Chapter 22 – Gauss Law

- Charge and Electric flux
- Electric Flux Calculations
- Gauss’s Law and applications
- Charges on Conductors

Child acquires electric charge by touching a charged metal sphere. Electrons coat each individual hair fiber and then repel each other.
1. Charge and Electric Flux

- A charge distribution produces an electric field ($\vec{E}$), and $\vec{E}$ exerts a force on a test charge ($q_0$). By moving $q_0$ around a closed box that contains the charge distribution and measuring $\vec{F}$ one can make a 3D map of $\vec{E} = \vec{F}/q_0$ outside the box. From that map, we can obtain the value of $q$ inside box.

- If we construct a boundary around a charge, we can think of the flow coming out from the charge like water through a screen surrounding a sprinkler.
(a) Positive charge inside box, outward flux $\vec{E}$

(b) Positive charges inside box, outward flux

(c) Negative charge inside box, inward flux $\vec{E}$

(d) Negative charges inside box, inward flux
Electric Flux and Enclosed Charge:

(a) No charge inside box, zero flux
(b) Zero net charge inside box, inward flux cancels outward flux.
(c) No charge inside box, inward flux cancels outward flux.

- There is a connection between sign of net charge enclosed by a closed surface and the direction of electric flux through surface (inward for \(-q\), outward for \(+q\)).

- There is a connection between magnitude of net enclosed charge and strength of net “flow” of \(\vec{E}\).

- The net electric flux through the surface of a box is directly proportional to the magnitude of the net charge enclosed by the box.
(a) A box containing a charge

\[ E \sim \frac{1}{r^2} \]

\[ r_1 = \text{distance of } q \text{ to surface of box}_1. \]

\[ r_2 = 2r_1 = \text{distance of } q \text{ to surface of box}_2. \]

(b) Doubling the enclosed charge doubles the flux.

\[ 2q \rightarrow 2E \]

(c) Doubling the box dimensions does not change the flux.

\[ q \rightarrow E \]

In (c), \( E_2 = E_1/4 \), since \( r_2 = 2r_1 \), but \( A_2 = 4A_1 \) \( \rightarrow \) net flux constant.

- Electric flux = (perpendicular component of \( E \)) \( \cdot \) (area of box face)

- The net electric flux due to a point charge inside a box is independent of box’s size, only depends on net amount of charge enclosed.

- Charges outside the surface do not give net electric flux through surface.
2. Calculating Electric Flux

Flux Fluid Analogy:

- If we considered flux through a rectangle, the flux will change as the rectangle changes orientation to the flow.

\[
\frac{dV}{dt} = vA \quad (v \perp A)
\]

\[
dV/dt = \text{volume flow rate}
\]
\[
v = \text{flow speed}
\]

\[
\frac{dV}{dt} = vA_{\perp} = vA \cos \varphi = v_{\perp} A = \vec{v} \cdot \vec{A}
\]
Flux of a Uniform Electric Field:

Units: N m²/C

\[
\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi = E \perp A
\]

We can define a vector area: \( \vec{A} = A \cdot \hat{n} \) with \( n \) being a unit vector \( \perp A \).

Flux of a Non-uniform Electric Field:

\[
\Phi_E = \int E \cos \phi \, dA = \int E_\perp \, dA = \int \vec{E} \cdot d\vec{A}
\]
3. Gauss’s Law

- The total electric flux through any closed surface is proportional to the total electric charge inside the surface.

Point Charge Inside a Spherical Surface:

\[ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} \]

\[ \vec{E} \parallel dA \text{ at each point} \]

\[ \Phi_E = E \cdot A = \frac{1}{4\pi\varepsilon_0} \frac{q}{R^2} \left(4\pi R^2\right) = \frac{q}{\varepsilon_0} \]

- The flux is independent of the radius \( R \) of the sphere.
Point Charge Inside a Nonspherical Surface:

- Divide irregular surface into \( dA \) elements, compute electric flux for each \( (E \, dA \, \cos \phi) \) and sum results by integrating.

- Each \( dA \) projects onto a spherical surface element \( \rightarrow \) total electric flux through irregular surface = flux through sphere.

\[
\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}
\]

Integral through a closed surface

Valid for + / - \( q \)

If enclosed \( q = 0 \) \( \rightarrow \Phi_E = 0 \)
Point charge outside a closed surface that encloses no charge. If an electric field line enters the surface at one point it must leave at another.

- Electric field lines can begin or end inside a region of space only when there is a charge in that region.

**General form of Gauss’s law:**

\[
\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E \cos \varphi \, dA = \oint E_n \, dA = \frac{Q_{\text{encl}}}{\varepsilon_0}
\]

**Example:** Spherical Gaussian surface around \(-q\) (negative inward flux)

\[
\Phi_E = \oint E_n \, dA = \oint \left( \frac{-q}{4\pi\varepsilon_0 r^2} \right) dA = \left( \frac{-q}{4\pi\varepsilon_0 r^2} \right) 4\pi r^2 = \frac{-q}{\varepsilon_0}
\]
4. Applications of Gauss’s Law

- When excess charge (charges other than ions/e⁻ making up a neutral conductor) is placed on a solid conductor and is at rest, it resides entirely on the surface, not in the interior of the material.

- Electrostatic condition (charges at rest) $\Rightarrow E = 0$ inside material of conductor, otherwise excess charges will move.
5. Charges on Conductors

- Excess charge only on surface.

- Cavity inside conductor with $q = 0 \rightarrow E = 0$ inside conductor, net charge on surface of cavity = 0.

- Cavity inside conductor with $+q \rightarrow E = 0$ inside conductor, $-q$ charge on surface of cavity (drawn there by $+q$). Total charge inside conductor = 0 $\rightarrow +q$ on outer surface (in addition to original $q_c$).

Because $\vec{E} = 0$ at all points within the conductor, the electric field at all points on the Gaussian surface must be zero.
Field at the surface of a conductor:

\[ E_\perp A = \frac{\sigma A}{\varepsilon_0} \]

and

\[ E_\perp = \frac{\sigma}{\varepsilon_0} \]

Field outside a charged conductor is perpendicular to surface.

\[ \sigma = \frac{q}{A} \Rightarrow q = \sigma A \]
Ex. 22.2