Chapter 10
Simple Harmonic Motion and Elasticity

Goals for Chapter 10
• To follow periodic motion to a study of simple harmonic motion.
• To solve equations of simple harmonic motion.
• To use the pendulum as a prototypical system undergoing simple harmonic motion.
• To study how oscillations may be damped or driven.
• To study stress, strain and elastic deformation.
• To define elasticity and plasticity.

Periodic Motion
Periodic motion or oscillations: An object repeats the same motion over and over; e.g. rising of the sun, rotation of a bicycle wheel, mass on a spring, ...

The Ideal Spring and Simple Harmonic Motion

Hooke's Law: Restoring Force of an Ideal Spring

The restoring force of an ideal spring is

$$F_x = -kx$$
Simple Harmonic Motion and Uniform Circular Motion

- A ball is attached to the rim of a turntable of radius A
- The focus is on the shadow that the ball casts on the screen
- When the turntable rotates with a constant angular speed, the shadow moves in simple harmonic motion

Connection between Rotational and Oscillatory Motion

\[
\begin{align*}
\phi(t) &= \cos(Rx) \\
Y(t) &= R \sin(\phi) \\
X(t) &= R \cos(\phi)
\end{align*}
\]

Simple Harmonic Motion and the Reference Circle

DISPLACEMENT

\[
x(t) = A \cos(\theta) = A \cos(\omega t)
\]

\[
\theta = \omega t
\]

Amplitude

- Amplitude, A
  - The amplitude is the maximum position of the object relative to the equilibrium position
  - In the absence of friction, an object in simple harmonic motion will oscillate between ±A on each side of the equilibrium position

Period and Frequency

- The period, T, is the time that it takes for the object to complete one complete cycle of motion
  - From \( x = A \) to \( x = -A \) and back to \( x = A \)
- The frequency, \( f \), is the number of complete cycles or vibrations per unit time

\[
f = \frac{1}{T} \\
\omega = 2 \pi f = \frac{2 \pi}{T}
\]
Example

The graph shown in the figure closely approximates the displacement $x$ of a tuning fork as a function of time $t$ as it is playing a single note. What are

(a) the amplitude, $(A = 0.4\, \text{mm})$
(b) period, $(T = 2\, \text{ms})$
(c) frequency $(f = 1/T = 1/2\, \text{ms}) = 500\, \text{Hz}$

Example: Maximum Speed of a Loudspeaker Diaphragm

The frequency of motion is 1.0 KHz and the amplitude is 0.20 mm.
- What is the maximum speed of the diaphragm?
- Where in the motion does this maximum speed occur?

$v_x = -v_t \sin \theta = -A\omega \sin \omega t$

$v_{\text{max}} = A\omega = A(2\pi f)$

$v_{\text{max}} = (0.20 \times 10^{-3}\, \text{m})(2\pi)(1.0 \times 10^3\,\text{Hz}) = 1.3\, \text{m/s}$

The maximum speed occurs midway between the ends of its motion.

Example: Maximum Acceleration of a Loudspeaker Diaphragm

The frequency of motion is 1.0 KHz and the amplitude is 0.20 mm.
- What is the maximum acceleration of the diaphragm?

$a_x = -a_x \cos \theta = -A\omega^2 \cos \omega t$

$a_{\text{max}} = A\omega^2 = A(2\pi f)^2$

$a_{\text{max}} = (0.20 \times 10^{-3}\, \text{m})(2\pi)^2(1.0 \times 10^3\,\text{Hz})^2$

$a_{\text{max}} = 7.9 \times 10^7\, \text{m/s}^2$ huge

Simple Harmonic Motion (SHM)

For SHM: Restoring force is directly proportional to the displacement from equilibrium

Hooke's law force
- The force always acts toward the equilibrium position
  - It is called the restoring force
- The direction of the restoring force is such that the object is being either pushed or pulled toward the equilibrium position

Spring - mass system
example for Hooke's law
and simple harmonic motion
**Frequency of Vibration**

\[ x = A \cos \omega t \]
\[ a_x = -A\omega^2 \cos \omega t \]

\[ \sum F = -kx = ma_x \]
\[ -kA = -mA\omega^2 \]
\[ \omega = \sqrt{\frac{k}{m}} \]

**Oscillation frequency of a mass spring system**

- The force is given by Hooke’s Law
  \[ F = -kx = ma_x \]
- acceleration for simple harmonic motion
  \[ a_x = -\omega^2 x \]

\[ \Rightarrow -\frac{k}{m}x = -\omega^2 x \]
\[ \omega = 2\pi f = \sqrt{\frac{k}{m}} \]

\[ f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

- Frequency
  - Units are cycles/second or Hertz, Hz
- Period
  - This gives the time required for an object of mass \( m \) attached to a spring of constant \( k \) to complete one cycle of its motion

**Example: A Body Mass Measurement Device**

The device consists of a spring-mounted chair in which the astronaut sits. The spring has a spring constant of 606 N/m and the mass of the chair is 12.0 kg. The measured period is 2.41 s. Find the mass of the astronaut.

**Given \( k \) and \( T \), find \( m_{astro} \)**

\[ \omega = \sqrt{\frac{k}{m_{total}}} \]
\[ m_{total} = \frac{k}{2\pi T}^2 = m_{chair} + m_{astro} \]
\[ m_{astro} = \frac{k}{2\pi T}^2 - m_{chair} \]
\[ = \frac{(606 \text{ N/m})(2.41 \text{ s})^2}{4\pi^2} - 12.0 \text{ kg} = 77.2 \text{ kg} \]

**Summary: Simple Harmonic Oscillator (t)**

\[ x = A \cos \omega t \]
\[ v_x = -A \omega \sin \omega t \]
\[ a_x = -A \omega^2 \cos \omega t \]

**Initial Position and Phase Shift:**

The most general solution is \( x = A \cos(\omega t + \phi) \)

where \( A \) = amplitude
\( \omega \) = frequency
\( \phi \) = phase
Example
A mass \( m = 2 \text{ kg} \) on a spring oscillates with amplitude \( A = 10 \text{ cm} \).
At \( t = 0 \) its speed is maximum, and is \( v_{\text{max}} = +2 \text{ m/s} \).

What is the angular frequency of oscillation \( \omega \)?
What is the spring constant \( k \) ?

\[
\omega = \frac{v_{\text{max}}}{A} = \frac{2 \text{ m/s}}{10 \text{ cm}} = 20 \text{ s}^{-1}
\]

Also:
\[
k = m \omega^2
\]

So \( k = (2 \text{ kg}) \times (20 \text{ s}^{-1})^2 = 800 \text{ kg/s}^2 = 800 \text{ N/m} \)

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Energy and Simple Harmonic Motion

A compressed spring can do work.

\[
W_{\text{elastic}} = \left( F \cos \theta \right) s
\]

\[
= \frac{1}{2} (kx_a + kx_f) \cos \theta (x_a - x_f)
\]

\[
W_{\text{elastic}} = \frac{1}{2} kx_a^2 + \frac{1}{2} kx_f^2
\]

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Energy and Simple Harmonic Motion

DEFINITION OF ELASTIC POTENTIAL ENERGY

The elastic potential energy is the energy that a spring has by virtue of being stretched or compressed. For an ideal spring, the elastic potential energy is

\[ PE_{\text{elastic}} = \frac{1}{2} kx^2 \]

SI Unit of Elastic Potential Energy: joule (J)

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Conceptual Example 2
Are Shorter Spring stiffer Spring?

A 10-coil spring has a spring constant \( k \). If the spring is cut in half, so there are two 5-coil springs, what is the spring constant of each of the smaller springs?

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Final covers up to page 296.