V\cos\omega t}$$

$$\boxed{i = I\cos\omega t}$$

v, i = instantaneous potential difference / current.
V, I = maximum potential difference / current → voltage/current amplitude.



$\omega = 2\pi f$



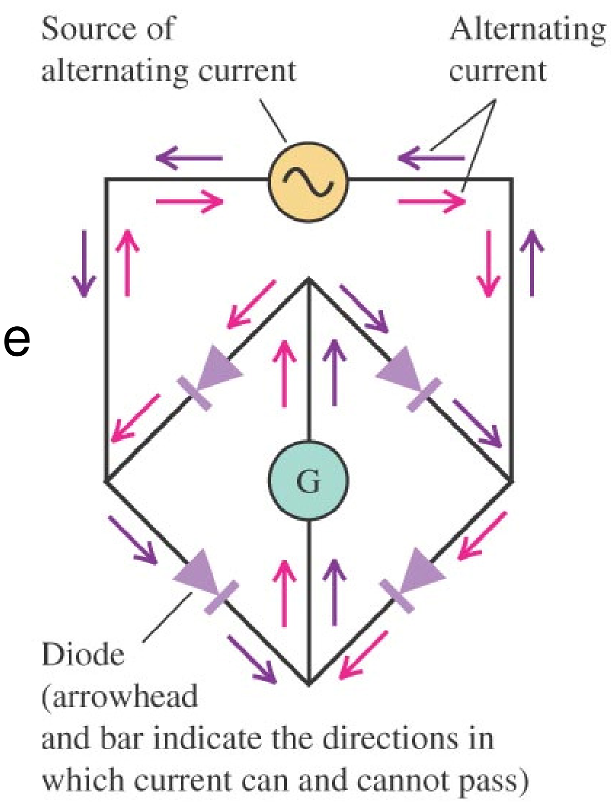
Phasor Diagrams

- Represent sinusoidally varying voltages / currents through the projection of a vector, with length equal to the amplitude, onto a horizontal axis.

- Phasor: vector that rotates counterclockwise with constant ω.

- Diode (rectifier): device that conducts better in one direction than in the other. If ideal, R = 0 in one direction and R = ∞ in other.

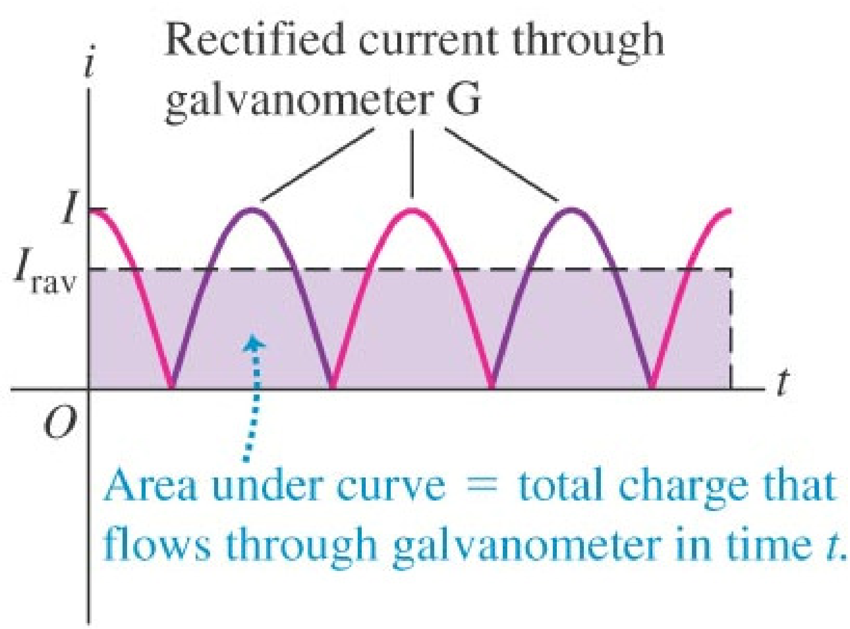
full wave rectifier circuit



Rectified average current (I_{rav}): during any whole number of cycles, the total charge that flows is same as if current were constant (I_{rav}).

$$\boxed{i_{rav} = \frac{2}{\pi} I}$$

average value of Icos ωtl or Isin ωtl



Root-Mean Square (rms) values:

$$\boxed{i_{rms} = \sqrt{(i^2)_{av}} = \frac{I}{\sqrt{2}}}$$

$$\boxed{V_{rms} = \frac{V}{\sqrt{2}}}$$

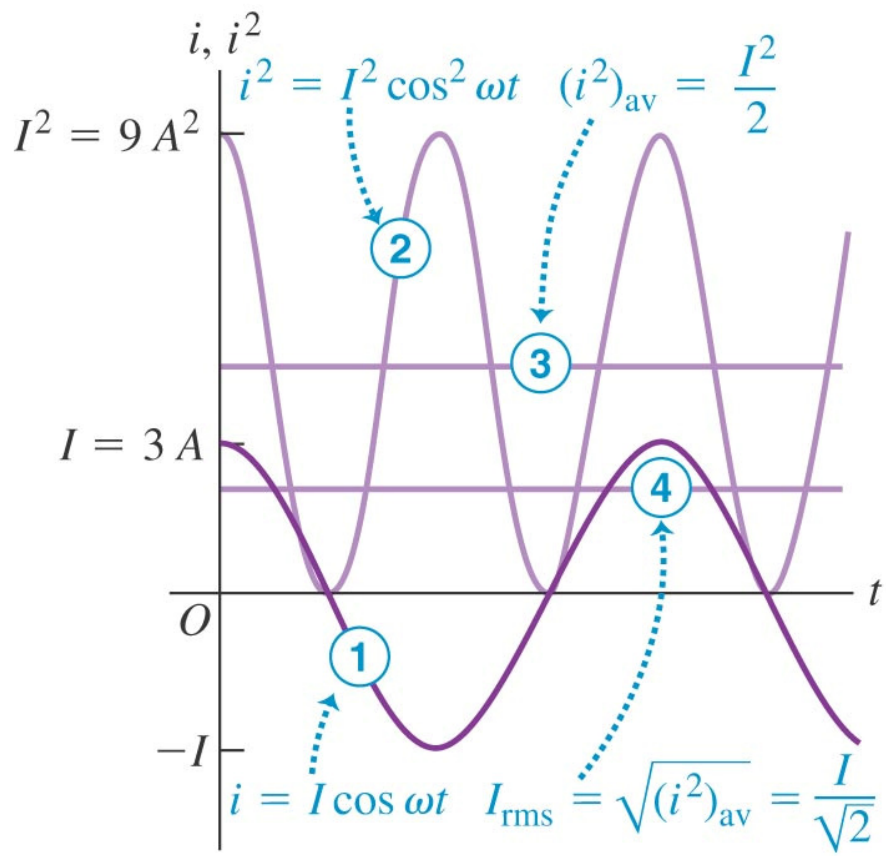
$$i^2 = I^2 \cos^2 \omega t$$

$$\cos^2 \omega t = 0.5 \cdot (1 + \cos 2\omega t)$$

$$i^2 = 0.5 I^2 + 0.5 I^2 \cos(2\omega t)$$

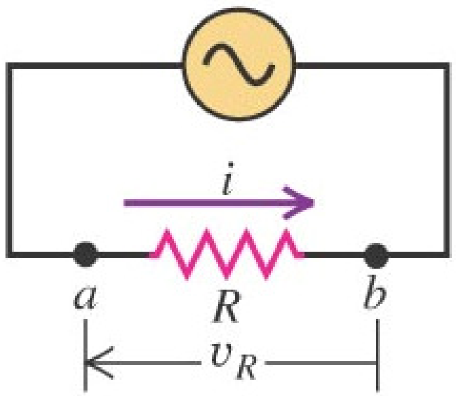
Meaning of the rms value of a sinusoidal quantity (here, ac current with $I = 3$ A):

1. Graph current i versus time.
2. *Square* the instantaneous current i.
3. Take the *average* (mean) value of i^2.
4. Take the *square root* of that average.



2. Resistance and Reactance

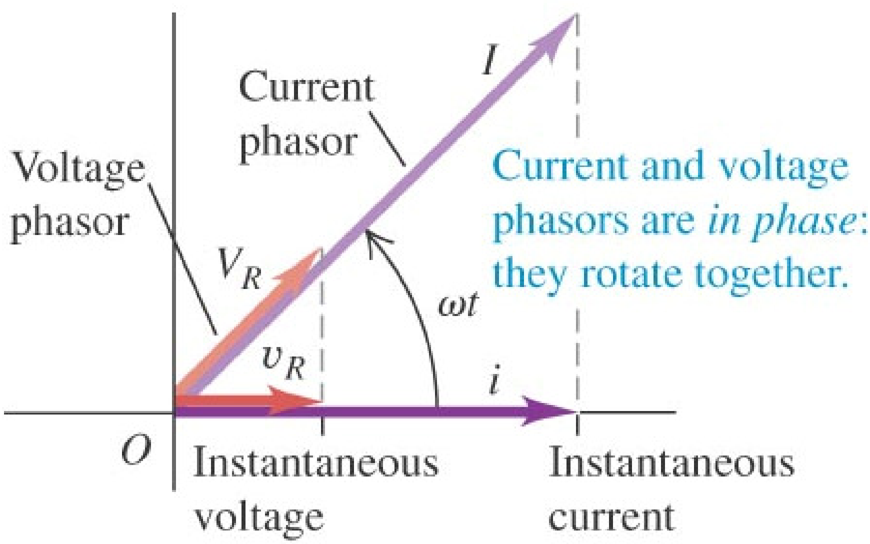
Resistor in an ac circuit

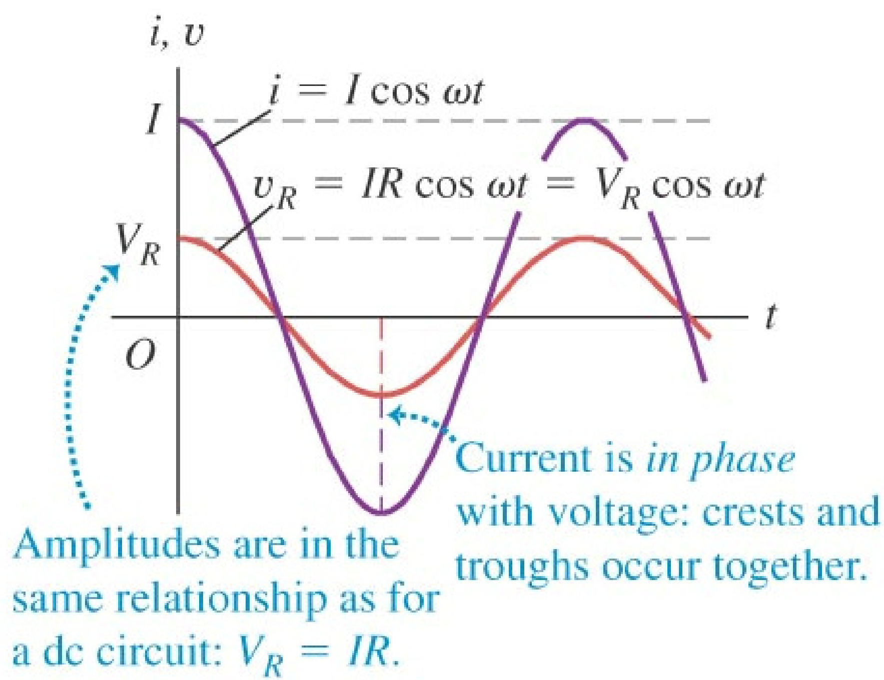


$$\boxed{v_R = iR = (IR)\cos\omega t = V_R \cos\omega t}$$ (instantaneous potential)

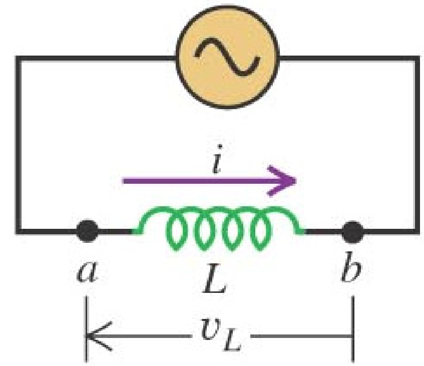
$$\boxed{V_R = IR}$$ (amplitude –max- of voltage across R)

- Current in phase with voltage → phasors rotate together





Inductor in an ac Circuit



- Current varies with time → self-induced emf → di/dt > 0 → ε < 0

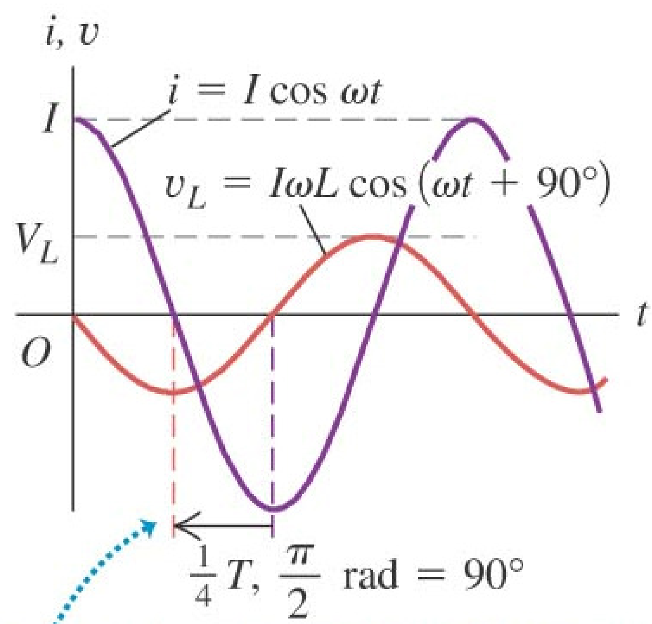
$$\varepsilon = -L\frac{di}{dt}$$

$V_a > V_b \rightarrow V_{ab} = V_a - V_b = V_L = L\, di/dt > 0$

$$v_L = L\frac{di}{dt} = L\frac{d}{dt}(I\cos\omega t)$$

$$\boxed{v_L = -I\omega L\sin\omega t = I\omega L\cos(\omega t + 90^\circ)}$$

v_L has 90° "head start" with respect to i.



Voltage curve *leads* current curve by a quarter-cycle (corresponding to $\phi = \pi/2$ rad $= 90°$).

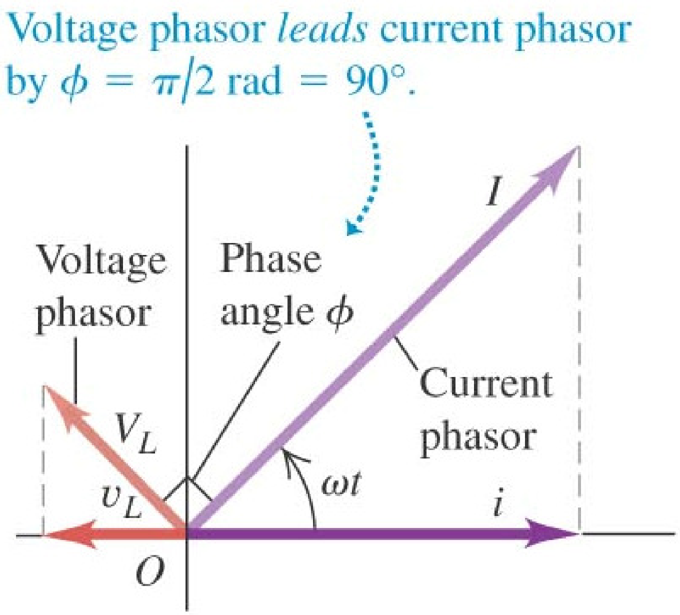
Inductor in an ac circuit

$$i = I \cos \omega t$$

$$v_L = \boxed{I \omega L} \cos(\omega t + 90^\circ)$$

V_L

$$v = V \cos(\omega t + \varphi)$$



φ = phase angle = phase of voltage relative to current

Pure resistor: φ = 0

Pure inductor: φ = 90º

Inductive reactance: $\boxed{X_L = \omega L}$

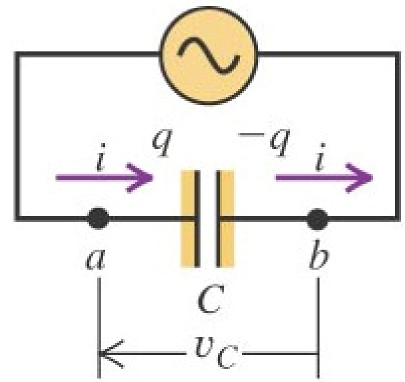
$$I = \frac{V_L}{\omega L}$$

High ω → low I
Low ω → high I

Voltage amplitude: $\boxed{V_L = I X_L = I \omega L}$

Inductors used to block high ω

Capacitor in an ac circuit

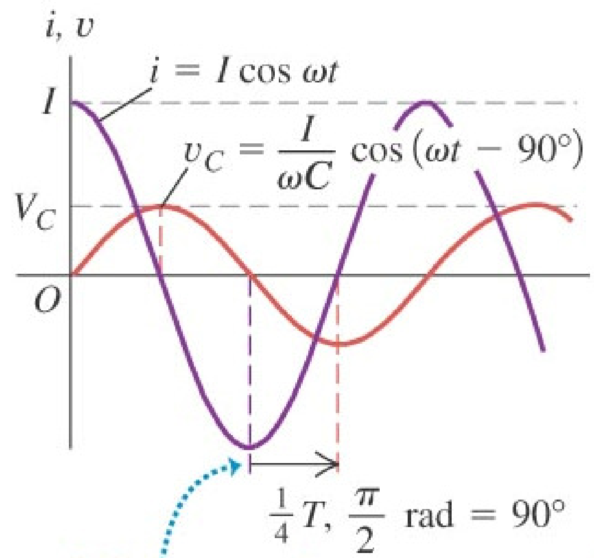


As the capacitor charges and discharges → at each t, there is "i" in each plate, and equal displacement current between the plates, as though charge was conducted through C.

$$i = \frac{dq}{dt} = I\cos\omega t \quad \rightarrow \int dq = \int I\cos\omega t\,dt$$

C = q / v_C

$$\boxed{q = \frac{I}{\omega}\sin\omega t}$$



Voltage curve *lags* current curve by a quarter-cycle (corresponding to $\phi = \pi/2$ rad = 90°).

$$\boxed{v_c = \frac{q}{C} = \frac{I}{\omega C}\sin\omega t = \frac{I}{\omega C}\cos(\omega t - 90^\circ)}$$

$$\boxed{V_C = \frac{I}{\omega C}}$$

Pure capacitor: φ = 90º

v_c lags current by 90º.

Capacitive reactance: $\boxed{X_C = \dfrac{1}{\omega C}}$

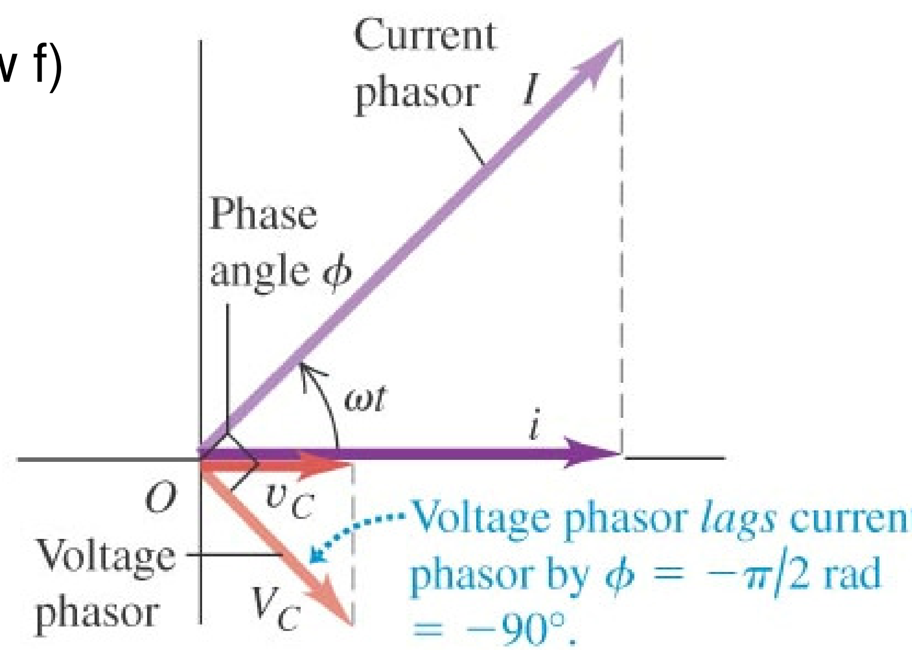
$\boxed{V_C = I X_C}$ (amplitude of voltage across C)

$I = V_C \omega C$

High ω → high I
Low ω → low I

Capacitor in an ac circuit

Capacitors used to block low ω (or low f)
→ high-pass filter



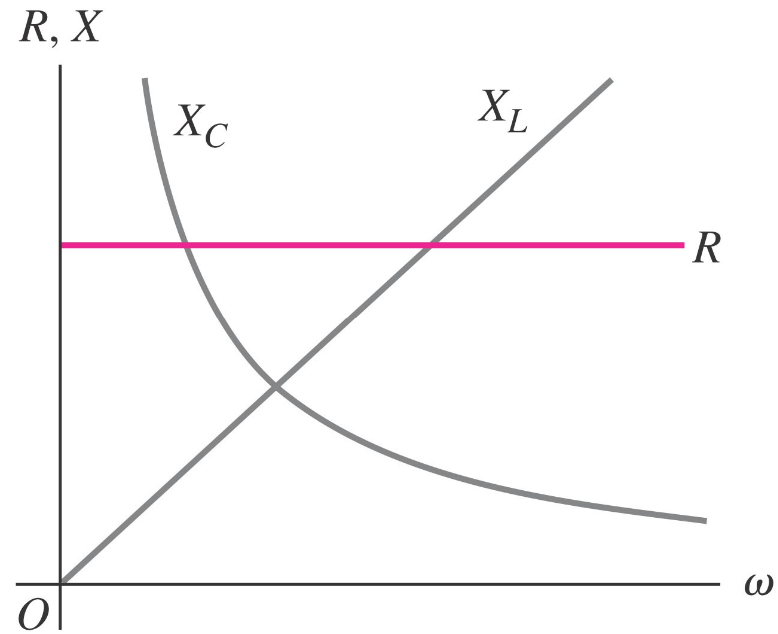
Comparing ac circuit elements:

- R is independent of ω.
- X_L and X_C depend on ω.
- If ω = 0 (dc circuit) → $X_c = 1/\omega C \rightarrow \infty$
  → $i_c = 0$

  $X_L = \omega L = 0$

- If $\omega \rightarrow \infty$, $X_L \rightarrow \infty \rightarrow i_L = 0$

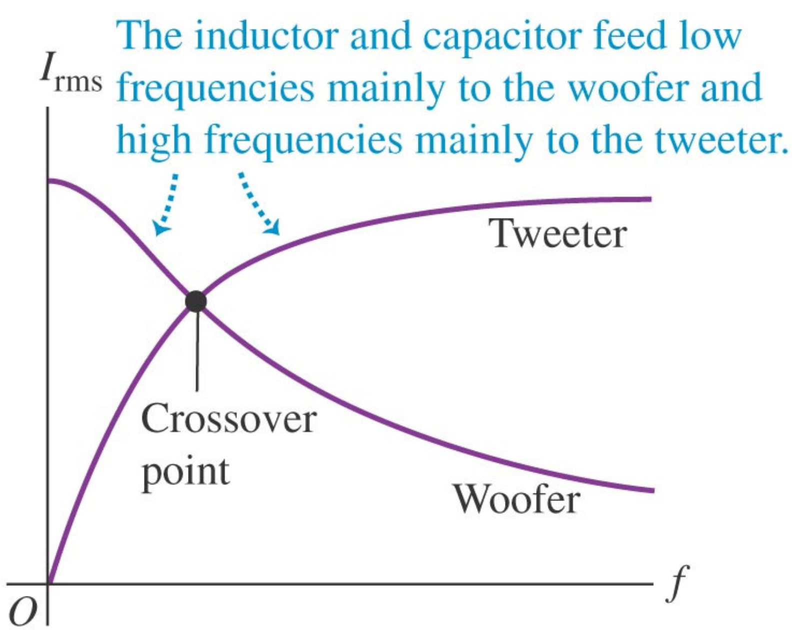
  $X_C = 0 \rightarrow V_C = 0$ → current changes direction so rapidly that no charge can build up on each plate.



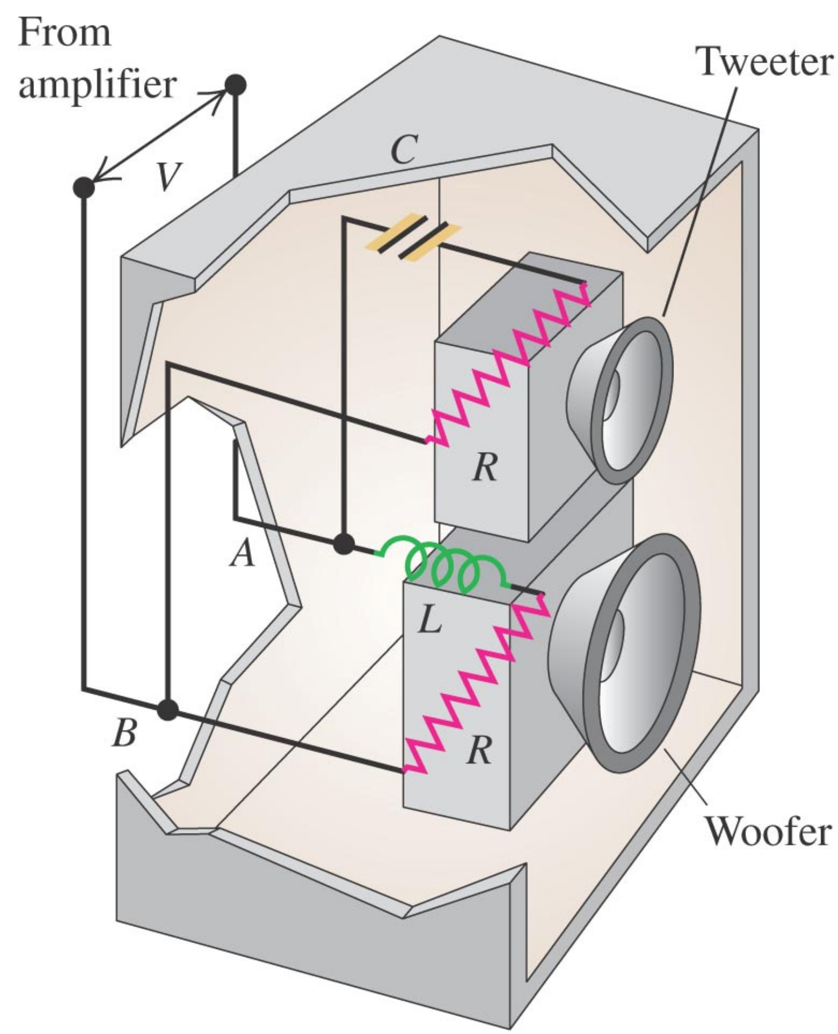
Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°

Example: amplifier → C in tweeter branch blocks low-f components of sound but passes high-f; L in woofer branch does the opposite.

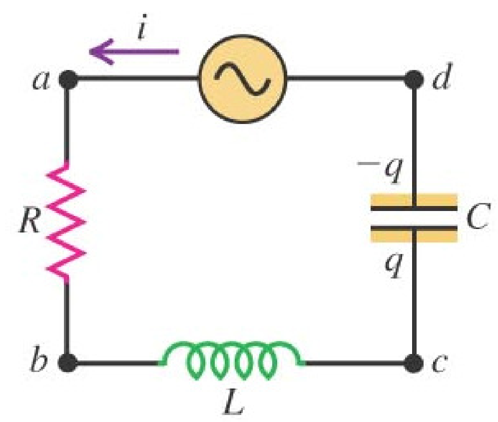
Graphs of rms current as functions of frequency for a given amplifier voltage



A crossover network in a loudspeaker system



3. The L-R-C Series Circuit

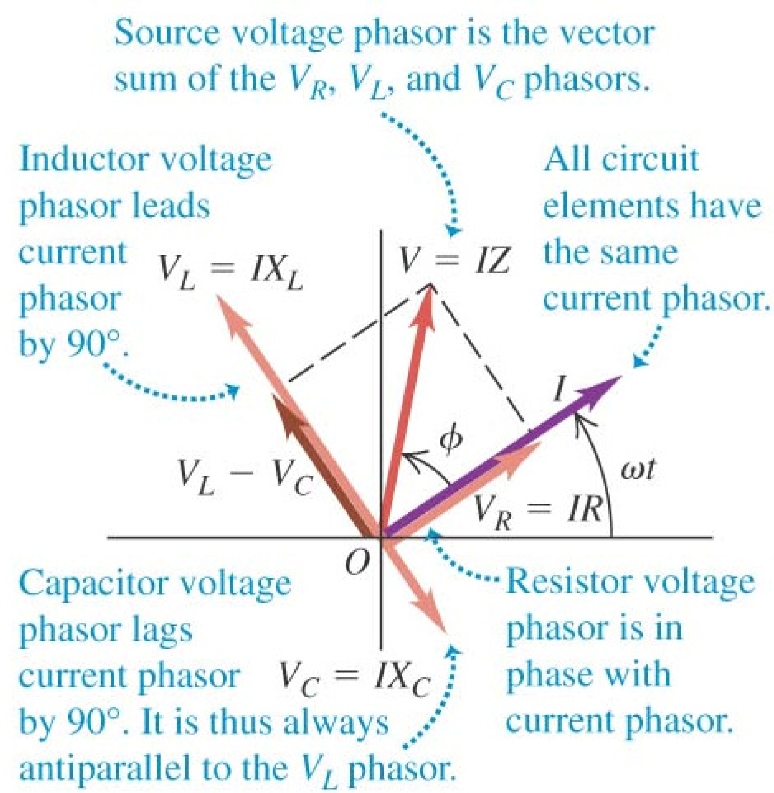


- Instantaneous v across L, C, R = v_{ad} = v source

- Total voltage phasor = vector sum of phasors of individual voltages.

- C, R, L in series → same current, i = I cosωt → only one phasor (I) for three circuit elements, amplitude I.

- The projections of I and V phasors onto horizontal axis at t give rise to instantaneous i and v.



$$V_C = IR$$

$$V_L = IX_L$$

$$V_C = IX_C$$

(amplitudes = maximum values)

-The instantaneous potential difference between terminals a,d =
= algebraic sum of v_R, v_C, v_L (instantaneous voltages) =
= sum of projections of phasors V_R, V_C, V_L
= projection of their vector sum (V) that represents the source voltage v and instantaneous voltage v_{ad} across series of elements.

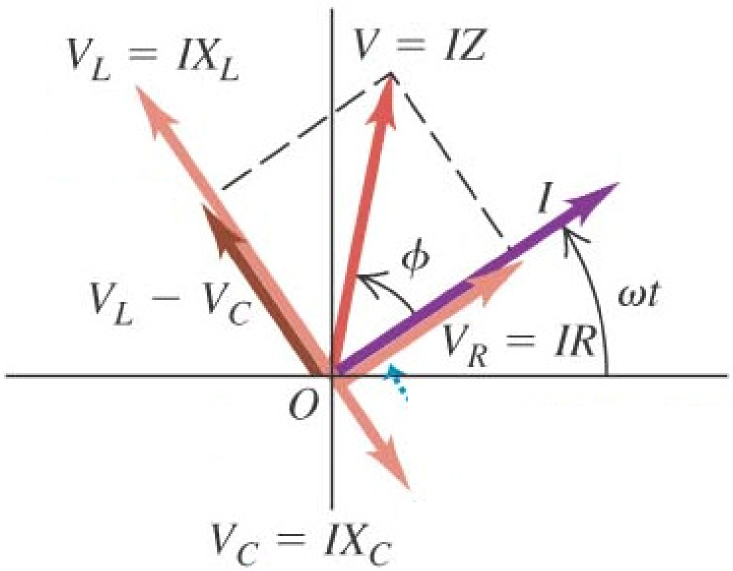
$$V = \sqrt{V_R^2 + (V_L - V_c)^2} = \sqrt{(IR)^2 + (IX_L - IX_c)^2} = I\sqrt{R^2 + (X_L - X_c)^2}$$

Impedance: $$\boxed{Z = \sqrt{R^2 + (X_L - X_c)^2}}$$

$$\boxed{V = IZ}$$

$$\boxed{Z = \sqrt{R^2 + [\omega L - (1/\omega C)]^2}}$$

Impedance of R-L-C series circuit



$$\boxed{\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{I(X_L - X_C)}{IR} = \frac{X_L - X_C}{R}}$$

Phase angle of the source voltage with respect to current

$$\boxed{\tan\varphi = \frac{\omega L - 1/\omega C}{R}}$$

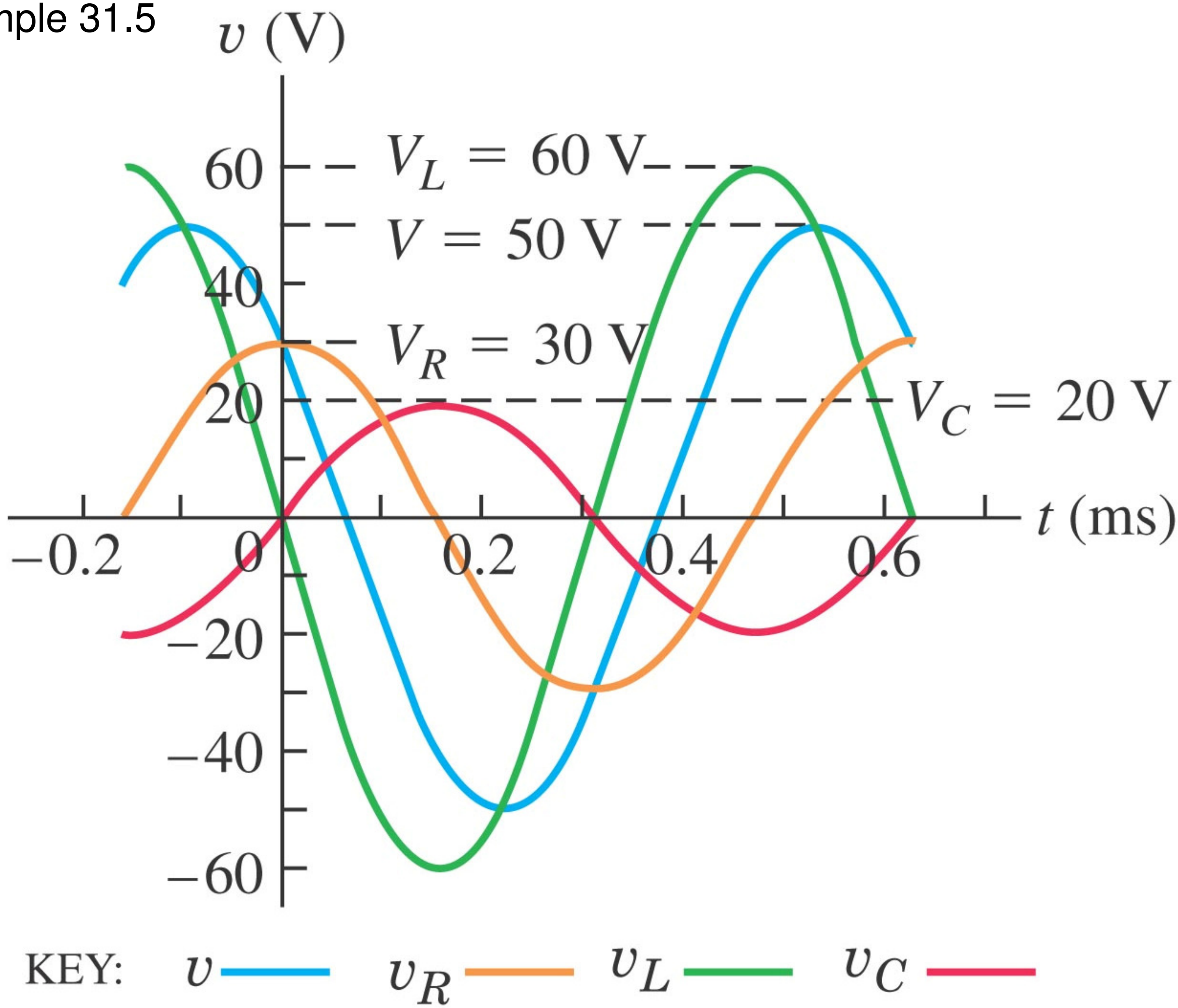
$$i = I\cos\omega t$$

$$v = V\cos(\omega t + \varphi)$$

$$\boxed{V_{rms} = I_{rms} Z}$$

$$\frac{V}{\sqrt{2}} = \frac{I}{\sqrt{2}} Z$$

Example 31.5

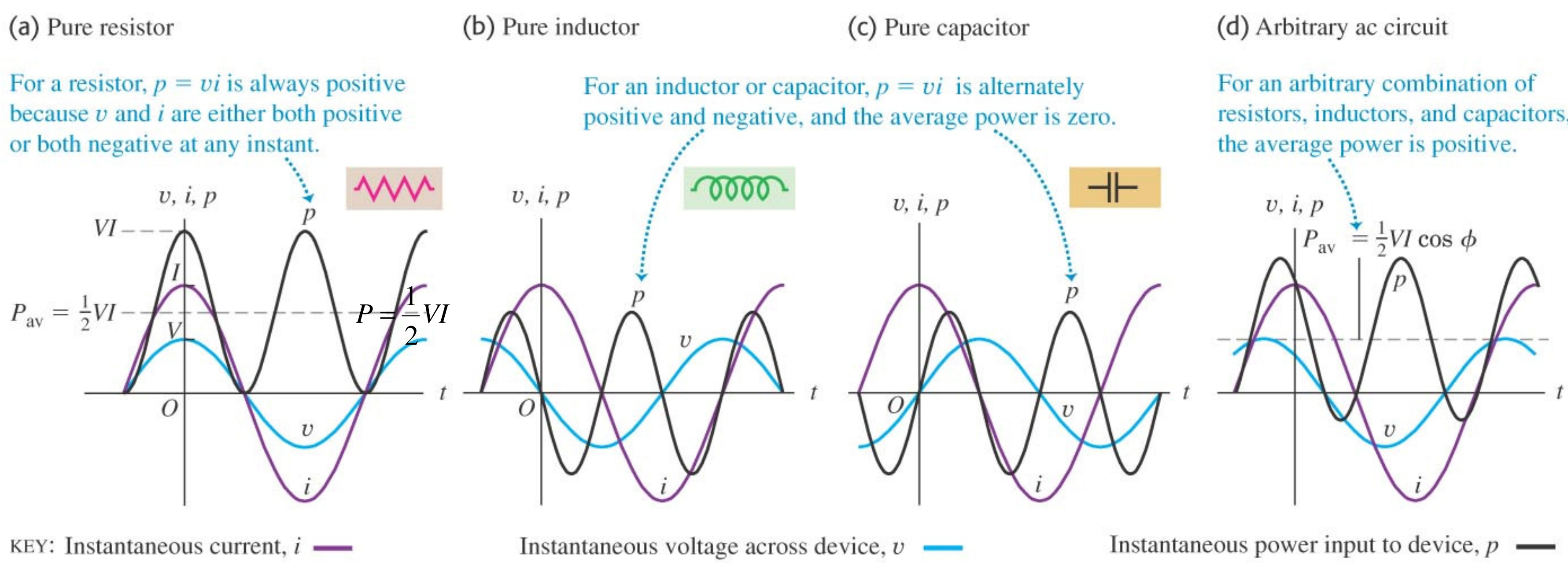


v (V)
60
40
20
−20
−40
−60
$V_L = 60$ V
$V = 50$ V
$V_R = 30$ V
$V_C = 20$ V
t (ms)
−0.2
0
0.2
0.4
0.6
KEY: v v_R v_L v_C

4. Power in Alternating-Current Circuits

$$P = \frac{1}{2} VI$$

$$P_{av} = \frac{V}{\sqrt{2}} \frac{I}{\sqrt{2}} = V_{rms} I_{rms} = I_{rms}{}^2 R = \frac{V_{rms}{}^2}{R}$$



Power in a General Circuit

$$P = vi = [V\cos(\omega t + \varphi)][I\cos\omega t] = [V(\cos\omega t\cos\varphi - \sin\omega t\sin\varphi)][I\cos\omega t]$$

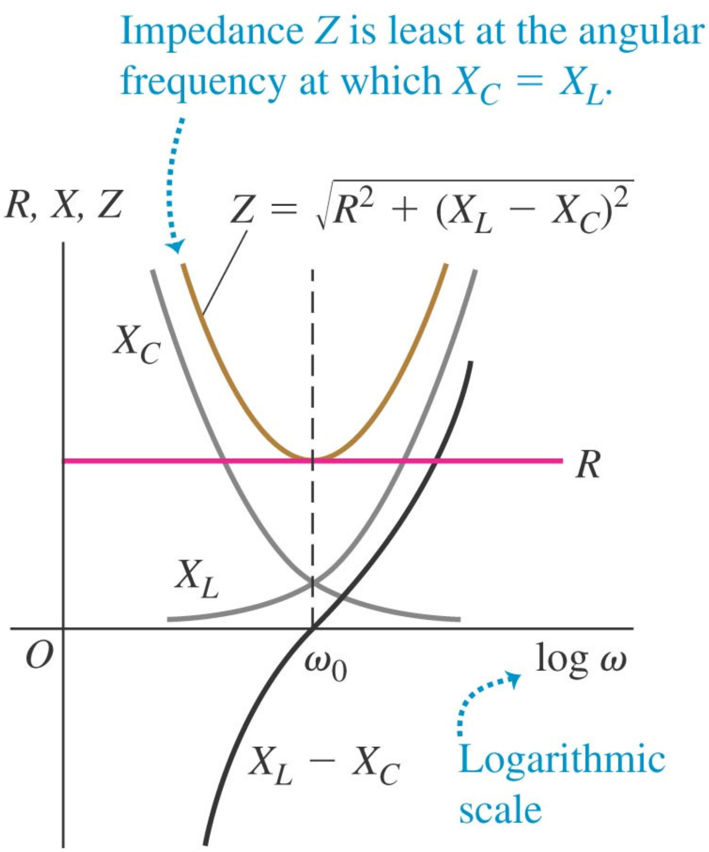
$$= VI\cos\varphi\cos^2\omega t - VI\sin\varphi\cos\omega t\sin\omega t$$

$$P_{av} = \frac{1}{2}VI\cos\varphi = V_{rms}I_{rms}\cos\varphi$$

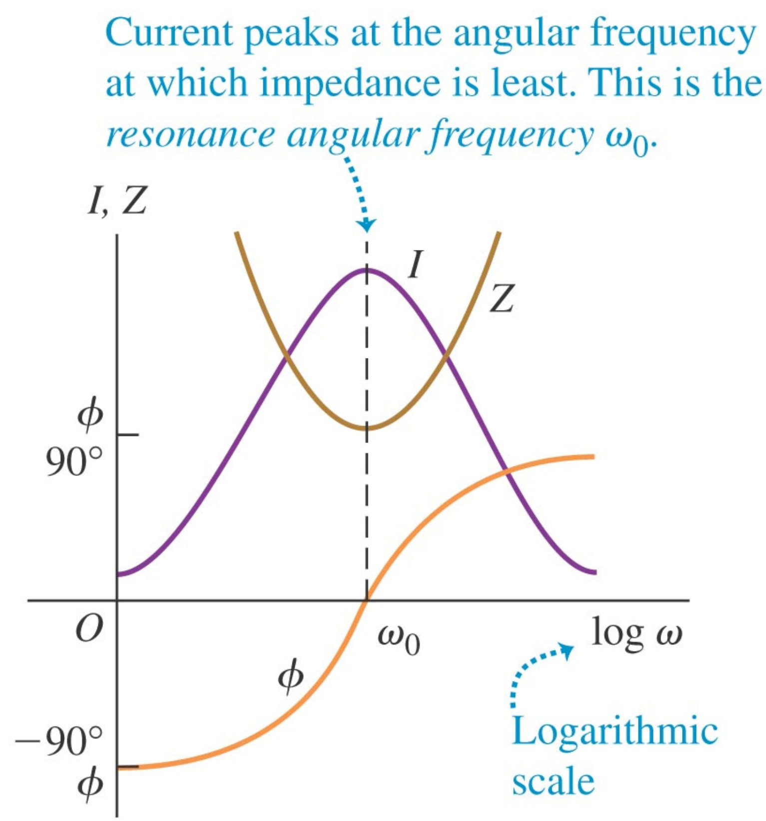
5. Resonance in Alternating-Current Circuits

$$X_L = X_C \qquad \omega_0 L = \frac{1}{\omega_0 C} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Reactance, resistance, and impedance as functions of angular frequency



Impedance, current, and phase angle as functions of angular frequency



6. Transformers

$$\mathcal{E}_1 = -N_1 \frac{d\Phi_B}{dt} \qquad \mathcal{E}_2 = -N_2 \frac{d\Phi_B}{dt}$$

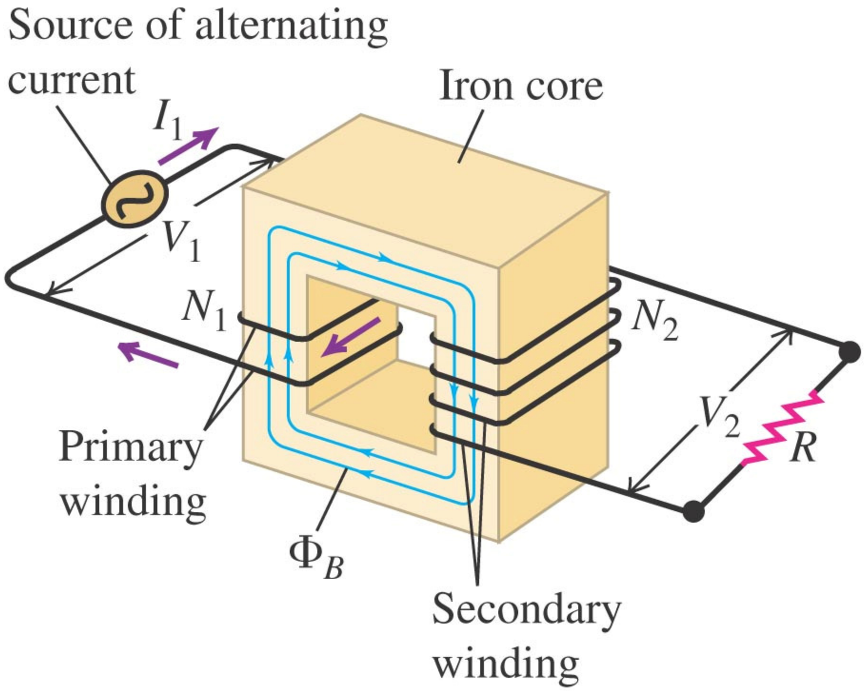
$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}$$

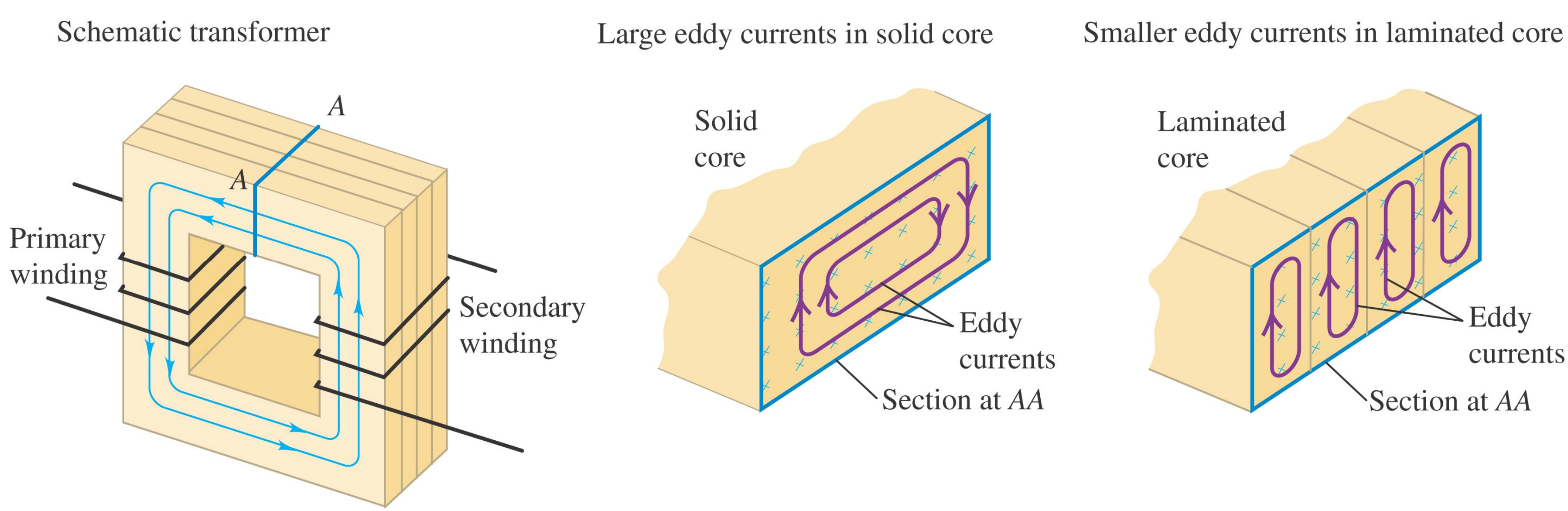
$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

$$\frac{V_2}{I_1} = \frac{R}{(N_2 / N_1)}$$

The induced emf *per turn* is the same in both coils, so we adjust the ratio of terminal voltages by adjusting the ratio of turns:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$




Schematic transformer
A
A
Primary
winding
Secondary
winding
Large eddy currents in solid core
Solid
core
Eddy
currents
Section at AA
Smaller eddy currents in laminated core
Laminated
core
Eddy
currents
Section at AA