

# Terahertz/Millimeter Wave Characterizations of Soils for Mine Detection: Transmission and Scattering

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Received: 11 April 2008 / Accepted: 19 May 2008 /  
Published online: 3 June 2008  
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**Abstract** The detection of anti-personnel mines depends on the soil transmission and scattering at a given wavelength. Transmission spectra were measured over the range 90–4200 GHz for 19 soil samples that span a number of soil orders that have extensive worldwide distribution using a vector network analyzer (90–140 GHz) and a Fourier spectrometer (120–4200 GHz). Transmission drops to nearly zero for wavelengths shorter than the characteristic particle size of the sample as a consequence of scattering. This interpretation is supported by a fit to a standard scattering model with physically reasonable fitting parameters. Transmission spectra were also measured for various liquids (90–600 GHz) for possible index matching. These liquids were mixed with the soil sample and were found to reduce scattering and increase transmission through the soil at higher frequencies. This work is relevant to mine detection using terahertz and millimeter wave radiation for high resolution images through the soil.

**Keywords** Millimeter wave measurements · Scattering · Soil measurements · Spectroscopy · Terahertz radiation

## 1 Introduction

The ability to detect and image buried objects depends on the soil transmission and scattering at a given wavelength. Ground Penetrating Radars operate in the MHz to low GHz range (10's of centimeters to meters wavelength) to maximize penetration through the soil up to tens of meters depending on soil conditions [1]. With the increase of penetration at longer wavelengths, the ability to detect and resolve the buried objects decreases. Operating at shorter wavelengths will increase the resolution of the system, since the limit

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of resolution is proportional to wavelength. Anti-personnel mines are buried in the first few centimeters of the soil and the ability to discriminate the mine from other objects is necessary to reduce the false alarm rate. An investigation of the transmission of soils at frequencies in the millimeterwave (mmW) and terahertz (THz) regions is needed to determine the shortest wavelength range where the penetration of the soil is still high. Soils with particle sizes larger than the wavelength are expected to have low transmittance due to scattering [2]. Reflections occur at the boundaries between the soil particles and air causing the light to scatter. Similarly, J. Pearce *et al* [3–4] found the scattering due to a collection of Teflon spheres the weakest at longer wavelength in the THz region. The reflections at the soil particle/air boundaries can be reduced by replacing the air surrounding the soil with an index matching fluid. An index matching fluid with low absorption in the THz/mmW ranges will reduce scattering and increase transmission through the soil. This paper reports the transmittance of different soils across the THz/mmW range, the transmittance of soil mixed with index matching liquids, and a model describing the attenuation due to scattering of the soil grains. Fit of the spectra to a standard scattering model with physically reasonable fitting parameters confirms that the gross characteristics are in all cases due to scattering.

## 2 Experimental methods

Transmission spectra were measured for soils in the 90–4200 GHz range. A Bomem DA8 Fourier Transform Infrared (FTIR) spectrometer equipped with a Hg arc lamp, mylar pellicle beamsplitter, and Infrared Labs Si bolometer [2] collected transmission spectra from 120 to 4200 GHz. Spectra consisted of 100 co-added scans at a spectral resolution of 60 GHz. The temperature during the scans was 296 K. The entire beam path was evacuated to a pressure of 0.2 Torr. Immediately after the recording of the sample spectrum, a reference spectrum of the empty sample holder was recorded. The transmittance spectrum was obtained as the ratio of the sample- to reference-spectrum. Measurements were similarly collected in the 120–450 GHz range using a 100  $\mu\text{m}$  mylar beamsplitter at 60 GHz resolution with a 1.8 K Si bolometer, in the 300–1500 GHz range measured at 60 GHz resolution using a 50  $\mu\text{m}$  mylar beamsplitter and a 4 K Si bolometer and in the range 1200–4200 GHz using a 12  $\mu\text{m}$  mylar beamsplitter and the 4 K Si bolometer. The transmission above 1500 GHz was found to be very low for all soil samples studied.

An Anritsu ME7808A vector network analyzer (VNA) equipped with 16 degree horn antennas [2] collected transmission spectra from 90–140 GHz at 0.125 GHz resolution. The radiation was collimated onto the sample using high density polyethylene (HDPE) lenses. The temperature during the scans was 296 K. The samples were measured with 100 averaged scans each at atmospheric pressure. Immediately after the recording of the sample spectrum, a reference spectrum of the empty sample holder was recorded. The transmittance spectrum was obtained from the ratio of the sample- to reference-spectrum, removing the absorption features due to the sample holder and the atmosphere. This high resolution data was smoothed using adjacent point averaging to reduce oscillations in the baseline due to etalon resonances in the sample cell.

Two sample cells were used for transmittance measurements. One sample was a HDPE cell that provided 1 mm path length through the soil. The HDPE flanges were wedged to inhibit etalon resonances. The second cell was made of polystyrene (PS) with 2.2 cm path length through the soil. This cell was oriented horizontally in front of the down looking spectrometer detector to allow the soil to lay flat in the cell. This helped to prevent the soil from settling to the bottom of the holder in the upright position. The air gap created by the

settled soil would cause the transmittance to increase giving an incorrect measurement if the cell was not completely filled. All of the dry soil samples were measured in the PS sample cell, along with all of the VNA measurements. The liquid samples were measured in the FTIR using the wedged HDPE sample cell with the o-ring.

The mmW and far-infrared transmission spectra were collected over the range 90–4200 GHz for 12 soil samples obtained from National Resource Conservation Service (NRCS). These samples include surface and subsurface soil from Wakeland, Doakum, Montevallo, Angola, Minvale, and Sibley. These samples were pulverized with a mortar and pestle and sieved for maximum particle size of 250  $\mu\text{m}$ . The soil samples were maintained in a laboratory environment with ~40% relative humidity.

Geologists recognize 12 distinct soil orders [5]. The NRCS samples belong to six of these. The orders are differentiated by chemical and other characteristics of their horizons. Our “Surface” samples are A-horizon, where much of the original rock structure has been obliterated, where soluble minerals have been leached out, and where humified organic matter has been accumulated. Anti-personnel mines would exclusively be located within A-horizons. Our “Subsurface” samples correspond to upper B-horizon, which occurs below the A-horizon. Though, much of the original rock structure is also obliterated, the B-horizon is compositionally distinct from the A-horizon. Anti-tank mines could conceivably be buried in B-horizon soils.

The “Sibley” samples are Mollisols, which usually have thick dark surface horizons and are characteristic of steppes, most extensively at mid-latitudes [5]. The “Minvale” samples are representative of the Ultisol order and are often acid forest soils in older stable landscapes [5]. The soil samples “Wakeland” are representative of the Entisol order found in steep rocky settings, which is characterized by the absence of distinct subsurface diagnostic horizons within 1 m of the surface [5]. The soil samples “Doakum” are representative of the Aridisol order, which is characteristic of arid climates with little or no leaching [5]. The “Montevallo” samples are representative of the Inceptisol order found on fairly steep slopes and have weakly developed subsurface horizons [5]. The samples “Angola” are representative of the Alfisol order found in temperate, humid, and sub-humid regions and are moderately leached with subsurface clay accumulation [5]. The six remaining soil orders were not studied at this time due to their unusual locations or low abundance on earth.

Millimeter wave and far-infrared transmission spectra were collected (90–4200 GHz) for 6 different soil samples from Night Vision and Electronic Sensors Directorate Mine Lane Facility (NVESD) Fort Belvoir, VA [6] using the FTIR and VNA. These samples contained a wide range of particle sizes, including rocks of up to centimeter dimensions. Data were collected for these samples in their original mixed state, after sieving for a maximum particle size of 0.5 mm, and of the coarse remainder from the sieving process. Large rocks from the coarse remainder that did not fit in the sample holder were excluded from the measurement to ensure proper filling of the holder.

The six soil samples obtained from the NVESD were Magnetite (Lane 1), Loam (Lane 2), Crusher Run Gravel (Lane 3), Bank Run Gravel (Lane 4), Virginia Clay (Lane 5), and White Beach Sand (Lane 6). Magnetite is a poorly graded gray sand containing no sizable rocks. It contains 74% Quartz, 20% Magnetite, 4.1% Ilmentite, 1.1% Plagioclase, and 1% total Phyllosilicates [6]. Magnetite has a higher amount of iron oxide ( $\text{Fe}_2\text{O}_3$ ) than the other soils. Loam is a low-plasticity brown clay containing some rocks a few mm wide. It contains 89% Quartz, 6.1% total Phyllosilicates, 2.6% K-Feldspar, and 1.2% Plagioclase [6]. Crusher Run Gravel is a brown gravelly silty sand containing many rocks from 0.5 to 4 cm wide. It contains 46% Plagioclase, 22% quartz, 18% total Phyllosilicates, 9.2%

K-Feldspar, 2.8% Dolomite, and 1.1% Calcite [6]. Bank Run Gravel is a gravelly low-plasticity reddish brown clay containing many rocks 0.1 to 5 cm wide. It contains 73% Quartz, 16% total Phyllosilicates, 7.2% Goethite, 2.7% K-Feldspar, and 1.5% Plagioclase [6]. Virginia Clay is a reddish brown gravelly silty sand containing many rocks 0.1 to 2.5 cm wide. It contains 89% Quartz, 8.4% total Phyllosilicates, 1.9% K-Feldspar, and 0.5% Plagioclase [6]. White Beach Sand is a light gray poorly graded sand containing small traces of rocks a few mm wide. It contains 98% Quartz and 1.8% total Phyllosilicates [6].

One sample was sourced locally from Orlando, FL. Florida sand is mostly composed of quartz and is free from rocks or debris and has uniform particle sizes of  $\sim 250$   $\mu\text{m}$ . Transmission spectra were taken for the Florida sand from 90–140 GHz using the VNA and from 120–1500 GHz using the FTIR. Transmission spectra were collected for some organic materials using the vector network analyzer in the range 90–140 GHz. Materials studied include wet and dry sod, organic debris (leaves, roots, sticks, *etc*) and water.

Due to the high scattering loss of soil at high frequencies, various liquids were tested to determine potential candidates for index matching liquids to fill in the air gaps around the soil producing higher transmission. The liquids measured were WD-40, Nujol mineral oil, vegetable oil, extra virgin olive oil, canola oil, Fluorolube, and silicone oil for these index matching experiments from 90–600 GHz - using the VNA from 90–140 GHz and the FTIR from 120–600 GHz. The Florida sand soil samples were fully saturated with the index matching liquids and placed into the sample cells. Transmission spectra were taken for these index matching liquids mixed with the Florida sand from 90–600 GHz using the VNA and FTIR systems.

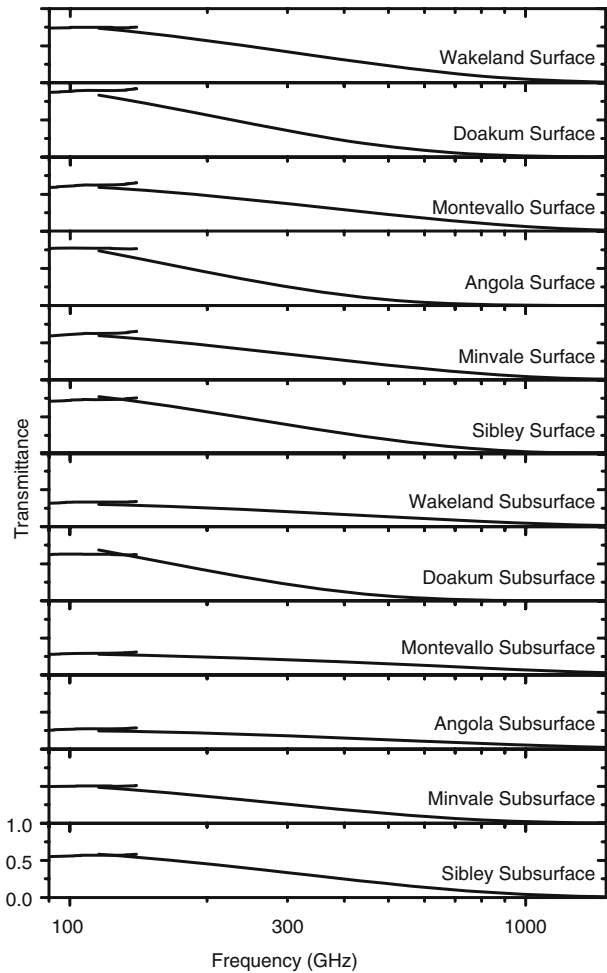
### 3 Transmission results

Transmittance spectra of the 12 NRCS soil samples for the range 90–1500 GHz are shown in Fig. 1. Each soil sample was measured from 90–140 GHz with the VNA and from 120–1500 GHz with the FTIR. The transmittance spectra from the VNA and FTIR systems were combined together to show the full 90–1500 GHz range with an overlap from 120–140 GHz. The absolute transmittance levels measured by the two instruments generally agree within 7.5% in this overlap range. The high frequency range is dominated by a roll-off, which we interpret as due to scattering. The characteristic cutoff wavelength is similar to the characteristic particle size, as determined using an optical microscope. For instance, the surface samples tend to zero transmittance for frequencies in the range 900–1200 GHz, which corresponds to wavelengths in the range 333–250  $\mu\text{m}$ . This correlates well with the 250  $\mu\text{m}$  maximum particle size obtained by sieving. Soil transmittance is seen to increase at lower frequencies because decreased scattering allows more signal to be collected by the bolometer. This occurs when the wavelength is longer than the particle size. At the low frequency end of the measured band (90 GHz), the transmittance ranges from 0.2 to 0.9.

The subsurface specimens tend to have a lower cutoff frequency and smaller transmittance than the corresponding surface samples. This may be due to small differences in the particle size distribution but it may also be due to the difference in chemical composition. We note that [7] near surface soils tend to be leached of soluble or quickly altered minerals. Surface soils are rich in insoluble quartz, clay minerals, and iron oxide alteration products, but they still are poor in calcium carbonate.

Transmittance spectra in the 90–140 GHz range for NVESD samples Loam, Virginia Clay, Bank Run Gravel, Crusher Run Gravel, are presented in Figs. 2 and 3 respectively. The sieved samples all have transmittance in the range 30% to 60%, which is higher than

**Fig. 1** Transmittance spectra for 1 mm thick surface and subsurface soil samples using a VNA from 90–140 GHz and a FTIR from 120–1500 GHz.

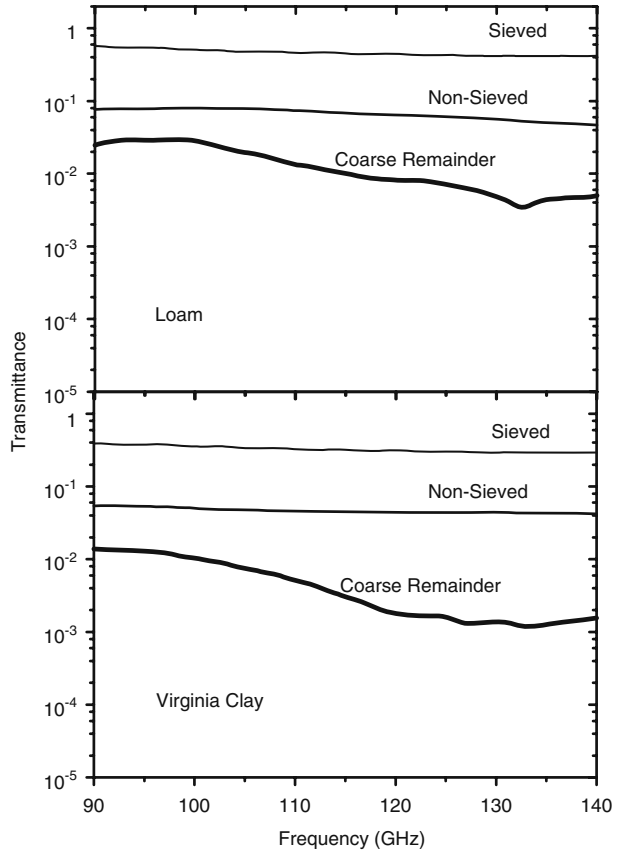


for the non-sieved samples by  $\sim 1\text{--}2$  orders of magnitude. The coarse after-sieving remainder has transmittance even lower, by at least an order of magnitude. These results show that large particle sizes in the range 0.5 mm to a few centimeters cause strong scattering of millimeter waves (2–3 mm wavelength). This observation is in agreement with that already made for the NRCS samples, namely that transmittance is high when the wavelength exceeds the soil particle size.

Figure 4 presents spectra of the White Beach Sand and Magnetite samples. Sand tends naturally to have a uniform particle size distribution. When isolated, the coarse remainder from the White Beach Sand sample has transmittance of 45–65%; however the abundance of coarse material in the non-sieved sample is insufficient to lower that samples transmittance much below 90%. We note that the 2–3 mm wavelengths are significantly greater than the particle size in this case, in agreement with our previous observations.

The magnetite results do not follow the trend of transmittance vs. particle size noted earlier. Non-sieved, sieved, and coarse remainder all have transmittance below 0.01%. The latter two segregated samples both have higher transmittance than the non-sieved sample.

**Fig. 2** Transmittance spectra for 2.2 cm thick Loam (top) and Virginia Clay (bottom) soil samples.



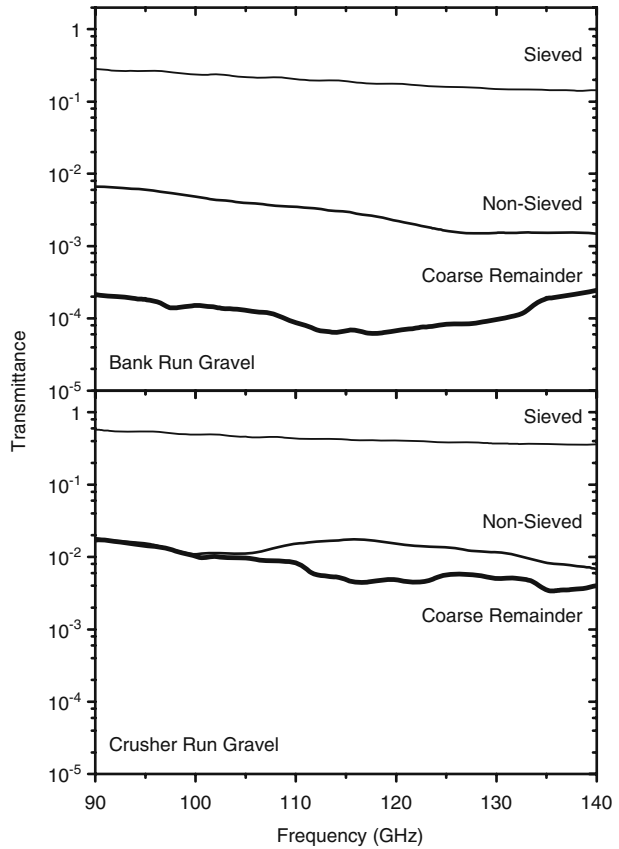
The magnetite sample morphology was very different from that of the other NVESD samples. It contained a light colored sandy component of uniform sub-mm size distribution and a dark magnetic component that tended to stick together in clumps. The non-sieved sample contained few large particles; the fraction that passed through the mesh openings during sieving contained both the light and dark components.

The transmittance of some organic materials in the 90–140 GHz frequency range is presented in Fig. 5. Organic matter is a strong source of attenuation possibly because of effective scattering by relatively large particle sizes. A spectrum of water is also presented in Fig. 5, and water is found to be a strong attenuator, which explains the relative transmittance of dry and wet sod samples. The dry sod has a slightly higher transmittance than the wet sod. Note that both wet and dry sod samples have lower transmittance than plain water, so that scattering by inhomogeneities in the organic samples is apparently more important than absorption by moisture.

#### 4 Index matching liquids

Transmittance spectra of some liquid samples over the range 90–600 GHz is shown in Fig. 6. The transmittance spectra from the VNA and FTIR systems were combined together

**Fig. 3** Transmittance spectra for 2.2 cm thick Bank Run Gravel (top) and Crusher Run Gravel (bottom) soil samples.

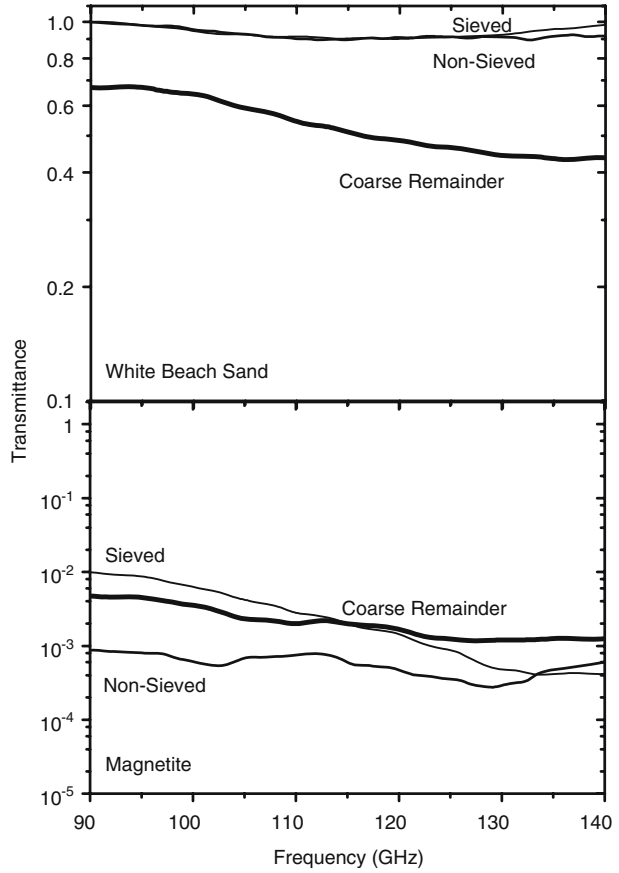


to show the full 90–600 GHz range with an overlap from 120–140 GHz. The maximum transmittances of the potential index matching liquids have a range from 0.75 to almost 1. The path length through the sample is 1 mm. Some of the liquids have good transmittance and hence are potential candidates for index matching.

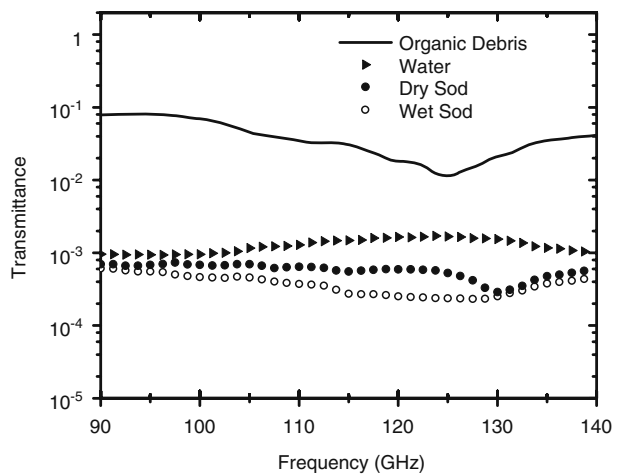
Several of the liquids were used to saturate one of the soil samples (Florida Sand) for the purpose of index matching. Transmittance spectra of the Florida sand sample for the range 90–1500 GHz is shown as the thin line in Fig. 7. The transmittance spectra from the VNA and FTIR systems were combined together to show the full 90–1500 GHz range with an overlap from 120–140 GHz. The high frequency range is dominated by a roll-off due to scattering. Figure 7 shows the transmittance spectra from 90–600 GHz of the saturated soil (thick line) compared to the transmittance of the dry soil (thin line). The transmittance spectra from the VNA and FTIR systems were combined together to show the full 90–600 GHz range with an overlap from 120–140 GHz. The Fluorolube sample was not used for index matching due to the high viscosity of the liquid and its inability to seep into the soil.

At high frequencies, the addition of index matching fluid reduces scattering and increases transmission. Even though index matching has increased transmission, there is still a roll-off at high frequency. This is because the liquids studied are not exact matches to the index of the particles ( $n_{\text{quartz}} \sim 2.113$  [8] and  $n_{\text{oil}} \sim 1.4679$  [9] at 900 GHz). Proper index

**Fig. 4** Transmittance spectra for 2.2 cm thick White Beach Sand (top) and Magnetite (bottom) soil samples.

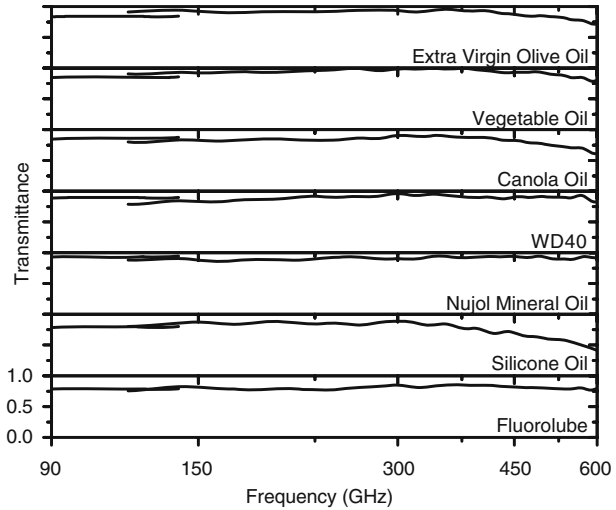


**Fig. 5** Transmittance spectra for 2.2 cm thick organic samples.





**Fig. 6** Transmittance spectra of 1 mm path length of potential index matching liquids in the 90–140 GHz range using a VNA and in the 120–600 GHz range using a FTIR.



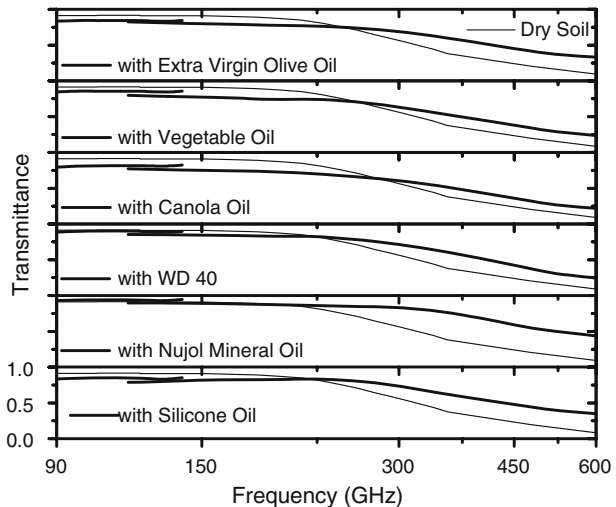
matching could potentially push the roll-off frequency higher, extending the useful transmission range further into the THz, but many of the liquids available that could provide a perfect index match are highly toxic and would not be safe to mix with the soil.

### 5 Soil scattering

#### A. Theory

As the electromagnetic radiation travels through the soil, energy is transmitted, absorbed, and scattered. To quantify the attenuation due to scattering [10], we consider that the

**Fig. 7** Transmittance spectra of 1 mm path length of several index matching liquids mixed with the Florida sand soil sample in the 90–140 GHz range using the VNA and in the 120–600 GHz range using the FTIR.



incident beam with power per unit area  $P_I$  encounters a volume of scatterers characterized by a radius,  $a$ , and a number per unit volume,  $N_s$ . The incident beam encounters  $N_s \, dL$  scatterers per unit area over a path length  $dL$ . The transmitted power is defined as,

$$P = P_I - dP \quad (1)$$

where  $dP$  is the power lost per unit area by the scatterers in the path length  $dL$ . The soil particles are assumed spherical with scattering cross sectional area,  $A_s$ . The scattered power is expressed as

$$dP = N_s \cdot A_s \cdot P_I \cdot dL \quad (2)$$

which means the power will decline exponentially as the length through the material increases according to

$$P = P_I \cdot e^{-N_s \cdot A_s \cdot L} = P_I \cdot e^{-\alpha_{sct} \cdot L} \quad (3)$$

where  $\alpha_{sct}$  is the power scattering attenuation coefficient.

The cross sectional area using the Rayleigh approximation [11] is given as

$$A_s = \frac{8\pi}{3} \left( \frac{2\pi n_{med} \nu}{c} \right)^4 a^6 \left( \frac{m_s^2 - 1}{m_s^2 + 2} \right)^2 \quad (4)$$

where  $\nu$  is the frequency,  $c$  is the speed of light,  $a$  is the particle radius, and  $m_s$  is the ratio of the refractive index of the spherical particle ( $n_{sph}$ ) to that of the surrounding medium ( $n_{med}$ ). Values for  $N_s$  and  $a$  were determined by fitting the model to the experimental data. Initial estimate for  $N_s$  was determined from packing density  $\rho$  according to

$$N_s = \frac{3\rho}{4\pi a^3}, \quad (5)$$

where  $\rho$  should have values between 0.7405 for hexagonal close packing and 0.3401 for a tetrahedral lattice [12]. Initial estimate for  $a$  is determined using an optical microscope.

The total attenuation from the soil is a combination of the scattering attenuation  $\alpha_{sct}$  plus the material absorption loss  $\alpha_{abs}$ .

$$\alpha_{total} = \alpha_{sct} + \alpha_{abs} \quad (6)$$

Therefore, the total transmitted power is given as

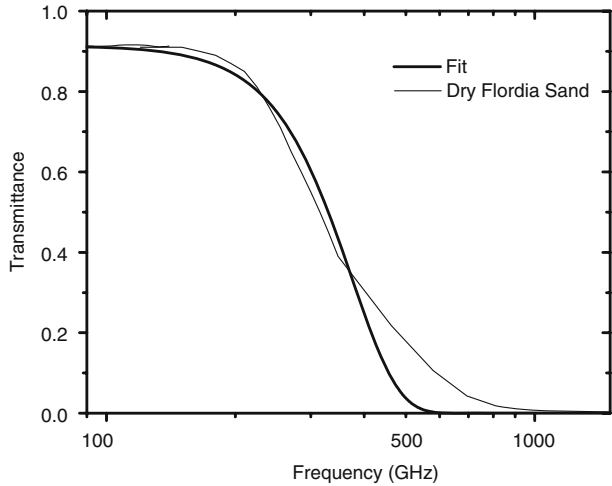
$$P = P_I \cdot e^{-\alpha_{total} \cdot L} \quad (7)$$

## B. Results

The model was fit to the Florida sand spectrum. Florida sand is primarily quartz with index 2.113 and absorption coefficient  $0.3 \text{ cm}^{-1}$  at 900 GHz [8]. The initial  $a$  value was set to 125  $\mu\text{m}$ . The path length ( $L$ ) was 3 mm. A value of 0.3401 for  $\alpha$  gave an initial  $N_s$  value of  $4.15 \times 10^{10} \text{ m}^{-3}$ . The best fit is compared to the data in Fig. 8, with  $a=143 \text{ }\mu\text{m}$  and  $N_s=4.3 \times 10^9 \text{ m}^{-3}$  and an  $R^2$  value of 0.9898.

The low frequency portion of the curve (90–150 GHz) coincides well with the attenuation due to absorption. As the frequency increases, the attenuation due to scattering increases and the transmittance starts to decrease. The theoretical roll-off due to scattering has the same trend as the measured data, but the tail end of the roll-off has a slightly different slope. This could be due to several factors. First, our theoretical model assumes spherical grains of sand composed of quartz, but the grains are not entirely spherical.

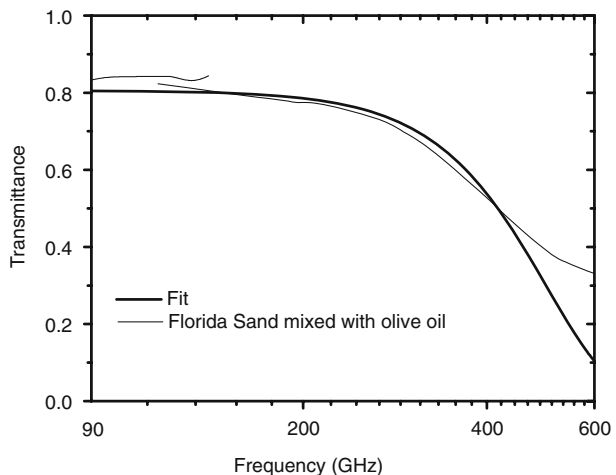
**Fig. 8** Theoretical and measured transmittance of 3 mm of Florida Sand.



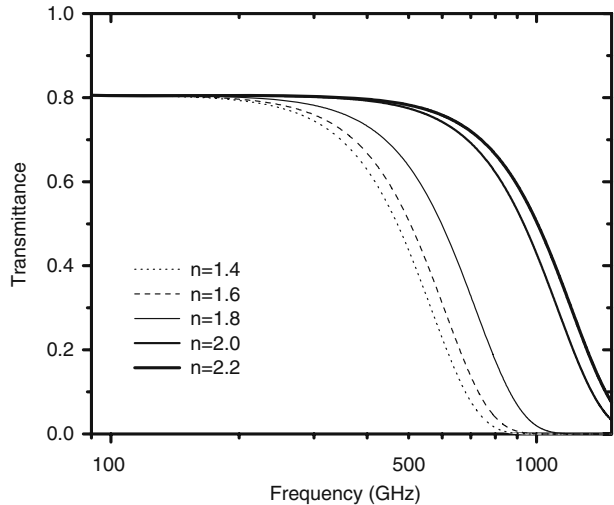
Second, the sample is not entirely quartz, but also contains other small amounts of organic material. Third, the Rayleigh scattering formula is being applied in a regime where wavelength is comparable to, and even less than, the particle size, but this contradicts the assumptions of its derivation, namely that each particle is in a uniform field. Forth, the fit assumes no dispersion, namely that  $m_s$  is independent of wavelength, though this assumption is doubtful in a region with phonon absorption. Nevertheless, the close agreement of our model to the experimental data with physically reasonable fitting parameters shows that the scattering from the soil grains explains the high frequency losses.

The model can also be applied to the index matching liquids data by changing  $m_s$ . An absorption coefficient for the liquid, determined for the Fig. 6 data, must also be added to the model. The data and fit for Florida sand mixed with olive oil are shown in Fig. 9. The

**Fig. 9** Theoretical and measured transmittance for Florida sand index matched with olive oil.



**Fig. 10** Theoretical transmittance of Florida sand mixed with index matching liquids from 1.4 to 2.2.



fit gives  $a = 135 \mu\text{m}$  and  $N_s = 5.505 \times 10^9 \text{ m}^{-3}$ , which are again reasonable values. The  $R^2$  value is 0.9367. The roll-off due to scattering occurs in the model at a similar frequency to the experimental data. The agreement between theory and experiment further supports a scattering origin for high frequency loss.

The use of higher index matching liquids can increase the transmittance through the soil to 1 THz. Figure 10 shows the theoretical transmittance through the soil as the index of refraction of the liquid increases from 1.4 to 2.2 with similar absorption to olive oil. The 50% transmission level changes from 315 GHz for dry sand to 1000 GHz for sand mixed with index matching liquid with an index of 2.2. The increase in transmittance at the higher frequencies due to the index matching liquids would allow for higher resolution images to be taken without the loss of penetration depth. However, a commercial low cost, high transmission, non-toxic, high index liquid that can easily seep into the ground was not found.

## 6 Conclusion

Transmission was measured over the 90–1500 GHz range for soil samples found around the world. The high frequency range is dominated by a roll-off due to scattering, which correlates with particle size. Samples with particles larger than the wavelength have low transmittance due to scattering. Organic materials and water are a strong source of attenuation. High transmission was noted through some soils types when the particles are sufficiently fine, showing promise for mmW/THz landmine detection, in regions of the world where such soils are found. Saturation with index matching liquids reduces scattering and increases transmission at higher frequencies. A transparent low toxicity fluid with index of refraction closer to 2 would have great promise for use with terahertz imaging for mine detection.

**Acknowledgment** This work was supported by a research contract between Northrop Grumman Corporation and Florida Photonics Center of Excellence. The authors would like to thank Ian McMichael of the US Army Night Vision Labs and Phil Jardine and Melanie Stewart from Oak Ridge National Labs for supplying some of the soil samples tested in this study.

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