MEMS clocking-cantilever thermal detector

Evan Smith[1], Javaneh Boroumand[1], Imen Rezadad[1], Pedro Figueiredo[1], Janardan Nath[1], Deep Panjwani[1], R. E. Peale[1], Oliver Edwards[2],
[1]Department of Physics, University of Central Florida, Orlando FL 32816
[2]Zybertec, Inc., 32789

ABSTRACT

We present performance calculations for a MEMS cantilever device for sensing heat input from convection or radiation. The cantilever deflects upwards under an electrostatic repulsive force from an applied periodic saw-tooth bias voltage, and returns to a null position as the bias decreases. Heat absorbed during the cycle causes the cantilever to deflect downwards, thus decreasing the time to return to the null position. In these calculations, the total deflection with respect to absorbed heat is determined and is described as a function of time. We present estimates of responsivity and noise.

Keywords: MEMS, thermal, bimorph, infrared

1. INTRODUCTION

This paper estimates the sensitivity and noise for a MEMS cantilever that transduces heat input into a measurement of time. The design is presented schematically in Fig. 1. For modeling, we consider a simplified geometry consisting of a simple rectangular cantilever of length L and width w, comprised of a metal on top of a material of small thermal expansion coefficient. A cantilever is supported on one end at a height z_0. In equilibrium the free end is in contact with the substrate surface, where it makes electrical contact with a surface tip pad. The top metal is electrically continuous with a surface metal of the same dimensions. A third co-dimensional metal plate is buried a distance d below the surface within the insulating substrate. For definiteness, we consider just two materials, gold and silicon dioxide, and we will assume the following nominal dimensions: $L = w = 100$ µm, $z_0 = 2$ µm, $d = 0.1$ µm. The thickness of the cantilever will be $t = 0.5$ µm.

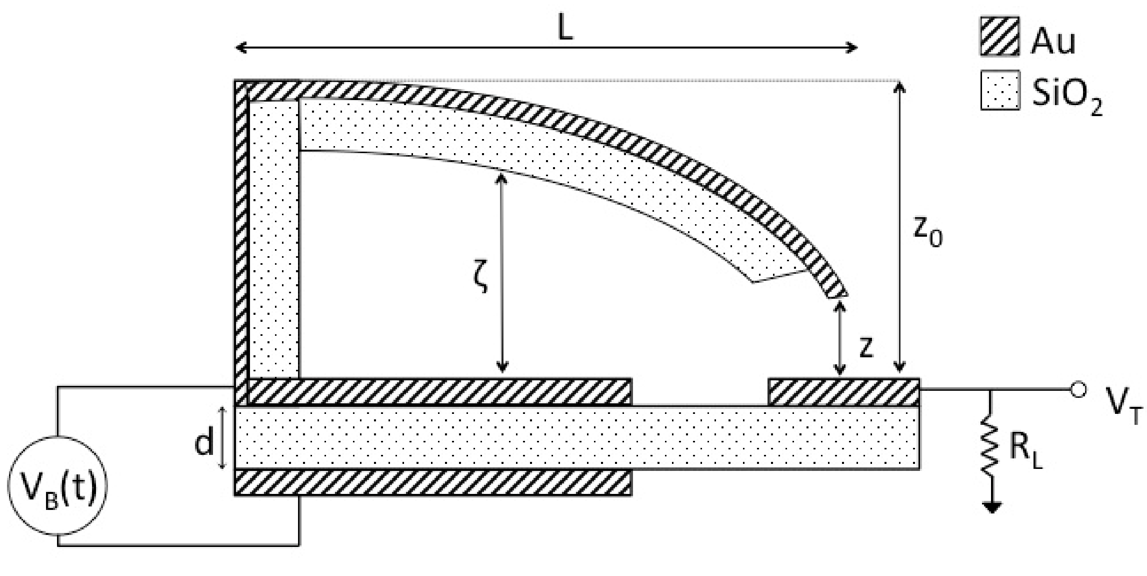

Fig. 1. Schematic of clocking cantilever heat sensor. A periodic saw-tooth bias V_B is applied between a buried electrical plate and a surface plate which is electrically connected to the flexible bimorph cantilever, which is comprised of gold and oxide. When the tip contacts the tip pad, the bias voltage appears at the output V_T.

Fig. 2 presents a schematic of the applied and measured voltage waveforms. When the cantilever is biased with V_B as in Fig. 1, a repulsive electrostatic force lifts the cantilever from the surface and breaks the tip contact. As V_B is ramped down, the time τ taken for the tip to return to the null position is measured via a surface contact. When heat is absorbed by the cantilever, thermal deformation of the bimorph causes the tip to return sooner by $\Delta\tau$, giving a temporal measurement of the heat flux.

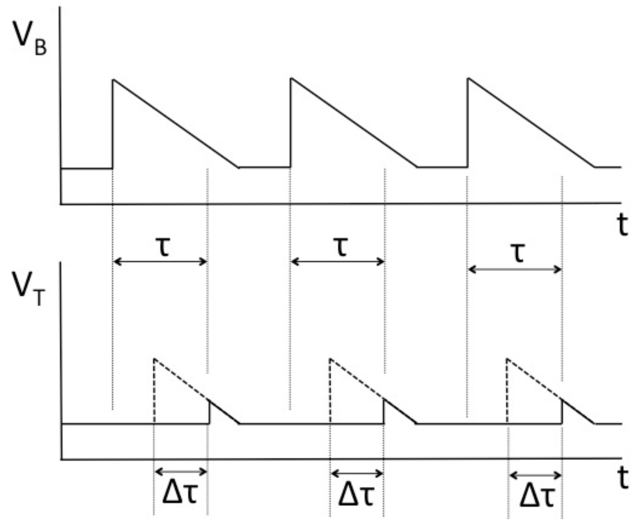

Fig. 2. Clocking and sensing waveforms. A periodic ramp voltage is applied to each pixel at the frame rate. Electrostatic repulsion breaks the tip contact, so that V_T is initially zero. As V_B decreases, the tip re-contacts after a time τ, when the remainder of the waveform appears at V_T. Downward deflection of the thermal bimorph shortens τ by $\Delta\tau$, which is a measure of the amount of absorbed heat.

2. RESPONSIVITY

We first consider the mechanical behavior of the cantilever. Fig. 1 defines the coordinate system. The equilibrium tip position is $z = 0$. If the cantilever bends upward, there is a restoring force $\boldsymbol{F}_k = -k\, z\, \mathbf{e_z}$, where the last factor is the unit vector in the vertical direction. For a force concentrated at the center of the cantilever, k has the value[1] $\frac{24EI}{L^3}$, where E is Young's modulus and I is the areal moment of inertia equal to $\frac{wt^3}{12}$. Oxide and gold have about the same value of Young's modulus, which we will take to be 75 GPa, and I has the value 1 μm^4 for the assumed dimensions. Thus, $k = 2$ N/m, approximately.

Next we consider the force of electrostatic repulsion, which is found assuming parallel motion of the upper plate using energy methods for three parallel plate conductors held at constant potential[2]. The result is

$$F_E = +\frac{\epsilon_0 A}{2}\left[\frac{1}{\zeta^2} - \frac{1}{(\zeta+d)^2}\right] {V_B}^2\, \boldsymbol{e_z}, \tag{1}$$

where $\zeta \sim (z+z_0)/2$ is the height of the cantilever's midpoint. Setting $F_E = F_k$ gives $f(z) = b\, V_B^2$, where $f(z)$ is a quadratic function of z^2. Graphical solution determines $z(V_B)$ to be nearly linear over the range of interest, according to $z = 0.033\, V_B$. The units of the coefficient are μm/V. For the assumed dimensions, about 60 V are required to raise the tip to the height of the support.

Next we consider the effect of raising the cantilever temperature by ΔT_c, which bends it downward, so that $z(V_B, \Delta T_c) = 0.033\, V_B - B(\Delta T_c)$, where B($\Delta T_c$) is proportional to both ΔT_c and L^2 via a function of the thicknesses, Young's moduli, and expansion coefficients of metal and oxide[3-6]. Assume the thickness of gold to be 100 nm and the thickness of oxide to be 400 nm, and assuming their Young's moduli to be about equal, we find B(ΔT_c) = 0.1 ΔT_c, approximately. The units of the coefficient are μm/C. Thus, the height of the tip is given by

$$z(V_B, \Delta T_c) = 0.033\, V_B - 0.1(\Delta T_c)\ . \tag{2}$$

We assume that the ramped part of V_B is about half the period, that the frame rate is 30 Hz, and that the maximum value of V_B is 60 V. Then, the bias ramp is given by

$$V_B(t) = 60\,V\,(1 - (60\,s^{-1})\,t). \qquad (3)$$

Combining (2) and (3), solving for *t*, and setting z = 0 gives the time to contact (Fig. 2)

$$\tau = \frac{1}{60}(1 - 0.05\,\Delta T_c). \qquad (4)$$

When ΔT_c is zero, τ is half the clock period. When $\Delta T_c > 0$, the tip contacts sooner. The difference is $\Delta\tau = 0.8\times 10^{-3}\Delta T_c$, which is the measure of absorbed heat. A temperature rise of 1 mK, gives an easily measureable 800 ns time difference.

In thermal equilibrium the absorbed power is balanced by heat flow out of the cantilever to its environment. Under equilibrium conditions, the cantilever temperature difference caused by an absorbed power increase ΔP is

$$\Delta T_c = \frac{\Delta P}{G}, \qquad (5)$$

where *G* is the thermal conductance to the environment. Heat is lost via conduction through the air and support and by radiation. We will suppose that the device is sealed in an evacuated package with pressure less than 50 mTorr, which allows us to neglect the conduction by air. Radiative conductance is also much smaller than conductance through the solid support, which is given by

$$G = \frac{gtw}{L_s}, \qquad (6)$$

where g is the thermal conductivity. We assume the structural material of the support to be oxide with only a thin metal coating for continuity. For silicon dioxide, $g\sim 1$ W/m-K. For the assumed dimensions, we obtain $G = 25$ μW/K. Assuming a power input of 1 nW, the temperature rise under equilibrium conditions is 0.04 mK.

A larger temperature increase occurs when heating is adiabatic. This occurs if the thermal time constant significantly exceeds the clock period. High frame rate *f* and small *G* favor adiabatic heating of the cantilever with cooling only during null contact. Then

$$\Delta T_c \sim \frac{\Delta P}{Cf}, \qquad (7)$$

where *C* is the cantilever heat capacity. For an oxide cantilever of the assumed dimensions, the heat capacity is ~5 nJ/K. Thus, for 1 nW input, adiabatic heating give a temperature rise during one cycle of 7 mK.

The actual temperature rise will be between the values determined from Eqs. (5) and (7). Converting these to timing changes using Eq. (4), we find for 1 nW incident power, 33 ns < $\Delta\tau$ < 6 μs. The responsivity $\frac{\Delta t}{\Delta P}$ is therefore in the range 33 to 6000 sec/W.

3. NOISE

Timing noise is due to fluctuations in *z*, which are damped out each time the tip touches its pad. The steady-state rms amplitude provides an upper bound on the timing uncertainty. Fluctuations in *z* arise from fluctuations in the cantilever temperature due to their effect on the bimorph. The RMS temperature fluctuation is[7]

$$\sqrt{\overline{\Delta T^2}} = \frac{2T\sqrt{k_B \Delta f}}{\sqrt{G}}. \qquad (8)$$

For parameters assumed and the derived value for G, we obtain an rms temperature fluctuation of 3.4 μK. From Eq. (3), this translates to a timing uncertainty of 2 ns.

Thermomechanical noise (TM), or "thermally induced lever noise"[8] is caused by the exchange of mechanical and heat energy. It differs from σ_{Th} by being unrelated to the bimorph effect. TM noise treats the cantilever as a resonant oscillator, with spring constant k, quality factor Q, and resonant frequency ω_0. The mean square fluctuation of the amplitude of the cantilever is

$$\sqrt{\overline{\partial z^2}} = \frac{2\sqrt{k_B T \Delta f}}{\sqrt{k Q \omega_0}} \ . \quad (9)$$

Based on assumed dimensions and a density for oxide of 2.6 g/cm^3, the mass of the cantilever is 1.3 x 10^{-11} kg. With k = 2 N/m, the resonant frequency is 4 x 10^5 s^{-1}. Assuming a Q of 2000[3], the TM noise is 2 x 10^{-8} microns. Using Eqs. (2) and (3), this translates into a timing uncertainty of 0.15 ns, which is negligible in comparison to that due to temperature fluctuations.

A timing uncertainty of 2 ns for a power input of 1 nW corresponds to a percent uncertainty in the range 0.03 to 6 %, with the smaller number holding for adiabatic heating. Thus, to obtain optimum values for both sensitivity and signal-to-noise ratio, thermal conduction through the support should be minimized.

4. SUMMARY

This paper presented an analysis of responsivity and noise for a MEMS cantilever device that converts heat input into a time measurement. Assuming a simplified structural model composed of metal and oxide and reasonable dimensions, we determined a responsivity for adiabatic heating of 6000 sec/W and a timing uncertainty due to noise of 2 ns. For 1 nW power input, the signal to noise ratio would be 3000.

REFERENCES

[1] Young, W.C., [Roark's Formulas for Stress and Strain], McGraw-Hill, New York, 137-191 (1989).
[2] Landau, L.D. and Lifshitz, E.M., [Electrodynamics of Continuous Media, 2nd Edition], Elsevier, Burlington, MA, 29-33 (2008).
[3] Datskos, P.G., Lavrik, N.V. and Rajic, S., "Performance of Uncooled Microcantilever Thermal Detectors," Rev. Sci. Instrum. 75(4), 1134-1148 (2004).
[4] Kruse, P.W., [Uncooled Thermal Imaging: Arrays, Systems and Applications], SPIE, Bellingham, WA, (2002).
[5] Shaver, P.J., "Bimetal Strip Hydrogen gas Sensor," Rev. Sci. Instrum. 40(7), 901-905 (1969).
[6] Timoshenko, S., "Analysis of Bimetal Thermostats," J. Opt. Soc. Am., 11, 233-255 (1925).
[7] Dereniak, E.L. and Boreman, G.D., [Infrared Detectors and Systems], John Wiley and Sons, New York, 177-183 (1996).
[8] Sarid, D., [Scanning Force Microscopy], Oxford Univ., New York, 46-49 (1991).