Electrodynamics III Exam 3 problems

Vol. 8 sec. 4. A conducting ellipsoid

1. Show that for roots of Eq. (4.1), the surfaces of constant  are ellipsoid, hyperboloid of one sheet, and hyperboloid of two sheets, respectively. Sketch these surfaces.
2. Consider a 2D example of Eq. (4.1), x2/(a2+u) + y2/(b2+u) = 1, a > b, with real roots   -b2  -a2. (a) show that that  = constant is an ellipse. (b) Show that this surface is confocal with the ellipse x2/a2 + y2/b2 = 1. (c)Plot some example surfaces for different constant .
3. Consider a 2D example of Eq. (4.1), x2/(a2+u) + y2/(b2+u) = 1, a > b, with real roots   -b2  -a2. (a) show that that  = constant is a hyperbola. (b) Show that this surface is confocal with the ellipse x2/a2 + y2/b2 = 1. (c)Plot some example surfaces for different constant .
4. For the solution u = x, show that Eqs. (4.4) satisfy Eq. (4.1).
5. Derive the expression for h1 in Eq. (4.5).
6. Derive Laplace’s equation (4.6) in ellipsoidal coordinates.
7. For given values of , , a, b, solve Eq. (4.7) graphically for u. (Oblate spherical coordinates). Plot surfaces of constant z and h for given a and c in (r,z) space. How does the shape of these surfaces change as   -c2 or   -c2.
8. Derive Eqs. (4.8) and (4.9).
9. Derive Eq. (4.16).
10. (a) Derive Eq. (4.17) for the potential of a charged prolate spheroid conductor. Useful substitutions: u = (a2 + z), s = u/(b2 – a2). Useful formulas: arctan(i x) = i arctanh(x), arctanh(-x) = - arctanh(x), arctanh() = - i /2. Tanh(a+b) = coth(a+b-i/2). WolframAlpha is also useful. (b) Show how the potential becomes that of a sphere when a = b = c.
11. Derive (4.18) for the capacitance of a conducting prolate spheroid. Show that this becomes the capacitance of a sphere when a = b = c.
12. How much charge can we put on a flat metal disk of radius 1 cm when it is held at 1 V potential with respect to ground? How much on two such disks held 0.1 mm apart with 1 V applied between them? What is the ratio?
13. Derive the differential equation for the trial function F() and show that Eq. (4.23) is one of the solutions.
14. Derive Eqs. (4.29) for sum of the depolarization factors.
15. A conducting sphere perturbs the uniform external field that it is placed in. In what way could you deform the sphere to minimize the perturbation? Explain based on the mathematics of the depolarization factors.

Section 8. The dielectric ellipsoid

1. Derive (8.1) and (8.2) for a dielectric sphere in a uniform external field.
2. Derive (8.3) and (8.4) for a dielectric cylinder in a perpendicular uniform external field.

section 9. The permittivity of a mixture

1. A bucket of fine glass spheres (n = 1.5, k = 0) has a volume fraction of air = 50%. What is the index of refraction of the mixture to 3 decimal places. Hint: see section 83.

section 12. Electrostriction of isotropic dielectrics

section 13. Dielectric properties of crystals

section 23. The contact potential

1. What is the maximum contact potential, magnitude and sign, for a penny and a nickel? For a silver dollar and a nickel? I press a nickel and a penny together and measure 0V with my voltmeter. Why?
2. You stack 10 dry coins, alternating between nickels and pennies. What is the electrostatic potential difference between the bottom and the top of the stack in terms of the work potentials of copper and nickel.

section 24. The galvanic cell

1. Two wires of the same metal connect a voltmeter to the two terminals of a battery. The two terminals are of metals A and B, and these are both dipped into an electrolyte solution with ions A+, B+, and X-. Show by considering work functions and contact potentials that the measured emf is non-zero if and only if A is not the same metal as B.
2. Identify the conductors A and B in a) lead-acid battery b) alkaline battery, and c)Li-ion battery. What ions pass into or out of solution in each case.
3. For three different metal electrodes (A,B,C) immersed in an electrolyte solution containing molecules AX, BX, CX, where X is any negative ion, show that the emfs between pairs of electrodes are related by AB + BC = AC.

Sec. 64. Excitation of currents by acceleration

Landau Problem 2. (requires doing also section 63 Landau Problem 3).

1. Show ****
2. Find original papers on the Stewart-Tolman effect and summarize how the experiment was performed and the effects observed. Find one other more recent paper that cites ST and summarize.
3. For a loop of 1-mm-radius gold wire, who much linear acceleration is needed for each length element to produce a current of 1 nA. Compare to gravity. If the radius of the loop is 10 cm and it starts from rest, how many rpm is there for this acceleration after 1 s.
4. What magnitude of current flows in a 1 mm radius Au wire formed in a loop of radius b = 10 cm which is stopped from  = 1000 rpm within 1 ms?

Section 87. Surface impedance of metals

Landau Problem

1. What is the conduction electron mean free path for Cu at 300 K? At what wavelength does the relation  = ) become invalid? (You will need the expression for electromagnetic penetration depth in SI units.)
2. Show that for non-magnetic metals, the imaginary part of the surface impedance is negative.
3. Derive the expression (87.13) for reflectance *R*| in both Gaussian and SI untis.
4. What is the normal incidence reflectivity of a superconductor and how does it depend on wavelength?
5. Find the minimum value of *R||* (87.16) and the angle *0* where this minimum occurs in terms of surface impedance.
6. Find the complex surface impedance for a metal from a measurement of normal incidence reflectance and the minimum reflectance angle (87.16) for *R||*.
7. From experimental complex surface impedance data for a metal, determine the complex permittivity in the optical range.
8. Use the permittivity spectra for gold to determin the complex surface impedance and the normal incidence reflectivity at wavelengths of 1 m and 100 m. (You need to use the polar form of a complex number, you have to distinguish one correct solution out of 4 possibilities, and use these data from “Longwave plasmonics on doped silicon and silicides,” R. Soref, R. E. Peale, and W. Buchwald, OPTICS EXPRESS 16, 6507 (2008).)
9. For gold at 1 m wavelength, at what angle of incidence is the reflectivity a minimum. Consider both polarizations and use the complex surface impedance found in the previous problem.

Section 88. The propagation of waves in an inhomogeneous medium

1. Work through the derivation of the wavefunction (vol. 3, section 46, Eq. (46.9)) for a particle in 1D motion in the quasiclassical case.
2. Show that the E-wave solution 88.7 is the same as for 1D quantum mechanical motion in a homogeneous exernal field, U = Fx. See vol. 3 section 24.
3. Derive Eq. (88.8). See v. 3, section 47 and problem 2 above.
4. Work through the derivation in v.3 Appendix b on the Airy function.
5. Consider a suspension of metal particles in a liquid with a concentration increasing with depth. Laser light is normally incident on the top surface. A detector is placed in the liquid. Discuss what might be observed as the vertical position of the detector is varied.
6. Similar situation as in problem 5, except TE polarized laser has variable angle of incidence. A linear array of detectors is mounted vertically on the inside wall of the container that holds the suspension, on the side opposite to the laser source, in the plane of incidence. Discuss what signal the array of detectors might record as the angle of incidence is varied.