HW 11 Computational Physics  
November 8, 2005  Due November 15

- Lea, Chapter 2, problems 7, 8(a), 10, 11, 14, 16. In 16, only take the derivative using $df/dz$.

1. The following integrals have a contour of integration $C$ which is the circle of radius 2 centered on the origin. Evaluate by hand, by substituting $z = 2e^{i\theta}$ and integrating over the real variable $\theta$.

   (a) $\oint \frac{dz}{z^2}$
   (b) $\oint \frac{dz}{z}$
   (c) $\oint dz$
   (d) $\oint z dz$

2. In this problem you will evaluate $\int_C \cos z \, dz$ several ways. Here $C$ is the line segment along $y = x$ beginning at $z = 0$ and ending at $z = 1 + i$.

   (a) Evaluate the integral using the indefinite integral and the endpoints.
   (b) Evaluate the integral by writing $z = x + iy$, using $y = x$, and turning the integral into an integral over the real variable $x$.
   (c) Now change the contour to a path from $z = 0$ to $z = 1$ along the real axis, followed by a vertical path from $z = 1$ to $z = 1 + i$. Do the integral of $\sin z$ along this two-step contour by hand (not using endpoints).

3. Let $f(z) = z^\frac{3}{2}$ have a branch cut along the negative real axis, and let $C$ be the semicircle of radius 1 beginning at $i$, moving counterclockwise, and ending at $-i$. This path crosses the cut. Evaluate $\int_C z^\frac{1}{2} \, dz$ using endpoints. Be careful when you cross the cut to stay within the correct branch. Then the integrand is discontinuous across the cut and as a result you need to consider four endpoints.