**ED1 Problems**

Section 1.

1. Obtain a copy of A. A. Michelson, Am. J. Sci. 122, 120-129 (1881). Summarize in no more than 200 of your own words plus one figure drawn by you (a) the details of his experimental apparatus, (b) the nature of his *raw data*, and (c) what *he* concluded from it.
2. Visit [www.scholar.google.com](http://www.scholar.google.com) and find one example of a *modern* Michelson-Morely experiment of as recent a vintage as you can. (Hint: Look for recent papers that cite the ancient paper by Michelson.) Copy the title, abstract, and citation. In your own words, what do they conclude?
3. Look for practical commercial devices that would not function without special relativity.

Section 2.

1. Draw a space-time diagram representing a game of catch for two people at rest 10 m apart, i.e. sketch the world line of the ball. Indicate also the light cone on the graph (need not be to scale).
2. Event A happens at (ct,x,y,z)=(15,5,3,0); event B at (5,10,8,0); both in the K system.
   1. What is the interval between the two events?
   2. Is there an inertial system K’ in which they occur simultaneously? If so what is its velocity vector relative to K?
   3. Is there an inertial system K’ in which they occur at the same spatial position? If so, what is its velocity vector relative to K?
3. Two observers K and K’ observe two events, A and B. The observers have a constant relative speed of 0.8 c. In units such that the speed of light is 1, observer K obtained the following coordinates: Event A, (x,y,z,t)=(3,3,3,3); Event B, (5,3,1,5). What is the space-time interval between these two events as measured by observer K’? Is it space-like or time-like? Can Event A have caused Event B?
4. Find a translation of the original work by H. Minkowski that introduced four-dimensional geometry. Hint: Look in Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity, by H A Lorentz, Albert Einstein, H Minkowski, Herman Weyl for the citation. Are there any obvious differences between Minkowski’s original presentation and the distilled version that appears in Landau & Lifshitz?

Section 3.

* + 1. The mean lifetime of muons = 2 s in their rest frame. Muons are generated in the upper atmosphere as cosmic ray secondaries. (a) Calculate the mean distance traveled by muons with speed 0.99 c assuming classical physics (c=infinity). (b) Calculate the mean distance using special relativity. (c) What percentage of muons produced at an altitude of 10 km reach the ground assuming they travel straight down at 0.99 c.
    2. In a laboratory experiment a muon is observed to travel 800 m before disintegrating. A student looks up the muon lifetime (2 s) and concludes that its speed was 4 x 108 m/s, which exceeds c. Identify the error, and find the actual speed of the muon.
    3. A rocket leaves earth at speed 3c/5. When a clock on the rocket says 1 hour has elapsed, the rocket ship sends a light signal back to earth. According to earth clocks, when was the signal sent? According to earth clocks, how long after the rocket left did the signal arrive back on earth? According to the rocket observer, how long after the rocket left did the signal arrive back on earth?
    4. A muon is traveling through the lab at 3c/5. How long does it last if its lifetime in its own rest frame is 2 s?
    5. Tau leptons are observed to have an average half-life of t1 in the frame S1 in which the leptons are at rest. In an inertial frame S2, which is moving at a speed v12 relative to S1, the leptons are observed to have an average half-life of t2. In another inertial reference frame S3, which is moving at a speed v13 relative to S1 and v23 relative to S2, the leptons have an observed half-life of t3. Which of the following is a correct relationship among two of the half-lives, t1, t2, and t3? A) t2=t1Sqrt[1-v122/c2]; B) t1=t3Sqrt[1-v132/c2];C) t2=t3Sqrt[1-v232/c2]; D)t3=t2Sqrt[1-v232/c2]; E)t1=t2Sqrt[1-v232/c2];

Section 4.

1. Inertial system K’ moves at constant velocity **V** = V Cos[] **x^** + V Sin[] **y^** with respect to inertial system K. The axes of the K and K’ systems are parallel. Their origins coincide at t = t’ = 0. For convenience, use abbreviations  = V/c and  = 1/Sqrt[1-]. Find the Lorentz transformation matrix , i.e. find  such that (ct,x,y,z) = (ct’, x’, y’, z’).
2. Show that the Galileo transformation does not leave the interval between events invariant? Show that the Lorentz transform leaves (ct)2-x2 unchanged.
3. Which of the following represents a Lorentz transformation, assuming c=1? (x’,y’,z’,t’)= A) (4x, y, z, t/4); B) (x - 3t/4, y, z, t); C) (5x/4 - 3t/4, y, z, 5t/4-3x/4); D) (5x/4 – 3t/4, y, z, 3t/4 – 5x/4), E) None of the above.
4. A bus of rest length 5 m passes through a garage of rest length 4 m. Due to Lorentz contraction, the bus is only 3 meters long in the garage’s rest frame. What is the velocity of the bus in the garage’s rest frame? What is the length of the garage in the bus’s rest frame?
5. The coordinate systems K1 and K2 move along the X-axis of a reference coordinate frame K, with velocities v1 and v2 respectively, referred to K. The time measured in K for the hand of a clock in K1 to go around once is t. What is the time interval t2 measured in K2 for the hand to go around, in terms of t, v1, v2, and c. (Hint: The time dilation formula approach is messy since the velocity of K2 relative to K1 is *not* simply equal to v2-v1. Use Lorentz transformation formulas instead.)
6. A bar lies along the X’ axis and is stationary in the K’ system. Show that if the positions of its ends are observed in K at instants which are simultaneous in K’, its length deduced from these observations will be greater than its length in K’ by a factor (1-V2/c2)-1/2.

Section 5.

1. Show that the sum of two velocities, each smaller than c, is also smaller than c.
2. For V<<c, verify expression (5.3) for the transformation of the velocity vector and the equations preceding it.
3. Derive the expression (5.4) for the change in the direction of the velocity on transforming from one reference system to another.
4. Derive the formula for the aberration of light (5.7) and the formula that precedes it.
5. A 0 meson (rest-mass energy 135 MeV) is moving with velocity 0.8 c in the z direction in the laboratory rest frame when it decays into two photons, 1 and 2. In the 0rest frame, 1 is emitted forward and 2 is emitted backward relative to the 0 direction of flight. What is the velocity of 2 in the laboratory rest frame?
6. In the frame K, at t = 1, a particle leaves the origin O and moves with constant velocity in the XY-plane having components vx = 5c/6, vy = c/3. What are the coordinates (x,y) of the particle at any later time t? If the velocity of K’ relative to K is V = 3c/5, calculate the coordinates (x’, y’) of the particle at time t’ in K’ and deduce that the closest approach of the particle to O’ occurs at time t’ = 220/113.
7. Obtain the transformation equations for the acceleration **a** by differentiating the transformation equations for **v**.
8. A nucleus is moving with speed 3c/5. It emits a -particle in a direction perpendicular to the line of motion of the nucleus as observed from the reference frame of the nucleus. The speed of the -particle in this frame is 3c/4. Find the velocity and direction of motion of the -particle as seen by a stationary observer in the lab frame.
9. A luminous disk of radius *a* has its center fixed at the point (x’,0,0) of the K’-frame, which moves at speed V along the common X,X’ axes relative to the K-frame. The plane of the disk is perpendicular to the X’-axis. It is observed from the origin of the K-frame, at the instant the origins of K and K’ coincide, that the disk subtends an angle 2. If *a*<<x’, show that tan = tan’ Sqrt[(c-V)/(c+V)]. If the disk is moving away from the K observer, does the disk appear larger or smaller than to the K’ observer? What if the disk is moving toward the K observer?
10. Atoms at rest emit photons isotropically. For an observer watching a beam of atoms moving at speed 0.9 c relative to the lab, does the emission appear isotropic to an observer in the lab? For photons emitted at angles 0,30,60,90,120,150,180 in the reference frame of the atoms, what angles of emission does the observer see? Describe qualitatively and draw the distribution of emission. Will atoms appear brighter as they approach or recede from the observer?

Section 6. Landau problems 6.1 and 6.2 in book.

1. Prove that the symmetry (or antisymmetry) of a tensor is preserved by Lorentz transformation.
2. Use the e tensor method to prove the product rule **** x (f**A**) = f(**** x **A**) – **A** x (****f), where f is a scalar function of coordinates and **A** is a vector function of coordinates.
3. Consider an integral over a surface in 3-space. Let the vectors d**r** = dx **x** and d**r**’ = dy **y** define an area element.
   1. Find the tensor df that gives the projections of the area element on the xx planes.
   2. What is the projection of the area element on the x1x2 = xy plane? On the x1x3 plane?
   3. Find the vector df that is dual to df. What is its magnitude and direction relative to the area element?
4. Consider a volume element determined by the vectors d**r**=dx **x**, d**r**’=dy **y**, d**r**”=dz **z**. Show that the determinant of 3rd rank formed from the components of these vectors gives the volume of the parallelepiped spanned by these vectors.
5. Show that the Lorentz transform leaves the 4-D volume element unchanged.
6. Show that (1/2) (dSi Aik/xk – dSk Aik/xi) = dSi Aik/xk, where Aik is antisymmetric.
7. Show that dfki Ai/ xk = (1/2) dfik (Ak/ xi - Ai/ xk), which is analogous to the (surface element \* Curl) expression in Stokes theorem.
8. Show that in 2D, the general orthogonal transformation as matrix A given by {{cos, sin}, {-sin, cos}}. Verify that det[A] = 1 and that the transpose of A equals its inverse. Let Tij be a tensor in this space. Write down in full the transformation equations for all its components and deduce that Tii is an invariant.
9. Aijk is a tensor, all of whose components are zero, except for A111 = A222 = 1, A212 = -2. Calculate the components of the vector Aiji. A necessary condition for a transformation to be orthogonal is that its determinant = 1 and that its transpose equals its inverse. Show that the transformation x’1 = (1/7) (-3 x1 – 6 x2 – 2 x3), x’2 = (1/7) (-2 x1 + 3 x2 – 6 x3), (1/7) (6 x1 – 2 x2 – 3 x3) has these properties. Calculate component A’123 in the x’-frame. If Bij is a tensor whose components in the x’ frame all vanish except B’13 = 1, calculate B12.
10. Show that the components of the metric tensor are the same in all coordinate systems. Hint: See L&L Problem 1.

Section 7.

1. A car travels along a 45 deg line in K at speed v = 2c/Sqrt[5].
   1. Find components of ordinary velocity vector **v**.
   2. Find components of the 4 velocity ui.
   3. System K’ moves in the X direction at speed V=Sqrt[2/5]c relative to K. Use the velocity transformation law to find the velocity of the car in K’, i.e. find **v**’.
   4. Find u’i for the car in K’ using the Lorentz transform for 4 vectors. Check that the result agrees with Eq. (7.2).
2. Consider a particle in hyperbolic motion, x[t] = Sqrt[b2 + (ct)2], y=z=0. Find the proper time t’ for the particle as a function of t, assuming the clocks are synchronized at t=t’=0. Find x and v (ordinary velocity) as functions of t’. Find the 4-velocity as a function of t.
3. From the transformation properties of the four-velocity of a particle, derive the transformation equations for the components of its ordinary three-dimensional velocity (5.1).
4. Find expressions for the 4-acceleration wi = (w0,**w**) in terms of the 3-D velocity **v** and 3-D accelerations d**v**/dt. What are the dimensions of the 4-acceleration?

Section 8

1. From its relativistic Lagrangian, determine a free particle’s equation of motion.

Section 9

1. Derive the relativistic Newton’s 2nd law (9.2) in the case that the force acting on a particle is perpendicular to its velocity. Derive the relativistic Newton’s 2nd law in the case that the force is parallel to the velocity (9.3). Show that each expression is a special case of **f** = (m/(1-v2/c2)) d**v**/dt + (**f****v**) **v**/c2.
2. If a particle with non-zero mass is ultra relativistic, show that its momentum is approximately its total energy divided by c.
3. Derive the expression (9.18) for the force 4-vector in terms of the usual 3-D force vector.
4. A monoenergetic beam consists of unstable particles with total energies 100 times their rest energy. If the particles have rest mass m, their momentum is most nearly A) mc; B) 10 mc; C) 70 mc; D) 100 mc; E) 104 mc. Explain.
5. A free electron (rest mass me = 0.5 MeV/c2) has a total energy of 1.5 MeV. What is its momentum in units of MeV/c?
6. A positive kaon (K+) has a rest mass of 494 MeV/c2, whereas a proton has a rest mass of 938 MeV/c2. If a kaon has a total energy that is equal to the proton rest energy, what is the speed of the kaon in units of c?
7. If a particle’s kinetic energy is n times its rest energy, what is its speed?
8. A fast charge *e* enters the space between the plates of a parallel plate capacitor at an angle  (see figure) at time t = 0 and at point y = x = 0. (a) What is the electric field vector between the plates in terms of the given parameters? (b) Solve the equations of motion for the relativistic momentum components for t > 0. (c) Find an expression for the kinetic energy kin as a function of time and initial kinetic energy 0. (Hint: Use a relation between kin and p). (d) Find an expression for the velocity components as a function of time. (Hint: use a relation between kin, **p**, and **v**.) (e) Find the parametric expressions for x(t) and y(t). Plot the trajectory in the x,y plane.

