**Electrodynamics III**

**Exam 1 problems 2017**

**LL8 section 103**

1. Derive (103.8)
2. If there is extraneous charge density ext(**r**) that varies with **r**, then  = pol + ext, **E**, **D**, ext and pol all vary spatially. Each can be expanded in a Fourier series with spatially oscillating factors ei**k.r**. Each Fourier component is related by (103.7): **D**(**k**) = (**k**) **E**(**k**). Show that (**k**)= ext(**k**)/(**k**) = ext(**k**)/(**k**). pol = polarization charge density.
3. Derive the Thomas-Fermi dielectric function (=0,k) = 1 + ks2/k2, where ks is the screening length, as follows. Consider a uniform gas of electrons of charge density –n0e and a uniform positive charge background n0e. Add some fixed non-uniformly distributed extraneous charge **ext. One of the Fourier components of the new fixed background is **+(x) = n0e + **ext(k)sinkx. The second term will perturb the free electron gas due to a potential ext given by Poisson’s equation. The corresponding Fourier component of the perturbed free electron charge density is **-(x) = - n0e + **pol(k) sinkx, where **pol is the polarization charge density induced by **ext. Find **pol(k) in terms of **ext (k) and (=0,k). Use the fact that the electrochemical potential energy = F(x) – e **(x) is constant everywhere for the electron gas. F = (hbar2/2m)(32n)2/3. Use results of Problem 2.
4. Write the Thomas-Fermi screening length in terms of the electron concentration and Bohr radius. Is this a strong function of concentration? What is the screening length for a typical metal with electron concentration 1022 cm-3? For a heavily-doped semiconductor with n = 1018 cm-3?
5. The static limit for the permittivity of an electron gas is given by the Thomas-Fermi dielectric function, see Problem 3. Use (83.4) to derive a dispersion relation valid for low frequency waves when the wavevector is not small. What is the large k limit? How does it compare to the dispersion for electromagnetic waves in vacuum?
6. Derive the screened Coulomb potential as follows. An excess point charge +e placed into a conductor is an extraneous charge in a sea of free electrons. The potential of the bare charge is ext(r) = e/r, but the free electrons shield the extraneous charge so that the total potential (extraneous positive charge plus cloud of shielding electrons) is (r). Find (r) as follows. Write the Poisson equation for ext. Write ext(r) as a Fourier integral. Use the Fourier representation of the delta function (LL2, sec 51). Find ext(k). Use the results of problems 2 and 3 to find the total potential (r). Graphically compare the screened and unscreened potentials of an electron in the field of unit positive charge with unity screening length.
7. Sound waves in are longitudinal compression waves of neutral particles. No electromagnetic fields are involved. All nearby particles move with the same phase. In a solid, there are matter waves where adjacent particles move with the same phase (acoustic phonons) or opposite phase (optical phonons). Besides waves with longitudinal displacements with respect to propagation direction (LA, LO), there are also waves with transverse displacements (TA, TO). If the solid is made up of different types of atom, bonding has an ionic component, and matter waves must have a co-propagating electromagnetic wave. An LA sound wave in a solid therefore is associated with a purely-electric longitudinal wave. If the solid is a conductor, charges move in this electric wave with spatial dispersion. Ions are also accelerated in this field, which provides a restoring force. All these effects combine to give a permittivity (,k). Use LL8 section 84. Argue that ions accelerate as though free (see LL8 section 78) and that the Thomas-Fermi permittivity (Problem 3) applies to the electrons to find the dispersion relation  vs k and the sound wave velocity in terms of the Fermi velocity (F = (1/2) m vF2) and the relevant masses.
8. Show from (103.7) and (103.12) for an isotropic homogeneous medium with spatial dispersion that a longitudinal electric wave has **D** = *l***E** and a transverse electromagnetic wave has D = *t***E**.

**LL8 section 106**

1. An electromagnetic plane wave in the far infrared can couple to transverse optical phonons in a non-homopolar crystal. The resulting resonant polarization changes the dispersion of the photon from  = ck. The coupled photon-phonon is called a **polariton**. Use the electromagnetic wave equation in a homogeneous linear medium to find a relation between **E**, **P**, , and k. Derive a second equation for these quantities from the interaction of photon and phonon, by considering the differential equation for a driven simple harmonic oscillator with natural frequency TO and reduced mass M, where the photon field provides a periodic driving force. P = dipole moment per unit volume. At low wavevectors, the TO phonon frequency TO is independent of wavevector. From the two equations derive the dispersion relation  vs. k and show that there are two branches. Plot both branches of the dispersion curve hbar  vs hbar ck, in units of eV vs. eV. Indicate asymptotes and intercepts and identify their significance. Assume GaP with hbar TO = 0.0455 eV, density 4.14 g/cm3, molar mass 101 g.
2. Show that the frequency dependent permittivity for an insulating ionic crystal driven by a transvers electromagnetic wave is () = () + [(0) – ()] / (1-2/T2). The natural frequency of oscillation for the ionic motion is T. Ionic reduced mass is M, ionic displacement = u, polarization P = Nqu = dipole moment per unit volumne, N = number of ion pairs, q their charge, u their normal mode displacement, () = n2 = contribution to the permittivity from ion cores, n = refractive index.
3. Use result of problem 2 and (83.7) to find a plot two branches of the polariton dispersion curve hbar  vs hbar ck for GaP. hbar TO = 0.0455 eV, (0) = 10.18, n = 3.2. Why is the result different than in Problem 1?
4. Raman scattering of a beam of monochromatic light excites transverse optical phonons (Stokes scattering). Energy and momentum of photons must be preserved. Show that usual Raman scattering with detector at 90 deg from the incident beam results only in phonons with large wavevector, which are far from the polariton region. What is the nature of the ionic motion when phonon momentum has it largest physically meaningful value? Plot dispersion curves of light and optical phonons for GaP on the same graph with the same scale.
5. How does one observe a polariton? Analyze and present C. H. Henry and J. J. Hopfield, “Raman Scattering by Polaritons”, Phys. Rev. Lett. **15**, 964 (1965), as follows. Associated with a TO phonon are oscillating dipoles at the TO frequency, so there has to be an EM wave at the same frequency. The photon energy for this EM wave is small, so its wavevector is also small ( = ck while c is large). The interaction resonance between the TO phonon and the EM wave is the polariton, which distorts the dispersion curves of the two excitations at their crossing point. Small angle Raman scattering measures the frequency of the Stokes scattered light S as a function of angle. Show that the wavevector of the excited phonon polariton is q  [(k/)=Lq2 + kLkSq2] where q = L-S, kL = k(L) = n(L) L/c, etc. Then from each pair of measured S and q, we can get a point (q,q) on the polariton dispersion curve. What happens if there is no normal dispersion in the refractive index?

LL8 section 113

1. An extraneous charged particle moves through a medium with non-relativistic velocity **v**. Show that the electric induction at the field point **r** is –grad (e/|**r** – **v**t|). (Put the origin at the initial position of the charge.)
2. Time average (80.2) for the instantaneous power distribution per unit volume by a changing polarization due to the passage of a charged particle, use (103.7), and assume D(,**k**) is real if E(,**k**) is real to show that the mean quantity of heat evolved per unit time and volume is Q = (-1/8)  Im(1/) D2(,**k**) for one spatio-temporal Fourier component of the field.
3. Plot the energy loss function -Im(1/) as a function of 1) energy and 2) wavelength over the near UV to near IR spectral range for gold. See refractive index.info for downloadable data and use section 83 to get (’,”) from (n,).
4. What is the maximum possible momentum that can be transferred to an electron in a crystal by an incident electron in a beam with 20 kV accelerating voltage? Does the corresponding wavevector q0 satisfy 0/*v* << q0 << 1/*a*? Here 0 is some mean frequency corresponding to the motion of the majority of electrons in the atom, *v* is the velocity of the electron in the incident beam, and *a* is the atomic dimension. Based on the definition of q, can it ever have this large value?
5. Suppose that the energy lost by electrons impinging on a solid surface is all converted to light, with each frequency component in the stopping power integral begin converted to photons of the corresponding frequency. Let q0 = 1/10a and a = 0.3 nm. Let the incident electron energy bet 20 keV, as in an ordinary SEM. Convert I() to I(), with the requirement that I()d = I()d, to show that I() = (const/3)(-Im[1/]) ln[(nm)/68]. The second factor is the energy loss spectrum from problem 3.
6. Plot I() for gold from  = 300 to 1000 nm, using the results from problems 5 and 3. Does the spectrum look like the cathodoluminescence spectrum of gold in [Cathodoluminescence study of silver and gold lamellar gratings](https://physics.ucf.edu/~rep/conf_pubs/ConfPubs2011/CL_SPIE2011final.pdf), Janardan Nath, Casey Schwarz, Yuqing Lin, Evan Smith, R. E. Peale, L. Chernyak, Walter R. Buchwald, Jane Lee, Proc. SPIE 8031 - 101 V. 5 (2011)?