Electrodynamics II Exam 1 problems Spring 2017

Section 1.

1. Suppose a conductor has a cavity inside it, and there is a point charge q somewhere in the cavity. Prove that the net charge on the wall of the cavity must be –q. If the net charge of the conductor is Q0, what is the net charge on its outer surface? Why must every field line in the cavity begin on q and end on the cavity wall?
2. Consider concentric spherical shells with radii ra < rb, held at potentials a, b. Find  everywhere. What are the surface charge densities a and b?

Section 2. Landau Problems 1-4.

1. Consider two spherical conductors of radii a1 and a2 and centers separated by d>>a1,a2. Estimate the coefficients C-1ab to lowest order in a/d. Invert C-1ab to find the capacity coefficients and electrostatic induction coefficients. Find the energy in the field. Find the mutual capacity C. Two metal spheres of radius 1 mm are separated by 1 cm. Find the mutual capacitance in Farads. How does the value compare to typical values for electronic components?
2. Two conducting plates are arranged in parallel, area A, separation d. One plate has charge +e and potential ; the other, -e and 0. Neglecting edge effects, find the E-field in terms of e, A, d. Find the mutual capacitance C = e/. Find the coefficients of capacity {Cab}. Find the energy U = (1/2) Cabab and compare to the usual expression in terms of  and C.
3. For the solution to L&L Section 2 Problem 4: if a = 1 mm and b = 10 cm, what is C in farads? Note that you must convert the solution to SI units!
4. Given a system of two conducting objects in vacuum. Conductor 1 is uncharged. Conductor 2 is grounded. Drive the potential of conductor 1.

Section 3. Landau Problems 1-8,10,11

1. Derive the expression for the surface charge induced on a planar conductor by a point charge e (3.2), = -(e/2) *a*/r3, where r is the distance from e to a point on the surface, and *a* is the closest distance from the charge to the conductor. Show that the total charge on the plane is –e.
2. Show that = e/r – e’/r’ = 0 on the surface of a conducting sphere if (l/l’) = (e/e’)2 and R2 = ll’, where R is the radius of the sphere, l (l’) is the distance of charge e (e’) from the center of the sphere, and r (r’) is the distance from charge e (e’) to the field point.
3. For a charge e outside a grounded spherical conductor, what is the surface charge density? For a charge e outside a grounded spherical conductor, determine the total induced charge on the sphere.
4. For the insulated uncharged conducting sphere show that the potential (3.6)  = (e/r) – (e’/r’) + (e’/r0) is constant on the surface and that the induced charge on the sphere is zero.
5. Show that the potential (3.10) satisfies Laplace’s equation with respect to the variable r’.
6. Show that inversion transforms the sphere (**r**-**r0**)2 = *a*2 to the sphere (**r**’ – **r0**’)2 = *a*’2, where **r0**’ and *a*’ are given by (3.13).
7. Consider charged conducting plates that intersect at right angles. Show that the complex potential w(z) = (x,y) – i A(x,y) = (2 b x y + 0) – i b (x2-y2) satisfies the boundary conditions. Show that real and imaginary parts satisfy Laplace’s equation. Show that the Cauchy-Riemann relations are satisfied. Determine and plot the equipotential curves and field lines. Find the surface charge density.
8. Consider the complex potential w(z) = 0 – i a z1/2. Express Re & Im parts in terms of x and y (Hint: write z in polar form and use trig identities.) Show that w(z) gives the solution to the problem of a charged conducting plate that occupies the half plane y = 0, x0, if the plate is at potential 0. Find the field and equipotential lines near the edge of the plate and sketch them.
9. For Section 3 Problem 1 in your text, sketch the resulting E-field.
10. Use the results of Section 3 Problem 7 in the text to find the capacitance per meter of usual radio antenna wire (parallel ~mm thick wire held ~1 cm apart.)
11. Use the results or Section 3 Problem 7 in the text to find the capacitance in pF per meter of usual coaxial cable (~1 mm inner conductor surrounded by a shield of ~3 mm radius.)
12. For the solution to Section 3 Problem 11 in the text, suppose a 10 F capacitor is formed by metalizing two sides of 1 m thick mylar. What is the correction to C (absolute and relative) due to edge effects?
13. Let r be any function that satisfies Laplace’s equation inside a spherical shell of radius r = R. Show that (r,) = (R/r)(R2/r, ) satisfies Laplace’s equation outside the spherical shell.

Section 5. Landau problems 1,2,3,4a.

1. Find the pressure exerted on the surface of a charged conductor in zero external field, and of an uncharged conductor in an external field.
2. Two large metal plates (each of area A) are held a distance d apart. Suppose each has a charge +e. What is the electrostatic pressure on the plates?
3. Use the results of problem 3 in the text, estimate the electrostatic force tending to split 238U, assuming the nucleus is a conductor, and compare to your weight.
4. Using the results of problem 1 in the text, find a numerical value for the maximum force if a = 10 cm, c = 1 mm, and  = 10 V and compare to your weight.
5. Use the results of problem 3 in the text. If the spherical conductor has radius 10 cm and the external field strength is 1 MV/m, what is the value of the force? Could you feel it?
6. Use the results of problem 4 in the text. The bulk modulus of steel is 140 x 109 N/m2. What is the relative volume change for steel in a field of 1 MV/m? By how much would a 1 m3 steel cube expand? Could you measure the change with an ordinary tape measure?
7. Calculate the force (magnitude and sign) between conductors in the parallel plate capacitor for (a) fixed charges on each conductor; and (b) fixed potential difference between the conductors.
8. A set of three parallel conducting plates is arranged and biased as shown. The relative position of the two lower plates is fixed. The upper plate can move vertically. What is the sign of the force on the upper plate? (a)First give your answer based on intuition and explain your reasoning. (b) Then see *"Micro electro mechanical cantilever with electrostatically controlled tip contact,"* Imen Rezadad, Javaneh Boroumand, Evan M. Smith, and Robert E. Peale, Appl. Phys. Lett. 105, 033514 (2014) and describe the presented analysis of this problem.



Section 6.

1. A cylinder of height h along the symmetry axis and radius 10 h is formed from a material with uniform permanent electric polarization. Calculate the E-field at the center of the cylinder. Sketch the E-field lines both inside and outside.
2. A thin dielectric rod of cross section A extends along the x-axis from x=0 to x=L. The polarization of the rod is along its length given by Px = a x2 + b. Find the volume density of polarization charge and the surface polarization charge on each end. Show explicitly that the total polarization charge vanishes.
3. A dielectric cube of side L has a radial polarization **P** = A **r**, A = constant. The origin is at the cube center. There is no extraneous charge. Find the volume charge density and the surface charge density on each surface. Show that the total charge is zero.
4. A dielectric rod in the shape of a right circular cylinder of length L and radius R is polarized in the direction of its length, **P** = P **e**z, P = constant. Find the electric field at points on the symmetry axis both inside and outside the rod as a function of z.
5. Show that the macroscopic electric field caused by a uniform polarization is equal to the electric field in vacuum of a fictitious surface charge density  = **n.P** on the surface of the body.
6. Suppose a homogeneous isotropic dielectric slab is placed into a uniform external electric field **E** and develops a polarization **P**. What is the macroscopic electric field **E** inside the dielectric in terms of **E** and **P**? Must **E** and **E** be in the same direction?
7. What are the S.I. units for polarization?
8. An external electric field **E** is applied to a crystal with cubic lattice structure. A polarization **P** is induced, and this can be attributed to the appearance of a dipole moment at the position of every atom in the lattice. A macroscopic electric field appears inside the crystal, which for thin slab geometry is **E** = **E** – 4**P**, generally non-zero. What is the microscopic field **e** at the position of one of the atoms due to all the other dipoles? Would **e** have a different value at an arbitrary point not on a lattice site?

Section 7. Landau problems 1-3,5

1. Two concentric conducting spherical shells, radii a and 2a, have charge +Q and –Q, respectively. The space between them is filled with a linear dielectric with permittivity (r) = 2a/(3 a – r). Determine the electric induction between the shells. Determine the bound charge density between the shells.
2. Two dielectric media with permittivities 1 and 2 are separated by a plane interface. There is no extraneous charge on the interface. Find the relationship between the angles 1 and 2, where these are the angles that an arbitrary line of electric induction makes with the normal to the interface: 1 in medium 1, 2 in medium 2.
3. A long cylindrical conductor of radius *a*, bearing the charge  per unit length, is immersed in a dielectric medium of constant permittivity . Find the electric field at distance r>*a* from the axis of the cylinder.
4. A coaxial cable of circular cross section has a compound dielectric between the conductors. The inner conductor has an outside radius *a*; this is surrounded by a dielectric sheath of permittivity 1 and of outer radius *b*. Next comes another dielectric sheath of permittivity 2 and outer radius *c*. The inner conductor is held at potential 0. The outer conductor is grounded. Calculate the polarization at each point in the two dielectric media.
5. Ferroelectrics such as BaTiO3 can have very large permittivity, more than 100000. The space between the plates of a capacitor is filled with such a dielectric. For a given extraneous charge stored on the capacitor, what is the electric field inside the dielectric in the limit that the permittivity becomes very large? How is the extraneous charge density related to the polarization charge density?

Section 10. Landau problem

1. Two parallel conducting plates in air are held at a potential difference 0 by a battery. The mutual capacitance in air is C0. Then the battery is disconnected and a dielectric () sheet that just fits the gap is inserted between the plates. What is the final potential difference ? What is the stored energy? Again the two plates in air are connected to the battery 0. While the battery is still connected, the dielectric sheet is inserted between the plates. What is the stored energy now?
2. Two coaxial thin-walled conducting tubes with radii a and b are dipped vertically into a dielectric liquid of susceptibility  and mass density . If a voltage difference 0 is applied to the tubes, the liquid rises to a height h in the space between the tube walls. What is h in terms of , 0, , g, b, and a? Temperature is held constant.
3. Consider a spherical linear dielectric shell (inner radius a, outer radius b, dielectric constant ) and a point charge e infinitely separated. Now bring the point charge to the center of the dielectric, allowing the dielectric to cool back to ambient temperature. Determine the change in energy of the system.

Section 11. Landau problem

1. For a space in which an electric field exists due to charges on some conductors, with dielectric bodies also in the space, the change in the free energy F due to changes in the field is F =  (F-E2/8) dV, where F is the free energy density with respect to charges on the conductors, and E is the electric field that would exist in the space (for the same charges) if the dielectric bodies were absent. The integral is over all space except the part occupied by the conductors. Show that F = -**P**E dV, where **P** is the polarization vector in the dielectrics and the integral is now only over their volume.