Chapter 27 – Magnetic Field and Magnetic Forces

- Magnetism
- Magnetic Field
- Magnetic Field Lines and Magnetic Flux
- Motion of Charged Particles in a Magnetic Field
- Applications of Motion of Charged Particles
- Magnetic Force on a Current-Carrying Conductor
- Force and Torque on a Current Loop
1) A moving charge or collection of moving charges (e.g. electric current) produces a magnetic field. (Chap. 28).

2) A second current or charge responds to the magnetic field and experiences a magnetic force. (Chap. 27).

1. **Magnetism**

Permanent magnets: exert forces on each other as well as on unmagnetized Fe pieces.

- The needle of a compass is a piece of magnetized Fe.

- If a bar-shaped permanent magnet is free to rotate, one end points north (north pole of magnet).

- An object that contains Fe is not by itself magnetized, it can be attracted by either the north or south pole of permanent magnet.

- A bar magnet sets up a magnetic field in the space around it and a second body responds to that field. A compass needle tends to align with the magnetic field at the needle’s position.
1. **Magnetism**

- Magnets exert forces on each other just like charges. You can draw magnetic field lines just like you drew electric field lines.

- Magnetic north and south pole’s behavior is not unlike electric charges. For magnets, like poles repel and opposite poles attract.

- A permanent magnet will attract a metal like iron with either the north or south pole.
Magnetic poles about our planet

The geomagnetic north pole is actually a magnetic south (S) pole—it attracts the N pole of a compass.

The geomagnetic south pole is actually a magnetic north (N) pole.

The earth’s magnetic axis is offset from its geographic axis.

Magnetic field lines show the direction a compass would point at a given location.

The earth’s magnetic field has a shape similar to that produced by a simple bar magnet (although actually it is caused by electric currents in the core).
Magnetic declination / magnetic variation: the Earth’s magnetic axis is not parallel to its geographic axis (axis of rotation) → a compass reading deviates from geographic north.

Magnetic inclination: the magnetic field is not horizontal at most of earth’s surface, its angle up or down. The magnetic field is vertical at magnetic poles.

Magnetic Poles versus Electric Charge

- We observed monopoles in electricity. A (+) or (-) alone was stable, and field lines could be drawn around it.

- Magnets cannot exist as monopoles. If you break a bar magnet between N and S poles, you get two smaller magnets, each with its own N and S pole.

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.

Breaking a magnet in two ... 

... yields two magnets, not two isolated poles.
In 1820, Oersted ran experiments with conducting wires run near a sensitive compass. The orientation of the wire and the direction of the flow both moved the compass needle.

Ampere / Faraday / Henry → moving a magnet near a conducting loop can induce a current.

The magnetic forces between two bodies are due to the interaction between moving electrons in the atoms.

Inside a magnetized body (permanent magnet) there is a coordinated motion of certain atomic electrons. Not true for unmagnetized objects.
2. Magnetic Field

Electric field:

1) A distribution of electric charge at rest creates an electric field $\mathbf{E}$ in the surrounding space.

2) The electric field exerts a force $\mathbf{F}_E = q \mathbf{E}$ on any other charges in presence of that field.

Magnetic field:

1) A moving charge or current creates a magnetic field in the surrounding space (in addition to $\mathbf{E}$).

2) The magnetic field exerts a force $\mathbf{F}_m$ on any other moving charge or current present in that field.

- The magnetic field is a vector field $\rightarrow$ vector quantity associated with each point in space.

\[ F_m = |q|v_\perp B = |q|v \ B \sin \varphi \]

\[ \mathbf{F}_m = q\mathbf{v} \times \mathbf{B} \]

- $\mathbf{F}_m$ is always perpendicular to $\mathbf{B}$ and $\mathbf{v}$. 


2. Magnetic Field

- The moving charge interacts with the fixed magnet. The force between them is at a maximum when the velocity of the charge is perpendicular to the magnetic field.

A charge moving parallel to a magnetic field experiences zero magnetic force.

\[ F = |q|v_B = |q|vB \sin \phi. \]

\( \vec{F} \) is perpendicular to the plane containing \( \vec{v} \) and \( \vec{B} \).

A charge moving perpendicular to a magnetic field experiences a maximal magnetic force with magnitude \( F_{\text{max}} = qvB \).
Right Hand Rule

Positive charge moving in magnetic field → direction of force follows right hand rule

\[ \vec{F} = q\vec{v} \times \vec{B} \]

\( \vec{v} \cdot \vec{B} \) plane

Force acts along this line.

Right hand!

Negative charge → \( F \) direction contrary to right hand rule.

\[ \vec{F} = (-q)\vec{v} \times \vec{B} \]

Units:

1 Tesla = 1 N s / C m = 1 N/A m

1 Gauss = 10^{-4} T
Right Hand Rule

Positive and negative charges moving in the same direction through a magnetic field experience magnetic forces in *opposite* directions.

If charged particle moves in region where both, $E$ and $B$ are present:

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$q_1 = q > 0$$

$$q_2 = -q < 0$$

$$\vec{F} = (-q)\vec{v} \times \vec{B}$$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$
Measuring Magnetic Fields with Test Charges

**Ex:** electron beam in a cathode X-ray tube.

- In general, if a magnetic field \( B \) is present, the electron beam is deflected. However this is not true if the beam is \( // \) to \( B \) \( (\varphi = 0, \pi \rightarrow F=0 \rightarrow \text{no deflection}) \).

(a) If the tube axis is parallel to the y-axis, the beam is undeflected, so \( \vec{B} \) is in either the +y- or the −y-direction.

(b) If the tube axis is parallel to the x-axis, the beam is deflected in the −z-direction, so \( \vec{B} \) is in the +y-direction.

Electron \( q<0 \) → \( F \) has contrary direction to right hand rule

\[ \text{No deflection } \rightarrow \vec{F} = 0 \rightarrow \vec{v} // \vec{B} \]

\[ \text{Deflection } \rightarrow \vec{F} \neq 0 \rightarrow \vec{F} \perp \vec{v}, \vec{B} \]
3. Magnetic Field Lines and Magnetic Flux

- Magnetic field lines may be traced from N toward S (analogous to the electric field lines).
- At each point they are tangent to magnetic field vector.
- The more densely packed the field lines, the stronger the field at a point.
- Field lines never intersect.
- The field lines point in the same direction as a compass (from N toward S).
- Magnetic field lines are not “lines of force”.

The direction of the magnetic force depends on the velocity $\vec{v}$, as expressed by the magnetic force law $\vec{F} = q\vec{v} \times \vec{B}$. 
- Magnetic field lines have no ends → they continue through the interior of the magnet.

(a) Magnetic field of a C-shaped magnet

Between flat, parallel magnetic poles, the magnetic field is nearly uniform.

(b) Magnetic field of a straight current-carrying wire

To represent a field coming out of or going into the plane of the paper, we use dots and crosses, respectively.

(c) Magnetic fields of a current-carrying loop and a current-carrying coil (solenoid)

Notice that the field of the loop and, especially, that of the coil look like the field of a bar magnet (see Fig. 27.11).
Magnetic Flux and Gauss’s Law for Magnetism

\[ \Phi_B = \int B_\perp dA = \int B \cos \varphi \cdot dA = \int \vec{B} \cdot d\vec{A} \]

- Magnetic flux is a scalar quantity.

- If \( \vec{B} \) is uniform: \[ \Phi_B = B_\perp A = BA \cos \varphi \]

**Units:** 1 Weber (1 Wb = 1 T m\(^2\) = 1 N m / A)

- Difference with respect to electric flux \( \rightarrow \) the total magnetic flux through a closed surface is always zero. This is because there is no isolated magnetic charge ("monopole") that can be enclosed by the Gaussian surface.

\[ \Phi_B = \oint \vec{B} \cdot d\vec{A} = 0 \]

- The magnetic field is equal to the flux per unit area across an area at right angles to the magnetic field = magnetic flux density.

\[ B = \frac{d\Phi_B}{dA_\perp} \]
4. Motion of Charged Particles in a Magnetic Field

- Magnetic force perpendicular to $\vec{v}$ → it cannot change the magnitude of the velocity, only its direction.

- $\vec{F}$ does not have a component parallel to particle’s motion → cannot do work.

- Motion of a charged particle under the action of a magnetic field alone is always motion with constant speed.

- Magnitudes of $\vec{F}$ and $\vec{v}$ are constant ($\vec{v}$ perp. $B$) → uniform circular motion.

$$\overrightarrow{F_m} = q\vec{v} \times \vec{B}$$

$$F = |q| \cdot \vec{v} \cdot B = m \frac{v^2}{R}$$

Radius of circular orbit in magnetic field:

$$R = \frac{mv}{|q|B}$$

+ particle → counter-clockwise rotation.
- particle → clockwise rotation.
Angular speed: $\omega = v/R \rightarrow \omega = v \frac{qB}{mv} = \frac{qB}{m}$

Cyclotron frequency: $f = \omega/2\pi$

- If $v$ is not perpendicular to $B \rightarrow v_\parallel$ (parallel to $B$) constant because $F_\parallel = 0 \rightarrow$ particle moves in a helix. (R same as before, with $v = v_\perp$).

This particle’s motion has components both parallel ($v_\parallel$) and perpendicular ($v_\perp$) to the magnetic field, so it moves in a helical path.

A charged particle will move in a plane perpendicular to the magnetic field.
5. Applications of Motion of Charged Particles

Velocity selector

- Particles of a specific speed can be selected from the beam using an arrangement of E and B fields.
- \( F_m \) (magnetic) for + charge towards right \( (q \ v \ B) \).
- \( F_E \) (electric) for + charge to left \( (q \ E) \).
- \( F_{\text{net}} = 0 \) if \( F_m = F_E \rightarrow -qE + q \ v \ B = 0 \rightarrow \ v = E/B \)
- Only particles with speed \( E/B \) can pass through without being deflected by the fields.

\[
F_E = qE \quad F_B = qvB
\]
Thomson’s $e/m$ Experiment

$$\Delta E = \Delta K + \Delta U = 0 \rightarrow 0.5 \, m \, v^2 = U = e \, V$$

$$v = \frac{E}{B} = \sqrt{\frac{2eV}{m}}$$

\[
\frac{e}{m} = \frac{E^2}{2VB^2}
\]

e/m does not depend on the cathode material or residual gas on tube $\rightarrow$ particles in the beam (electrons) are a common constituent of all matter.

Between plates $P$ and $P'$ there are mutually perpendicular, uniform $\vec{E}$ and $\vec{B}$ fields.

Electrons travel from the cathode to the screen.
Mass Spectrometer

- Using the same concept as Thompson, Bainbridge was able to construct a device that would only allow one mass in flight to reach the detector.

- Velocity selector filters particles with $v = E/B$. After this, in the region of $B'$ particles with $m_2 > m_1$ travel with radius ($R_2 > R_1$).

$$R = \frac{mv}{|q|B'}$$

Magnetic field separates particles by mass; the greater a particle’s mass, the larger is the radius of its path.
6. Magnetic Force on a Current-Carrying Conductor

\[ \vec{F}_m = q\vec{v}_d \times \vec{B} \]

- Total force:

\[ F_m = (nAl)(qv_d B) \]

\( n = \text{number of charges per unit volume} \)
\( A \ l = \text{volume} \)

\[ F_m = (nqv_d)(A)(lB) = (JA)(lB) = IlB \quad (B \perp \text{wire}) \]

In general:

\[ F = IlB_\perp = IlB \sin \phi \]

Magnetic force on a straight wire segment:

\[ \vec{F} = Il \times \vec{B} \]

Magnetic force on an infinitesimal wire section:

\[ d\vec{F} = Idl \times \vec{B} \]
Current is not a vector. The direction of the current flow is given by $\mathbf{dl}$, not $I$. $\mathbf{dl}$ is tangent to the conductor.

$$\mathbf{F} = I\mathbf{l} \times \mathbf{B}$$

**Reversing $\mathbf{B}$ reverses the force direction.**

**Reversing the current [relative to (b)] reverses the force direction.**
7. Force and Torque on a Current Loop

- The net force on a current loop in a uniform magnetic field is zero.

Right wire of length “a” → \( F = I \ a \ B \) \( (B \perp l) \)

Left wire of length “b” → \( F' = I \ b \ B \sin (90^\circ - \phi) \) \( (B \text{ forms } 90^\circ-\phi \text{ angle with } l) \)
\( F' = I \ b \ B \cos \phi \)

\( F_{\text{net}} = F - F + F' - F' = 0 \)

- Net torque \( \neq 0 \) (general).

\( \vec{\tau} = \vec{r} \times \vec{F} \)

\( \tau = r \cdot F \sin \alpha = rF_\perp \)

\( \tau_{F'} = r \cdot F \sin 0^\circ = 0 \)

\( \tau_F = F(b/2) \sin \phi \)
\[ \tau_{total} = \tau_{F'} + \tau_{-F'} + \tau_F + \tau_{-F} = 0 + 0 + 2(b/2)F \sin \varphi \]

\[ \tau_{total} = (IBa)(b \sin \varphi) = IBA \sin \varphi \]

Torque on a current loop

A = a \ b

The torque is maximal when \( \phi = 90^\circ \) (so \( \vec{B} \) is in the plane of the loop).

Torque is zero, \( \varphi = 0^\circ \)

\( \varphi \) is angle between a vector perpendicular to loop and \( \vec{B} \)
Magnetic dipole moment: $\vec{\mu} = I\vec{A}$

Direction: perpendicular to plane of loop (direction of loop’s vector area $\rightarrow$ right hand rule)

Magnetic torque: $\vec{\tau} = \vec{\mu} \times \vec{B}$

Potential Energy for a Magnetic Dipole: $U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos \varphi$

Electric dipole moment: $\vec{p} = q\vec{d}$

Electric torque: $\vec{\tau} = \vec{p} \times \vec{E}$

Potential Energy for an Electric Dipole: $U = -\vec{p} \cdot \vec{E}$
Magnetic Torque: Loops and Coils

If these loops all carry equal current “I” in same clockwise sense, F and torque on the sides of two adjacent loops cancel, and only forces and torques around boundary ≠ 0.

\[
\tau = NIBA \sin \varphi
\]

\(N\) = number of turns
\(\varphi\) is angle between axis of solenoid and \(B\)

Max. torque: solenoid axis \(\perp\) \(B\).

Torque rotates solenoid to position where its axis is parallel to \(B\).
Magnetic Dipole in a Non-Uniform Magnetic Field

- Net force on a current loop in a non-uniform field is not zero.

\[ d\vec{F} = I \, dl \times \vec{B} \]

Radial force components cancel each Other \( \rightarrow F_{\text{net}} \) to right.

If polarity of magnet changes \( \rightarrow F_{\text{net}} \) to left.
Magnetic Dipole and How Magnets Work

A solenoid and a magnet orients themselves with axis parallel to field.

Electron analogy: “spinning ball of charge” → circulation of charge around spin axis similar to current loop → electron has net magnetic moment.

- In Fe atom, large number of electron magnetic moments align to each other → non-zero atomic magnetic moment.

- In unmagnetized Fe piece → no overall alignment of μ of atoms → total μ = 0.

- Iron bar magnet → magnetic moments of many atoms are parallel → total μ ≠ 0.

- A bar magnet tends to align to B, so that line from S to N is in direction of B.
- South and North poles represent tail and head of magnet’s dipole moment, \( \mu \).

How can a magnet attract an unmagnetized Fe object?

1) Atomic magnetic moments of Fe try to align to \( B \) of bar magnet \( \rightarrow \) Fe acquires net magnetic dipole moment // \( B \).

2) Non-Uniform \( B \) attracts magnetic dipole.

The magnetic dipole produced on nail is equivalent to current loop (I direction right hand rule) \( \rightarrow \) net magnetic force on nail is attractive (a) or (b) \( \rightarrow \) unmagnetized Fe object is attracted to either pole of magnet.