KEY

Midterm 1
PHY 2049, January 29, 2004
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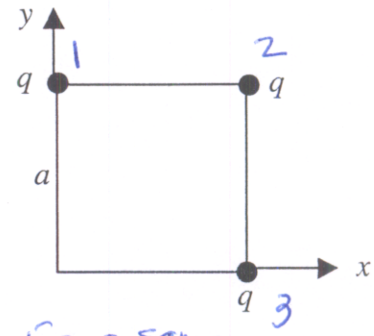
You may not use books, notes, or calculators. For full credit show your work. Simply writing down the answer will get little or no credit. If you use extra sheets of paper please put your name on each page.

1. Identical charges q are placed at three corners of a square of side a, as shown.



   a. What is the electric field at the lower left corner of the square? (10 pts)

$E_1 = \frac{kq}{a^2}$ $\quad E_{1x} = 0 \quad E_{1y} = -\frac{kq}{a^2}$

$E_2 = \frac{kq}{a^2+a^2}$ $\quad E_{2x} = -E_2\cos 45° = -\frac{kq}{2a^2}\frac{1}{\sqrt{2}}$ $\quad E_{2y} = \text{same}$

$E_3 = \frac{kq}{a^2}$ $\quad E_{3x} = -\frac{kq}{a^2} \quad E_{3y} = 0$

$E_x = E_{1x} + E_{2x} + E_{3x} = -\frac{kq}{a^2} - \frac{kq}{2\sqrt{2}a^2} = -\frac{kq}{a^2}\left(1 + \frac{1}{2\sqrt{2}}\right)$

$E_y = E_{1y} + E_{2y} + E_{3y}$ = " = " "

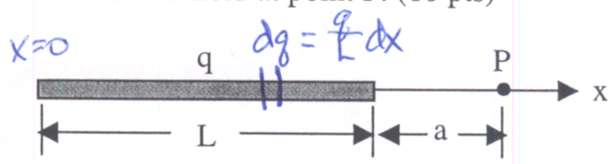
   b. What force would be felt by a charge $-q$ placed at the lower left corner? (5 pts)

$\vec{F} = (-q)\vec{E}$ where $\vec{E}$ is from (a).

$F_x = \frac{kq^2}{a^2}\left(1 + \frac{1}{2\sqrt{2}}\right)$

$F_y =$ same

2. A charge q is uniformly distributed along a thin non-conducting rod of length L. Calculate the electric field at point P. (10 pts)



$\vec{E} = \hat{\imath} E_x$

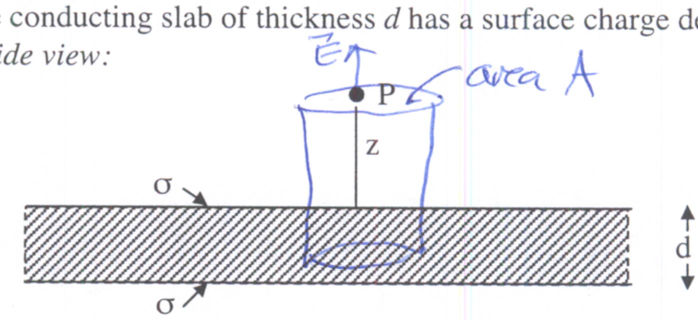
$$dE_x = \frac{k\,dq}{(L+a-x)^2} = \frac{kq}{L}\frac{dx}{(L+a-x)^2}$$

$$E_x = \frac{kq}{L}\int_0^L \frac{dx}{(x-a-L)^2} = \frac{kq}{L}(-1)\frac{1}{x-a-L}\Big|_0^L$$

$$= \frac{kq}{L}(-1)\left(\frac{1}{-a} - \frac{1}{-a-L}\right) = \frac{kq}{L}\left(\frac{1}{a} - \frac{1}{a+L}\right)$$

$$= \frac{kq}{L}\frac{L}{a(a+L)} = \frac{kq}{a(a+L)}$$

3. An infinite conducting slab of thickness d has a surface charge density σ on each surface. *Side view:*



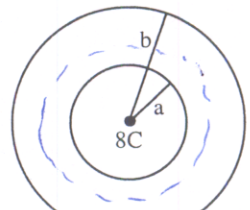
   a. What is the electric field within the slab? (5 pts) $\vec{E} = 0$ within a metal in equilibrium.

   b. Use Gauss's law to calculate the electric field at the point P. (10 pts)

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0} \qquad EA = \frac{\sigma A}{\varepsilon_0} \qquad E = \frac{\sigma}{\varepsilon_0}$$

$\vec{E} = \frac{\sigma}{\varepsilon_0}$, away from surface

4. A hollow spherical conducting shell has inner radius a and outer radius b. A point charge 8C is placed at the center of the cavity.



a. If the conductor is neutral, what is the net charge on the inner surface and on the outer surface? (5 pts)

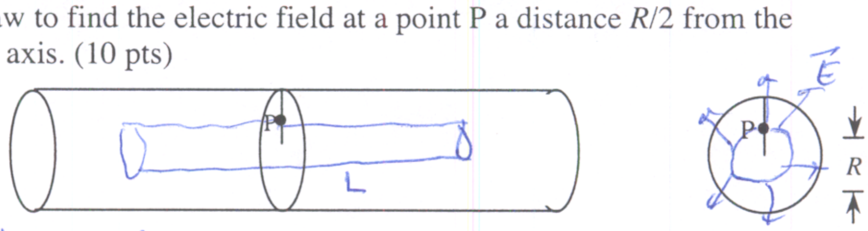
Gauss's law on ◌ requires $Q_{enc} = 0$ so have −8C on inner surface.

Then must have 8C on outer surface to stay neutral

b. Now suppose 3C of charge is added to the conductor (while keeping the 8C charge fixed at the center). What is the net charge on the inner surface and on the outer surface? (5 pts)

The same argument says still have −8C on inner surface. Then must have 11C on outer surface so that $(-8C) + (11C) = 3C$.

5. Charge of uniform density ρ fills an infinitely long cylinder of radius R. Use Gauss's law to find the electric field at a point P a distance $R/2$ from the cylinder's axis. (10 pts)



Use cylinder of length L & radius R/2 for Gaussian surface.

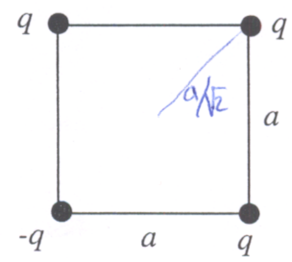
$\vec{E}$ radially outward $\Rightarrow$ $\oint \vec{E}\cdot d\vec{a} = E \cdot (\text{curved area}) = E \cdot 2\pi \frac{R}{2} \cdot L$

$Q_{enc} = \rho \cdot \pi \left(\frac{R}{2}\right)^2 \cdot L$

$$E \cdot 2\pi \frac{R}{2} \cdot L = \frac{1}{\varepsilon_0} \cdot \rho \cdot \pi \left(\frac{R}{2}\right)^2 \cdot L$$

$$E = \frac{1}{\varepsilon_0} \rho \cdot \frac{R}{4} \qquad \vec{E} = \frac{\rho R}{4\varepsilon_0} \text{ radially outward}$$

6. Four charges are arranged at the corners of a square of side a, as shown.



a. With V=0 at infinity, what is the net electric potential at the center of the square due to the four charges? (10 pts)

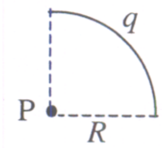
Each q gives $\frac{kq}{a/\sqrt{2}}$ so $V = (3-1)\cdot\frac{kq\sqrt{2}}{a}$

$-q$ gives $-(\text{ " })$ $= \frac{2\sqrt{2}\,kq}{a}$

b. What is the potential energy of the charges? (10 pts)

Pairs: $U = \frac{kq^2}{a} + \frac{kq^2}{\sqrt{2}a} - \frac{kq^2}{a} + \frac{kq^2}{a} - \frac{kq^2}{\sqrt{2}a} - \frac{kq^2}{a} = 0$

7. A charge q is spread along one quarter of a circle of radius R. What is the electric potential at point P, the circle's center? (10 pts)



Every bit at same distance R from P

so $V = \frac{kq}{R}$

8. If the electric potential as a function of position is $V(x,y)=3x^2y$, what is the electric field? (10 pts)

$$\vec{E} = -\vec{\nabla} V$$

$$E_x = -\frac{\partial}{\partial x}(3x^2y) = -6xy$$

$$E_y = -\frac{\partial}{\partial y}(3x^2y) = -3x^2$$

$$\vec{E} = -6xy\hat{\imath} - 3x^2\hat{\jmath}$$