HW 8 Computational Physics

October 20, 2005 Due October 25

1. The operator $\mathcal{L} = i\,d/dx$ has as its domain the set of all differentiable functions on the interval $0 \leq x \leq L$ with periodic boundary conditions $f(0) = f(L)$. Derive the adjoint operator $\mathcal{L}^\dagger$ and its domain.

2. The operator $\mathcal{L} = i\,d/dx$ has as its domain the set of all differentiable functions on the interval $0 \leq x \leq L$ with boundary conditions $f'(0) = 0 = f'(L)$ (notice the two derivatives). Derive the adjoint operator $\mathcal{L}^\dagger$ and its domain.

3. The operator $\mathcal{L} = d^2/dx^2$ has as its domain the set of all twice-differentiable functions on the interval $0 \leq x \leq L$ with boundary conditions $f'(0) = 0 = f(L)$ (notice the one derivative). Derive the adjoint operator $\mathcal{L}^\dagger$ and its domain.

4. Find the eigenfunctions and eigenvalues of the operator $\mathcal{L} = i\,d/dx$ on the interval [0,L] with periodic boundary conditions $y(L) = y(0)$. Show that, properly normalized, the eigenfunctions form an infinite orthonormal set.

5. Find the eigenfunctions and eigenvalues of the operator $\mathcal{L} = i\,d/dx$ on the interval [0,L] with boundary conditions $y'(0) = 0 = y'(L)$ (notice the two derivatives).

6. Find the eigenfunctions and eigenvalues of the operator $\mathcal{L} = d^2/dx^2$ on the interval [0,L] with boundary conditions $y'(0) = 0 = y(L)$ (notice the one derivative). Show that, properly normalized, the eigenfunctions form an infinite orthonormal set.