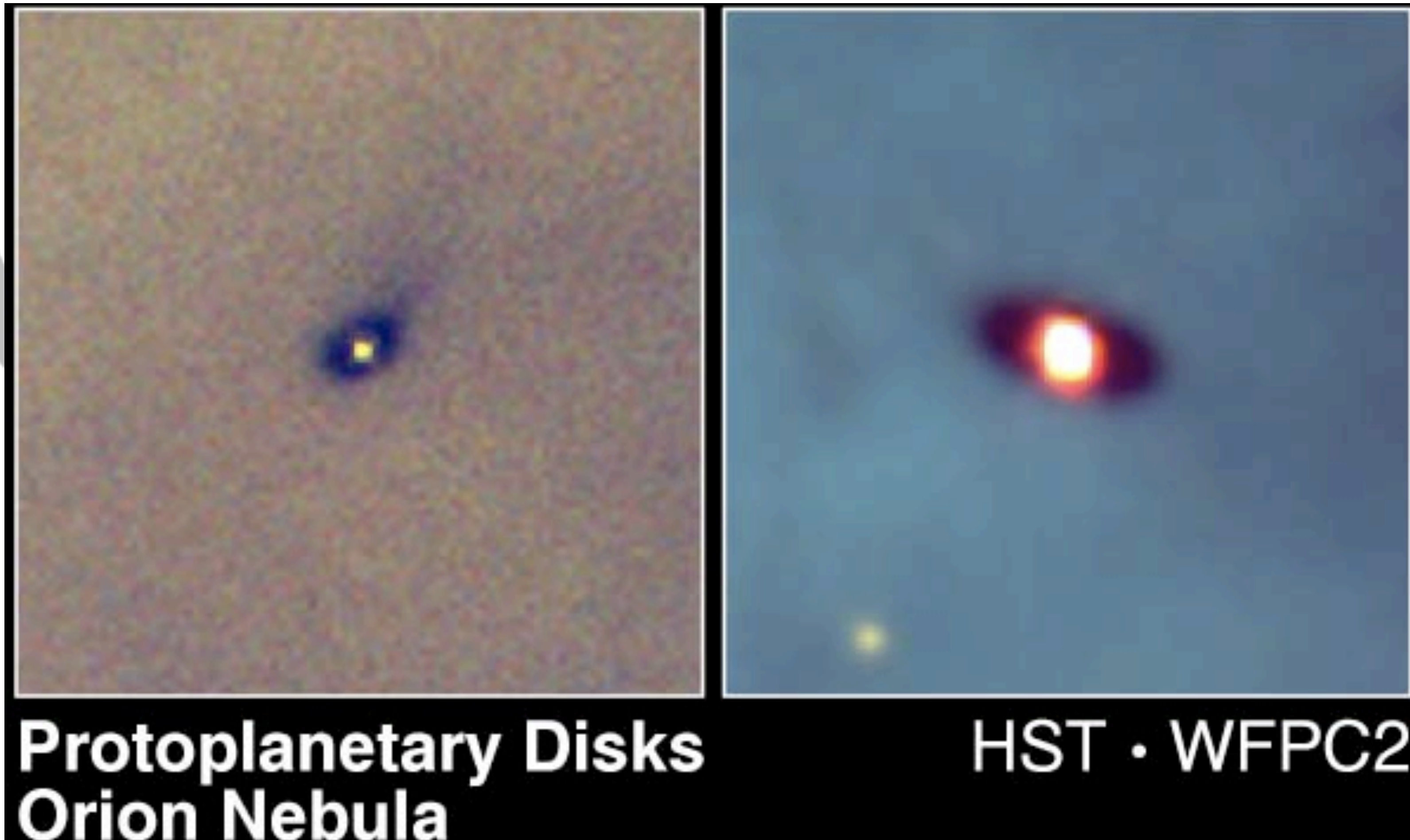
Circumplanetary Disk Formation

Ward & Canup 2010.



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Abril 21st, 2011.

Introduction:

Giant planet satellites suggest a formation from a circumplanetary disk.

Early giant planet formation studies employed a canonical MMSN model.

The authors develop a planet + disk + inflow evolution model.

Outline:

1 - Disk Formation:
- Inviscid disk
- Viscous disk

2 - Accreting Disk:
- Inflow stages:
  - Flux
  - Couple
- Planet contraction:
  - Stable rotation
  - Critical rotation

3 - Angular Momentum Inflow

4 - Planet-Disk Model:
- Inflow - Regimes
- Polytropic planet model
- Collapse phase

5 - Results:
- Final mass
- Contraction phase
- Disk profile
- Collapse phase

6 - Conclusions

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Average angular momentum delivered to the planet:

$$j_c \equiv dL/dM = \ell R_H^2 \Omega,$$ uncertain!

$$L = \int j_c dM = (3\ell/5) M R_H^2 \Omega.$$

As the planet contracts, it spins up:

$$\omega_c \approx (GM/R^3)^{1/2}$$

$$L_c = \lambda M R^2 \omega_c$$

Define a critical Radius: R_{rot}

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Contraction of an accreted planet:

No stress $\Rightarrow$ Shed material stays in orbit.

Mass and angular momentum conservation:

$$M_{\rm disk} = M_T[1 - (R/R_{\rm rot})^q]$$

For a Jupiter like planet: $M_{\rm disk} \sim 0.56\, M_J$.

- Too large.
- Probably unstable.

The source and magnitude is still debated.

Use alpha model: $\nu = \alpha c h$.

The shear due to viscosity generates a torque on the disk:

$$g = 3\pi\sigma\nu j$$

and drives a in-plane radial mass flux in the disk:

$$F\frac{\partial j}{\partial r} = -\frac{\partial g}{\partial r}$$

We conserve angular momentum again, but considering the viscous torque.

The mass of the disk:

$$M_{\text{disk}} = \frac{\lambda}{3} M_T \left(\frac{r_d^2}{\nu_d}\right) \left|\frac{\dot{R}}{R}\right| \left(\frac{R}{r_{dT}}\right)^{1/2} \left[1 - \left(\frac{R}{r_d}\right)^{1/2}\right]^2$$

Less than 5% of the total mass.

The centrifugal radius:

$$r_c = \frac{\ell^2}{3} R_H$$

Inflow rate of material $\mathscr{F}$ is accreted in range: $1/4\ r_c,\ \ 9/4\ r_c$

Two conditions determine the overall behavior:

1.- Is the planet in stable or critical rotation?

2.- Where is the material falling?

In equations, the in-plane flux is determined by:

$$\frac{dF}{dr} = \frac{\mathscr{F}}{2r_c^{1/2} r^{1/2}}$$

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$r_i, r_o \propto R_H$

S: no disk

C: shed material

S: $F_p \leq 0$

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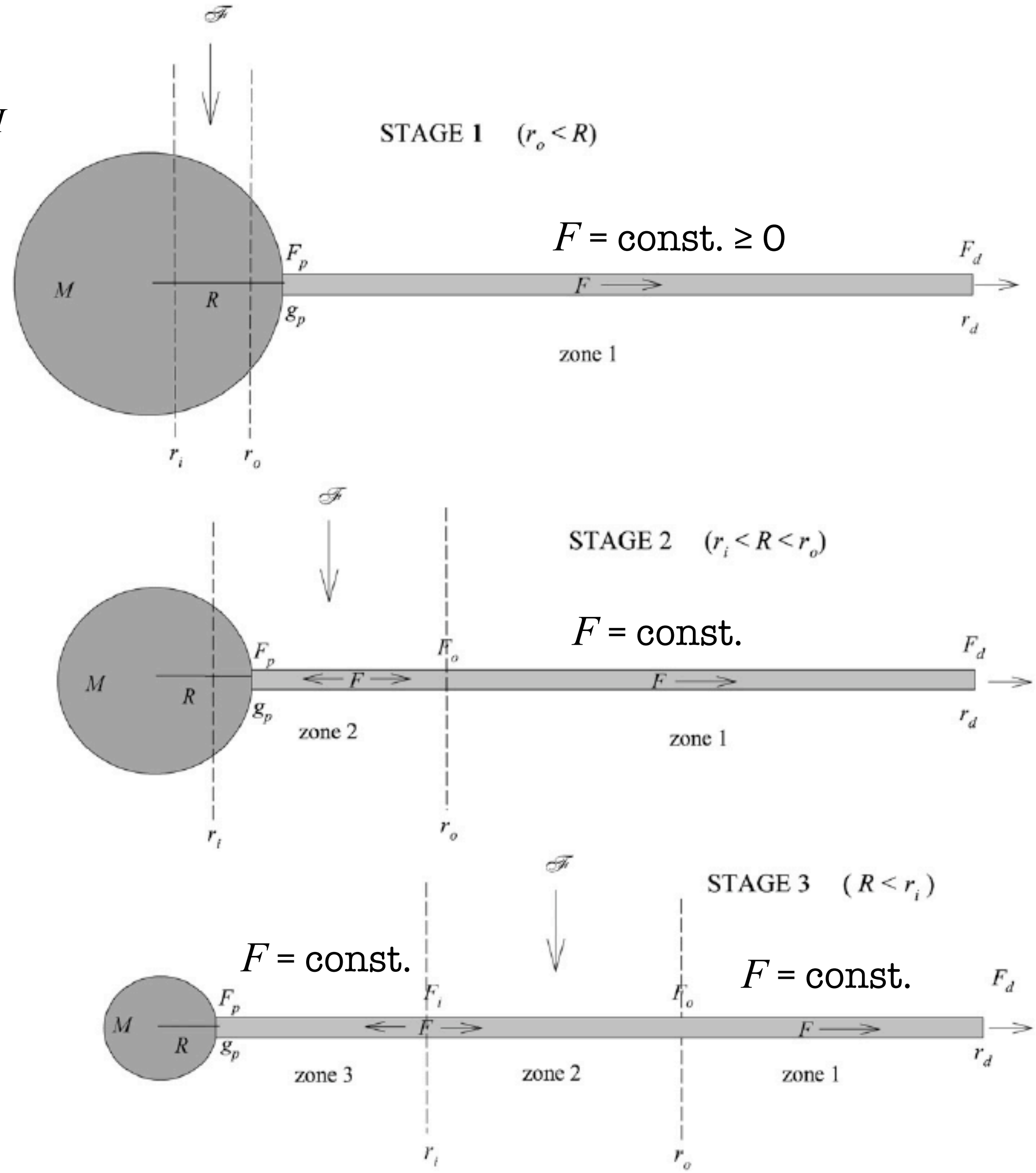

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They managed to write the flux in a general form:

$$F(r) = F_d - \mathscr{F}\left[\left(\frac{r_o}{r_c}\right)^{1/2} - \left(\frac{r_z}{r_c}\right)^{1/2}\right]$$

To determine F_d, its necessary to know the variation in the couple:

$$\frac{d}{dr}(Fj + g) = j\frac{dF}{dr}$$

$\ldots \Rightarrow$

$$g = g_p - F_p(j - j_p) - \mathscr{F}\left[\left(\frac{r_z}{r_c}\right)^{1/2} - \left(\frac{r_s}{r_c}\right)^{1/2}\right]j + \frac{\mathscr{F}}{2}\left[\frac{r_z}{r_c} - \frac{r_s}{r_c}\right]j_c.$$

They are in position of developing a model for the planet and the disk as a function of time:

Known the inflow mass rate, the planet's growth rate is:

$$\dot{M} = \mathscr{F}\left[\left(\frac{r_s}{r_c}\right)^{1/2} - \left(\frac{r_i}{r_c}\right)^{1/2}\right] - F_p$$

And also: $$\dot{M} = \mathscr{F} - F_d$$

Next, consider two cases:

Stable rotation: $g_p = 0$

Critical rotation: $\omega = \omega_c$

Solve equations of flux and couple:

$$3\pi\sigma\nu = \mathscr{F}\left[1-\left(\frac{r_c}{r_d}\right)^{1/2}\right]\left[1-\left(\frac{R}{r}\right)^{1/2}\right] \qquad \text{for } R < r < r_c/4$$

$$= \mathscr{F}\left(\left[1-\left(\frac{r_c}{r_d}\right)^{1/2}\right]\left[1-\left(\frac{R}{r}\right)^{1/2}\right] + \frac{1}{2}\left[1-\left(\frac{r}{r_c}\right)^{1/2} - \frac{1}{4}\left(\frac{r_c}{r}\right)^{1/2}\right]\right) \qquad \text{for } r_c/4 < r < 9r_c/4$$

$$= \mathscr{F}\left(\left[1-\left(\frac{r_c}{r_d}\right)^{1/2}\right]\left[1-\left(\frac{R}{r}\right)^{1/2}\right] - 1 + \left(\frac{r_c}{r}\right)^{1/2}\right) \qquad \text{for } 9r_c/4 < r$$

Solve equations again:

$$3\pi\sigma\nu = (\mathscr{F} - \dot{M})\left[\left(\frac{r_d}{r}\right)^{1/2} - 1\right] + \mathscr{F}\left[\frac{3}{2} - \left(\frac{r_z}{r_c}\right)^{1/2}\right] - \frac{\mathscr{F}}{2}\left[\frac{9}{4} - \frac{r_z}{r_c}\right]\left(\frac{r_c}{r}\right)^{1/2}$$

I believe that these last equations are among the main contributions of the paper.

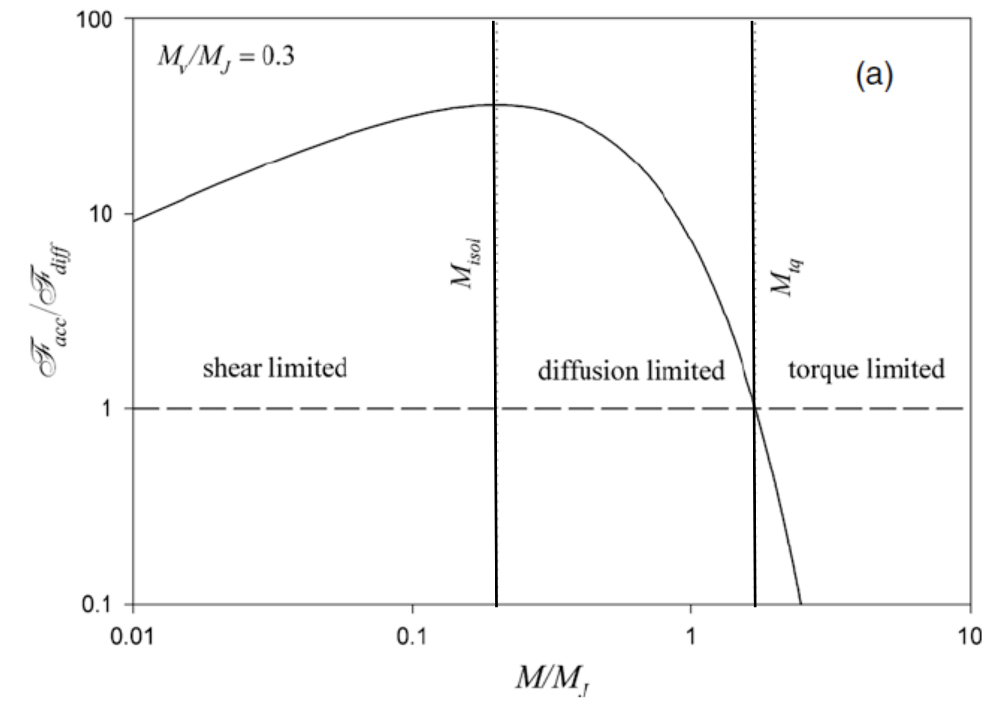
Equations (33–34) and (44–45) are among the main contributions of this paper. In the following sections, the critical rotation case will be applied to some specific examples. Before doing so, however, we will discuss the angular momentum of the inflow in more detail.

Three possible growth phases:

- Shear-limited

$$\frac{M_{\rm isol}}{M_\star} = \frac{8}{\sqrt{3}} \left(\frac{x_{\rm max}}{R_H}\right)^{3/2} \left(\frac{\pi \Sigma a^2}{M_\star}\right)^{3/2}$$

- Diffusion-limited
- Torque-limited



Shear − limited :

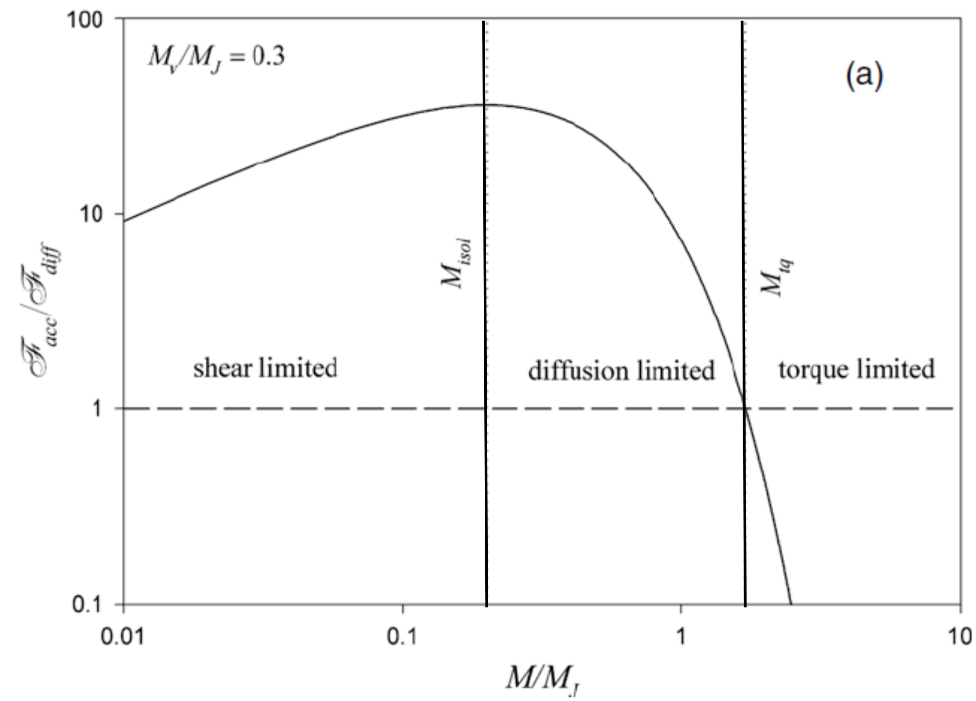
$$\mathscr{F}_{\rm acc} \approx 4\Sigma_o R_H^2 \Omega e^{-M/M_v} e^{-t/\tau_{\rm neb}} \quad \text{for} \quad M \leqslant M_{\rm isol}$$

Diffusion − limited :

$$\mathscr{F}_{\rm diff} \approx 3\pi \Sigma_o \nu_{\rm neb} e^{-t/\tau_{\rm neb}} \quad \text{for} \quad M_{\rm isol} \leqslant M \leqslant M_{\rm tq}$$

Torque − limited :

$$\mathscr{F}_{\rm acc} \approx 4\Sigma_o R_H^2 \Omega e^{-M/M_v} e^{-t/\tau_{\rm neb}} \quad \text{for} \quad M_{\rm tq} \leqslant M.$$



Model the planet as a polytrope:

$$P = K\rho^{1+1/n}$$

Pressure, density, temperature and energy are defined.

$$\dot{E} \approx -\mathscr{L} - F_p\,(-GM/2R)$$

$$\Rightarrow \qquad -2\frac{\dot{M}}{M} + \frac{\dot{R}}{R} = -\frac{\mathscr{L}R}{pGM^2} + \frac{1\,F_p}{2pM}$$

Expect molecular Hydrogen be predominant:

$$\Rightarrow \quad n = 2.5$$

- Planet contraction rises the central temperature.
- Above 3500 K Hydrogen dissociation occur.
- Enables the body to contract very fast.
- Past collapse, little change in radius.

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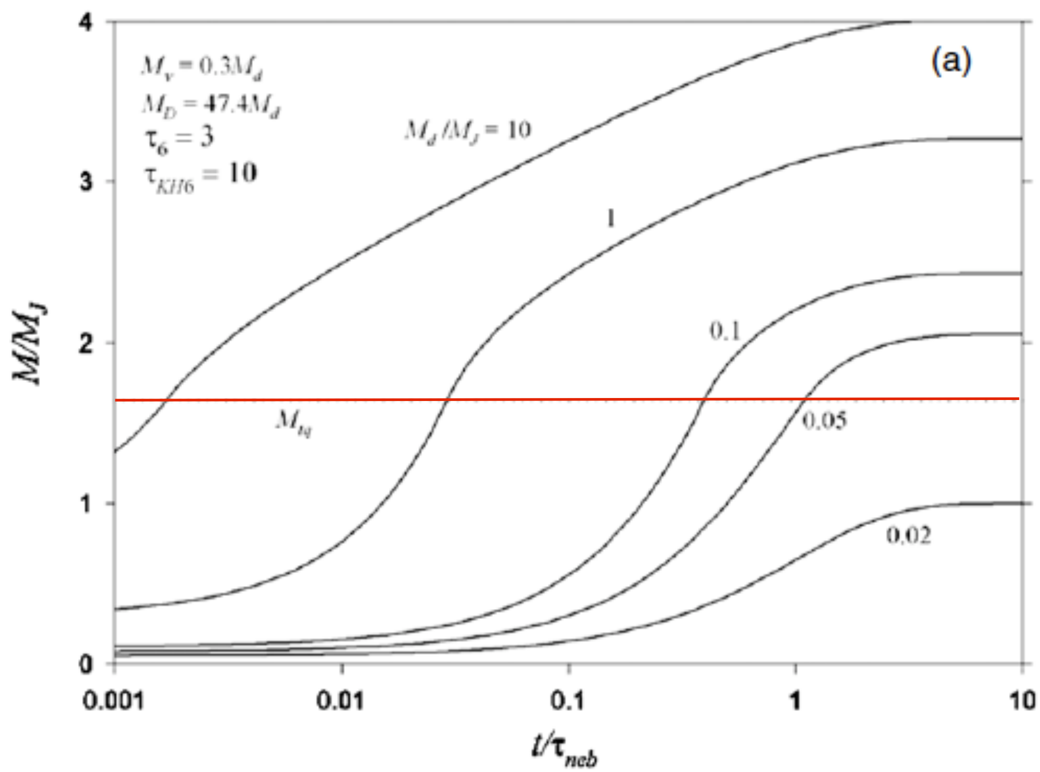
Mass versus time dependence:



Torque limited growth

Diffusion limited growth

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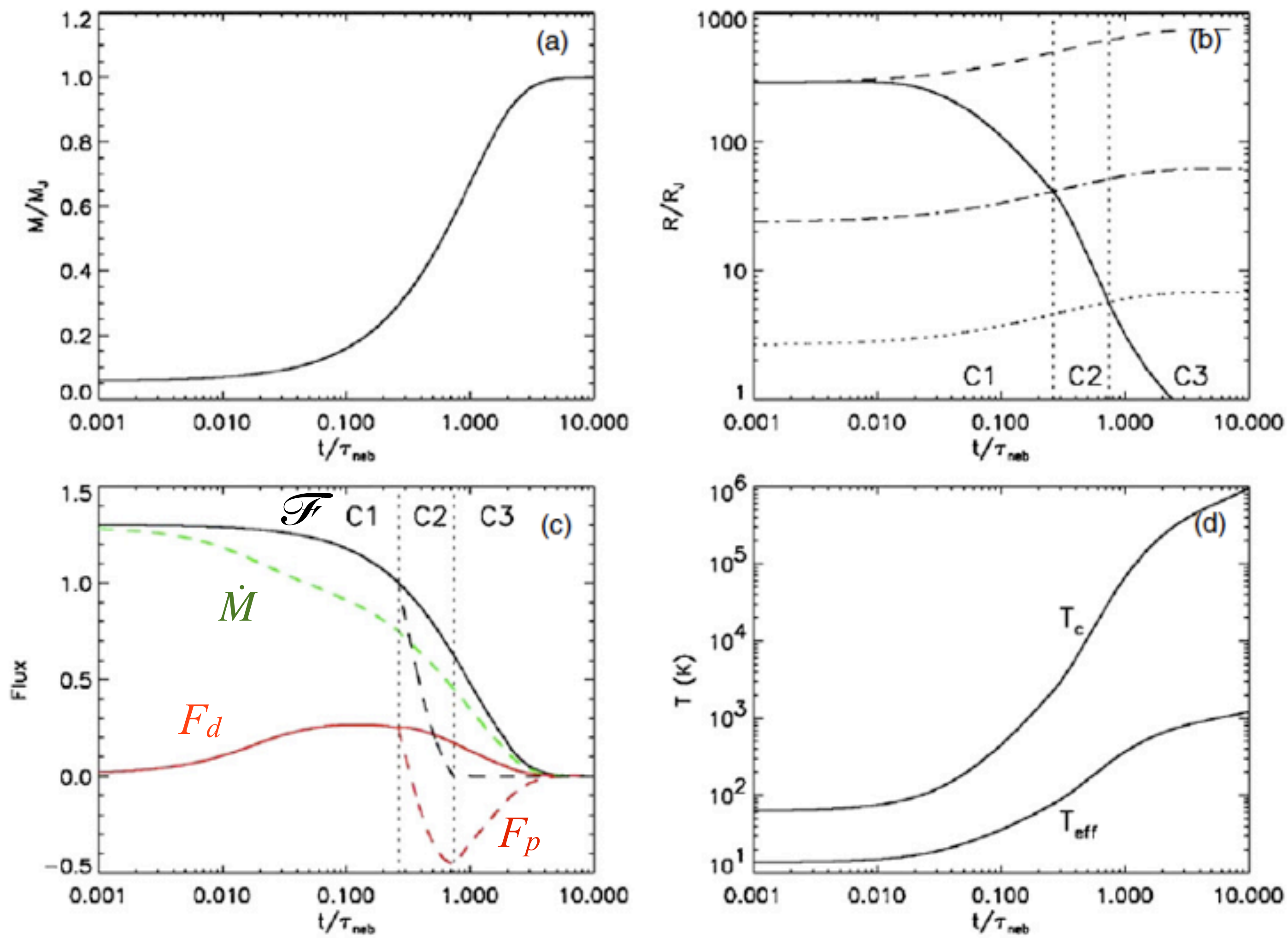

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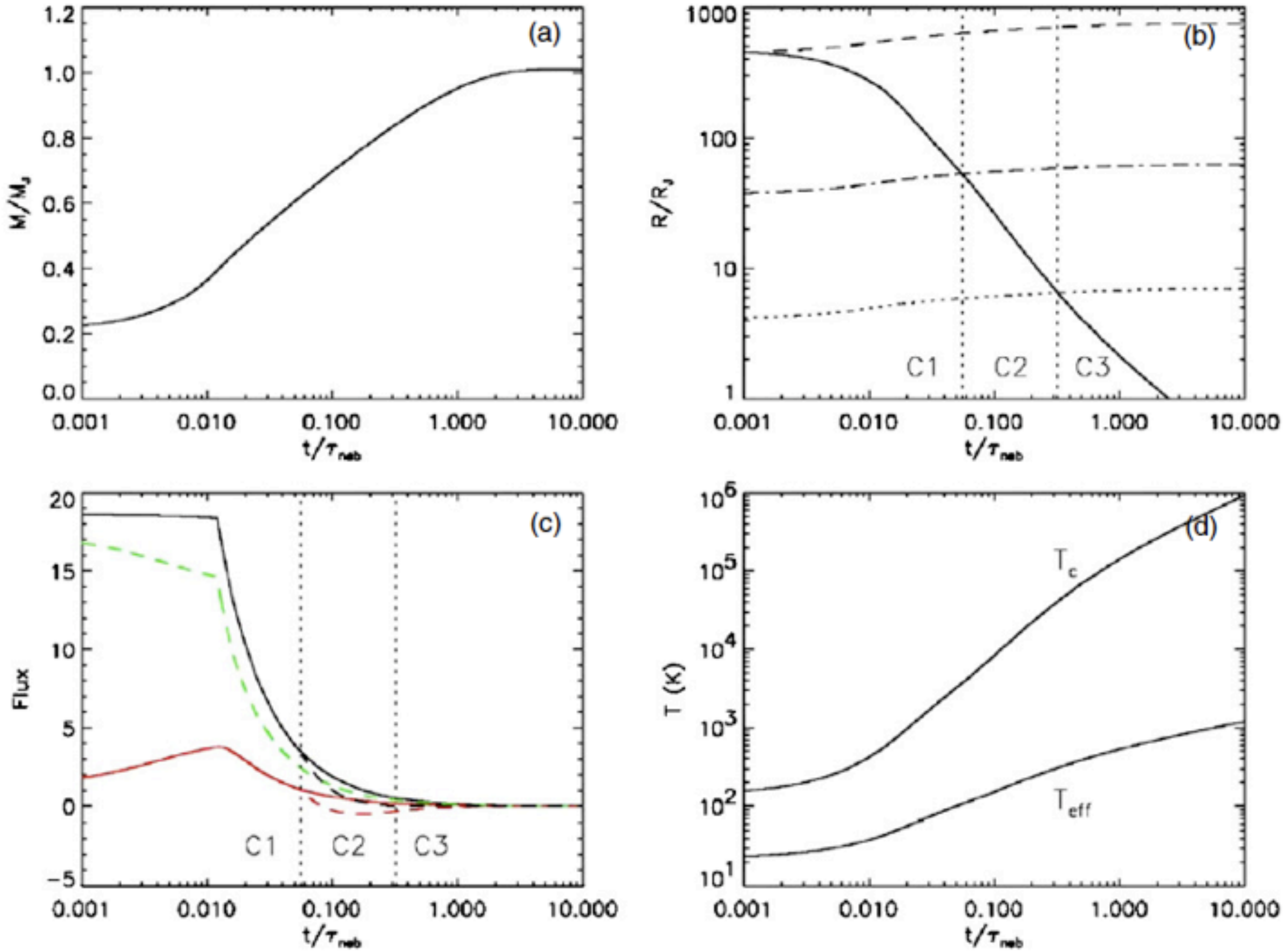

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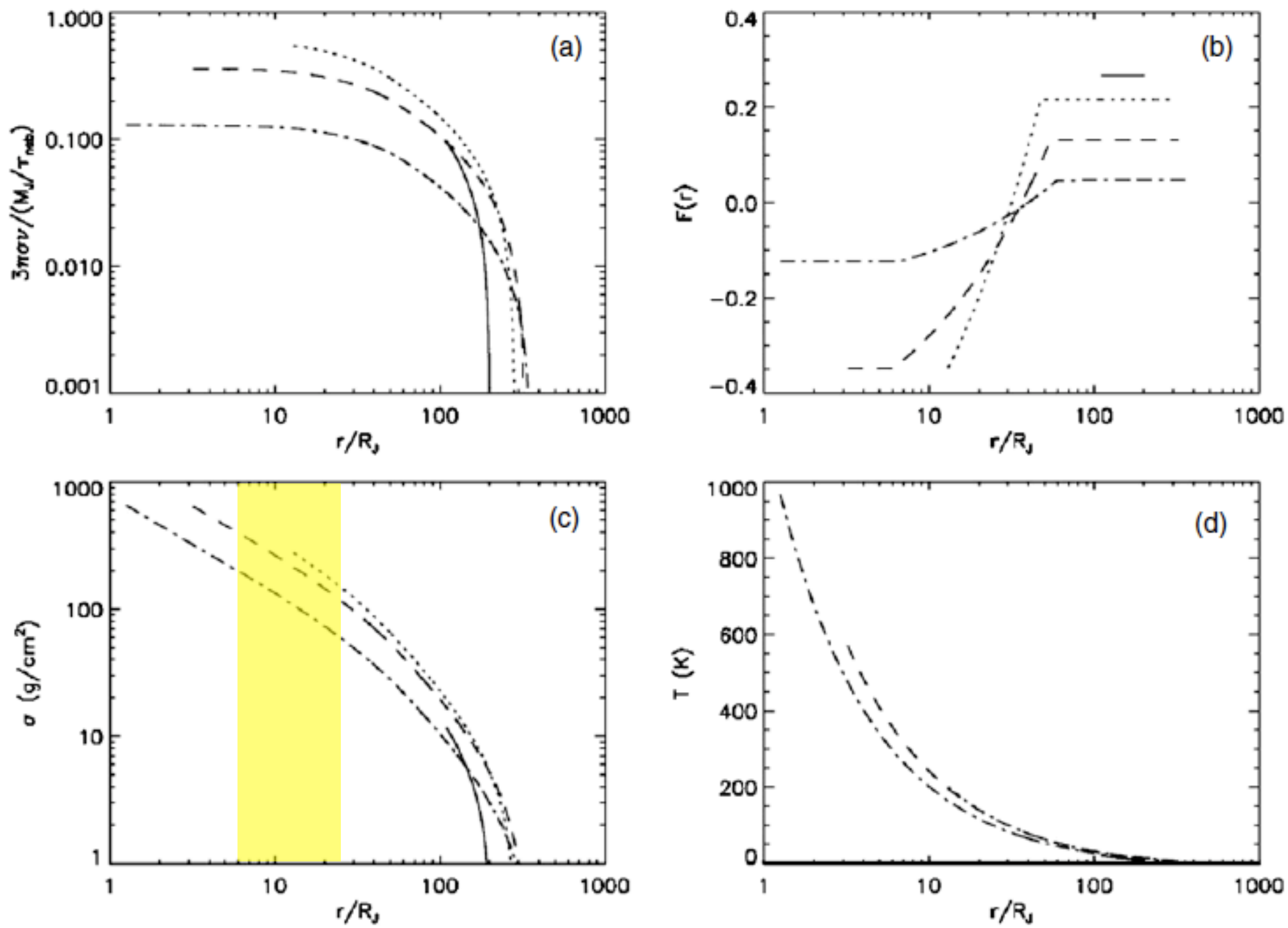

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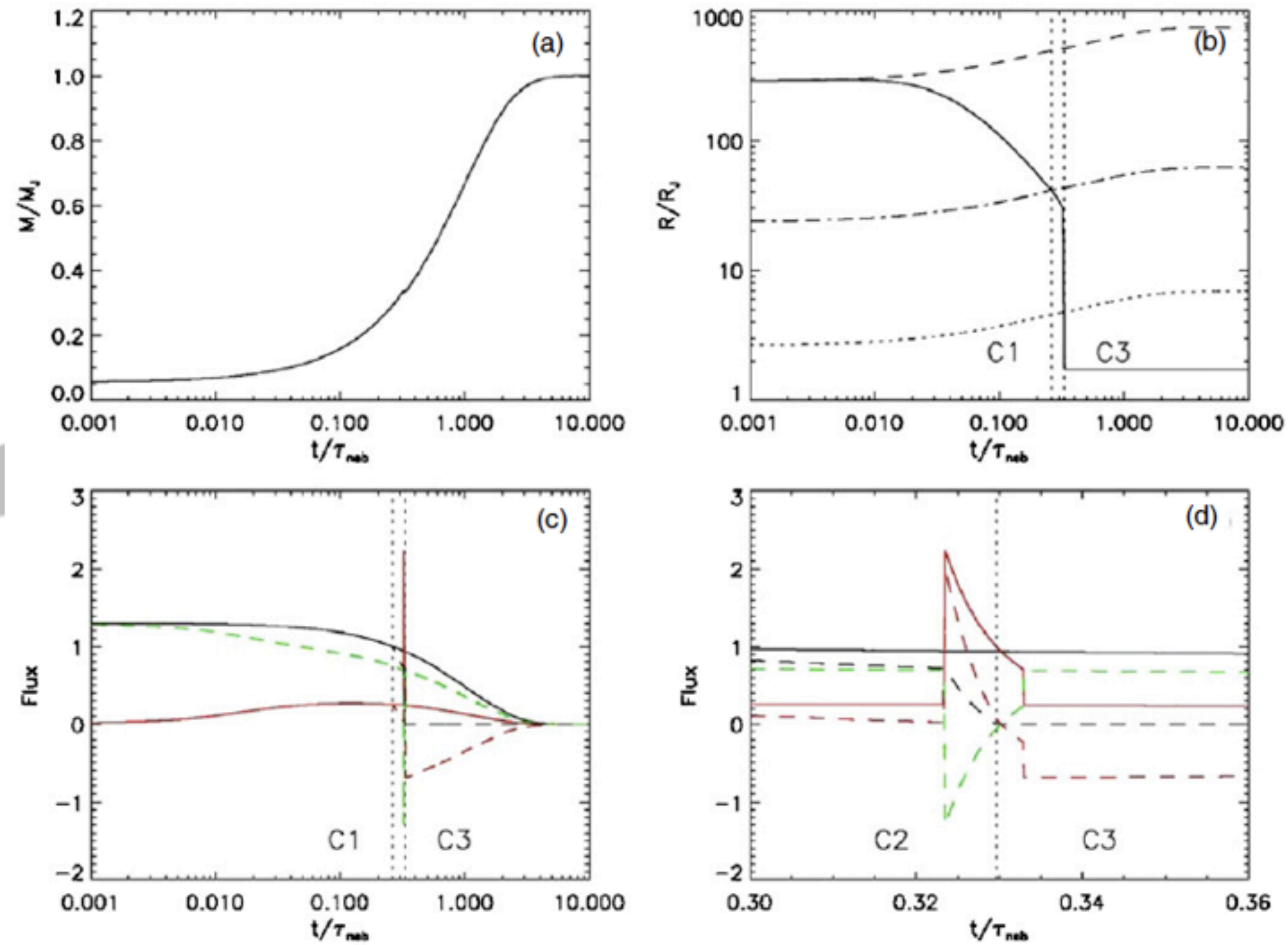

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1.- Circumplanetary disks form via spin-out from planet and inflow, then transitions to accretion.

2.- Peak gas surface densities are ~ 10^2-10^3 g/cm^2

3.- Temperatures in the disk are low enough to condense ices.

4.- During the evolution there might be a brief collapse phase.