Midterm
Condensed Matter Physics I
October 18, 2006

You may not use books, notes, or calculators. For full credit show all of your work and give complete explanations.

1. Why do crystals form (briefly)?

2. Describe the bcc lattice and its reciprocal lattice, giving or drawing primitive lattice vectors for each.

3. An X-ray beam traveling in direction $\hat{\mathbf{k}}$ has a range of wavelengths from λ_1 to λ_2. The beam strikes a crystal that is a Bravais lattice. Describe the scattering.

4. The single-particle model has Hamiltonian

$$H = \sum_{\ell=1}^{N} \left(-\frac{\hbar^2 \nabla_\ell^2}{2m} + U(\mathbf{r}_\ell) \right).$$

   Give expressions for N-electron energy eigenfunctions. Show that they are indeed eigenfunctions.

5. Single-particle energy eigenstates for some one-dimensional system have energies

$$\epsilon_k = a(1 - \cos ka).$$

   Derive the energy density of states.

6. Explain Bloch's theorem. (No proof is needed.)

7. Suppose the energy band eigenstates $\psi(\mathbf{r})$ are expanded in plane waves:

$$\psi(\mathbf{r}) = \sum_{\mathbf{q}} \psi(\mathbf{q}) e^{i\mathbf{q}\cdot\mathbf{r}}.$$

   The eigenfunction partial differential equation for $\psi(\mathbf{r})$ becomes a set of algebraic equations for the $\psi(\mathbf{q})$. Write down these equations. (No derivation is needed unless you cannot remember the equations.) Under what conditions are components $\psi(\mathbf{q})$ and $\psi(\mathbf{q}')$ strongly mixed?

8. Sketch the energy bands in a one-dimensional system in the presence of a periodic potential, in the extended zone scheme and in the reduced zone scheme.

9. List all of the point group operators for a rectangular (not square) two-dimensional Bravais lattice.

10. A one-dimensional lattice has the tight-binding Hamiltonian

$$\hat{H} = U \sum_{\ell} |\ell\rangle\langle\ell| + t \sum_{\ell} \left(|\ell\rangle\langle\ell + 1| + |\ell\rangle\langle\ell - 1| \right).$$

    Derive the energy eigenstates.

11. Describe how to derive the basis of states used in the Augmented Plane Wave method for calculating energy bands, in the case where there is one atom per unit cell. Do not actually carry out the derivation.

12. A square lattice with lattice constant a has Fermi wave vector $k_F = 1.2\,\pi/a$. Sketch the Fermi surface.