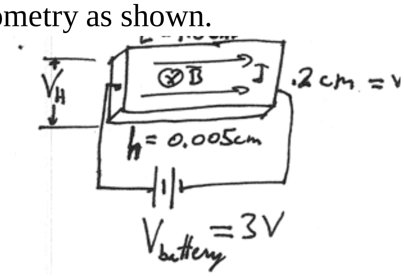
Electrodynamics II Exam 2 problems Spring 2014

Section 21. Landau Problems 1-3

1. Consider two coaxial conducting cylinders with radii a and 3a and length L. The region a<r<2a is filled with a material with conductivity σ_1, and the region 2a<r<3a has conductivity σ_2. Assume $\varepsilon_1 = \varepsilon_2 = 1$. The inner cylinder is held at potential ϕ_0 by a battery while the outer cylinder is grounded, so there is radial current I. Determine the resistance in terms of σ_1, σ_2, and L. Determine the surface charge density on the boundary at r = 2a.
2. A capacitor, filled with material of permittivity ε and conductivity σ, is charged to Q_0. (a) Find how the charge leaks off the plates with time. (b) Determine how much of the initial stored energy is converted to heat. (c) What is the characteristic time for discharge if the material is SiO_2 ($\varepsilon = 4.3$, $\rho = 10^{13}$ Ω-m)? (d)What is the implication for computer memory chips?
3. The leakage resistance of cable insulation is measured by immersing a length of insulated cable in salt water, applying a voltage between central conductor and solution, and measuring the current. If length = 3 m, voltage = 200 V, the insulation thickness is twice the central conductor radius, and a current of 2 nA is measured, what is the resistivity of the insulation?
4. A steady current with uniform density **j** flows through a flat interface between a medium with conductivity σ_1 and permittivity ε_1 and a medium with σ_2 and ε_2. Find the extraneous charge that accumulates on the interface.

Section 22. Landau Problem

1. A Hall probe is used to measure magnetic fields. A current I flows in a ribbon of n-type semiconductor in the B-field. In steady state, there is a voltage difference V_H between the edges of the ribbon, such that the net transverse force on an electron $e\mathbf{E}_\perp + e\,(\mathbf{v} \times \mathbf{B})_\perp$ is zero (SI units). From V_H, one determines B. If B = 0.1 T, what is V_H for the following conditions?: Si:As with n = 2 x 10^{15} electrons-cm^{-3}, resistivity = 1.6 Ohm-cm, L=1.0 cm, w = 0.2 cm, h = 0.005 cm, $V_{battery}$ = 3 V, and geometry as shown.



Section 23.

1. What is the maximum contact potential, magnitude and sign, for a penny and a nickel? For a silver dollar and a nickel? I press a nickel and a penny together and measure 0V with my voltmeter. Why?
2. You stack 10 dry coins, alternating between nickels and pennies. What is the electrostatic potential difference between the bottom and the top of the stack in terms of the work potentials of copper and nickel.

Section 24.

1. Two wires of the same metal connect a voltmeter to the two terminals of a battery. The two terminals are of metals A and B, and these are both dipped into an electrolyte solution with ions A+, B+, and X-. Show by considering work functions and contact potentials that the measured emf is non-zero if and only if A is not the same metal as B.
2. Identify the conductors A and B in a) lead-acid battery b) alkaline battery, and c)Li-ion battery. What ions pass into or out of solution in each case.
3. For three different metal electrodes (A,B,C) immersed in an electrolyte solution containing molecules AX, BX, CX, where X is any negative ion, show that the emfs between pairs of electrodes are related by $\varepsilon_{AB} + \varepsilon_{BC} = \varepsilon_{AC}$.

Section 29

1. Find the internal and external magnetic induction of a sphere with uniform magnetization and radius a. Use the magnetic scalar potential method and analogy with the uniformly polarized dielectric sphere.
2. Find the internal and external magnetic induction of a uniformly magnetized cylinder (radius a, length L) on the cylinder (z) axis. Use the magnetic scalar potential method and analogy with the uniformly polarized dielectric cylinder. Find B at the ends of the cylinder. Sketch B(z) vs. z.
3. In a large piece of material there is a uniform magnetic induction $\mathbf{B}_1$ and a parallel uniform magnetization $\mathbf{M}_1$. Find the induction $\mathbf{B}_2$ in the middle of cavities in the material of the following shapes: a thin disk, a long needle, a sphere. The symmetry axis of the cavity is parallel to $\mathbf{B}_1$ in each case.
4. Show that $\int\nabla(\mathbf{r}\bullet\mathbf{M})\,dV = \int(\mathbf{r}\bullet\mathbf{M})\,\mathbf{df}$, where the right integral is over the closed boundary surface of the integration volume. Show that $(\mathbf{M}\times\nabla)\times\mathbf{r} = -2\,\mathbf{M}$. Show that $\mathbf{r}\times(\mathbf{M}\times d\mathbf{f}) = (\mathbf{r}\bullet d\mathbf{f})\mathbf{M}-(\mathbf{r}\bullet\mathbf{M})d\mathbf{f}$. Show that $[\mathbf{r}\times(\nabla\times\mathbf{M})]_i =$ $\nabla_i(\mathbf{r}\bullet\mathbf{M}) + 2\,M_i - \nabla\bullet(\mathbf{r}M_i)$. Show that $-\int \nabla\bullet(\mathbf{r}M_i)\,dV = -\int [\mathbf{r}\times(\mathbf{M}\times d\mathbf{f})]_i - \int(\mathbf{r}\bullet\mathbf{M})\,\mathbf{df}_i$, where the right integrals are over the closed boundary surface of the integration volume. Finally show $\int \mathbf{r}\times(\nabla\times\mathbf{M})\,dV = -\int\mathbf{r}\times(\mathbf{M}\times d\mathbf{f}) - \int(\mathbf{M}\times\nabla)\times\mathbf{r}\,dV$, where the first integral on the right is over the closed boundary surface of the integration volume.
5. A right circular cylinder of length L has a uniform axial magnetization M and no net current. Find the mean value of the microscopic current density $\langle\rho\mathbf{v}\rangle_r$ inside the cylinder. Find also the surface current density $\mathbf{g}$. Compare with a solenoid.
6. A sphere of radius R centered at the origin has magnetization $\mathbf{M} = (a\,x^2 + b)\,\mathbf{x}$^, where a & b are constants. The net current density is zero. Find the mean value of the microscopic current density $\langle\rho\mathbf{v}\rangle_r$ inside the sphere and the surface current density $\mathbf{g}$ on the sphere. Try to make a sketch of the distribution of $\mathbf{g}$ on the sphere surface.
7. Given a spherical shell, inside radius R_1 and outside radius R_2, which is uniformly magnetized in the direction of the z-axis. The magnetization in the shell is $\mathbf{M}_0 = M_0\,\mathbf{z}$^. Find the magneto-static potential ψ for points on the z-axis, both inside and outside the shell.

Section 30. Landau Problems 1, 3,4

1. Show curl(**j**/R) = **j x R**/R^3, where **R** = **r-r'**, curl acts on coordinates **r**, and **j** = **j**(**r'**).
2. Derive from **A**=(1/c) ∫**j**/R dV the formula **A**=$\mathfrak{M}$ **x R**/R^3 for the vector potential of the field far from the currents, where the total magnetic moment of the system is $\mathfrak{M}$=∫**r x j** dV/2c.
3. The energy flux in a conductor is **S** = (c/4π) **E x H**. Show that in steady state div**S** = -**j.E**
4. Show ∇ × **A** = ∇A x **z^**, for the two dimensional problem in which **A** is a function only of x and y and is oriented in the **z^** direction, where **z^** is the unit vector in the z direction.
5. Show ∇ × ((1/μ) ∇ × **A)** = - **z**^ ∇ • ((∇**A**)/μ), for the 2D problem in which μ also depends only on x and y.
6. For the two dimensional problem of a medium infinite and piecewise homogeneous in the z^-direction, with current density **j** = j(x,y) z^, show that continuity of the tangential components of **A** and ((1/μ) ∇ × **A)** imply boundary conditions that A and (1/μ) ∂A/∂n be continuous at an interface, where n is the normal unit vector to the interface.
7. From the resulting general formula of text problem 1, determine the magnetic field on the axis of a circular current loop of radius *a* at a distance z from its center.
8. For the results of text problem 3, if J = 100 A, b = 5 cm, a = 1 cm, and h = 4 cm, what is the magnetic field in Tesla in the hole? How does this compare to the Earth's magnetic field?
9. Consider an infinite magnetic slab, susceptibility x, parallel to the xy plane, between z = -a and a, free conduction current density **j**(z) = j_0 (z/a) **x^**. What are **H**, **M**, and **B** everywhere?

Section 31

1. Show that **–E**•∇×**H** = ∇ • (**ExH**) – **H**•∇×**E.**
2. Show that $(\delta\tilde{\mathfrak{F}})_T$ = -(1/c) ∫ **A** • δ**j** dV.
3. What is the magnetic energy per unit length for a long cylindrical solenoid, radius *a*, tightly wound with n turns per unit length of wire carrying current J? Consider both an empty solenoid and one filled with a linear magnetic medium of permeability μ.
4. The magnetic induction in the gap between the poles of an electromagnet is maintained at B_0. A paramagnetic slab of susceptibility χ and cross section *A* experiences a force that draws it into the gap. Find the force.

Section 32

1. For a space in which exists a magnetic field due to currents in some conductors, with other conductors also in the space, the change in the free energy $\tilde{\mathbb{F}}$ due to changes in the field is $\delta\tilde{\mathbb{F}}$ = δ∫ ($\tilde{F}$ + $\mathbb{H}^2$/8π) dV, where F~ is the free energy density with respect to the currents in the conductors, and $\mathbb{H}$ is the magnetic field that would exist (for the same currents) in the absence of any conducting medium. The integral is over all space including the interior of all conductors, current

carrying or not. Show that $\delta\tilde{\mathbf{F}} = -\int \mathbf{M}\bullet\delta\mathbf{H}\, dV$, where M is the magnetization vector in the conductors, and the integral need only be extended over their volume.

Section 33.

1. For a system of two conductors carrying total currents J_a and J_b, prove that $L_{11}L_{22} \geq L_{12}^2$.
2. Show that in Gaussian units, inductance has units of length.
3. Two small circular loops of wire (of radii a and b) lie in the same plane a distance R_0 apart. What is the mutual inductance between the loops if the distance R_0 is sufficiently large that the dipole approximation may be used? Find the force between them if the current in each is held constant.
4. Two isolated superconducting circuits carry certain currents when they are positioned so that the mutual inductance is zero. Now they are moved so that their mutual inductance is M. If the circuits are identical and had the same initial currents J_0, find the final currents J. (Hint: Is the magnetic energy unchanged, or is the flux through a superconducting circuit unchanged by the reconfiguration of the circuits?)

Section 34. Landau problems 1 and 5.

1. Derive the boundary condition (34.2) $\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = 4\pi \mathbf{g} / c$.
2. A toroidal coil of N turns is wound on a non-magnetic form. If the mean radius of the coil is b and the cross sectional radius of the form is a, show that the self-inductance of the coil is $L = 4\pi N^2[b - \sqrt{(b^2 - a^2)}]$. If N = 150, b = 4 cm, a = 1.5 cm, what is L in Henries?
3. A circuit consists of two coaxial cylindrical shells of radii R_1 and $R_2 > R_1$, with common length l, connected by flat end plates. Current flows down one shell and back up the other. Find the self-inductance.
4. What is the self-inductance of the pictured rectangular solenoid?