Chapter 9
Rotational Dynamics

Goals for Chapter 9

- To study **torque**.
- To study how **torques add a new variable to equilibrium**.

- To relate **angular acceleration** and **torque**.
- To examine **rotational work** and include time to study rotational power.

- To understand **angular momentum**.
- To examine the implications of **angular momentum conservation**.

Force and Torque

- **Consider force required to open door. Is it easier to open the door by pushing/pulling away from hinge or close to hinge?**

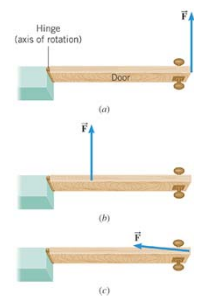
Farther from from hinge, larger rotational effect!

Physics concept: torque

Action of Forces and Torques on Rigid Objects

According to Newton's second law, a net force causes an object to have an acceleration.

What causes an object to have an *angular acceleration*?



TORQUE

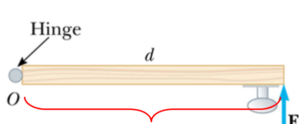
The amount of torque depends on where and in what direction the force is applied, as well as the location of the axis of rotation.

Torque

- **Torque, τ, is the tendency of a force to rotate an object about certain axis**

Door example:

$$\boxed{\tau = Fd}$$



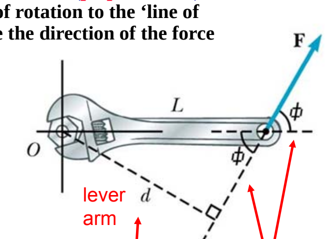
τ is the torque
- **d is the *lever arm* (or moment arm)**
- **F is the force**

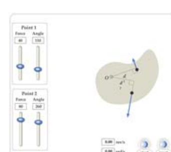
- **Torque (τ) is defined as the force applied multiplied by the lever (moment) arm.**

Lever Arm

- **The lever arm, d, is the shortest *(perpendicular)* distance from the axis of rotation to the 'line of action' drawn along the the direction of the force**

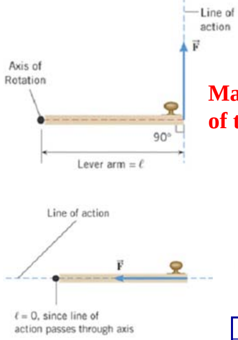
- **It is not necessarily the distance between the axis of rotation and point where the force is applied**





$$d = L \sin \phi$$

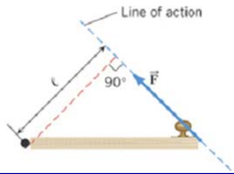
Torque and Lever Arm



DEFINITION OF TORQUE

$$\tau = F\ell$$

Magnitude of Torque = (Magnitude of the force) x (<u>Lever arm</u>)



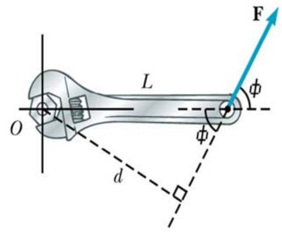
Units of Torque	
SI	Newton meter (Nm)
US Customary	Foot pound (ft lb)

Direction of Torque

- **Torque is a vector quantity**

We work with its magnitude

- The direction is perpendicular to the plane determined by the lever arm and the force
- **Direction and sign:**
  - **If the turning tendency of the force is counterclockwise, the torque will be positive**
  - **If the turning tendency is clockwise, the torque will be negative**



<u>Vector multiplication</u>

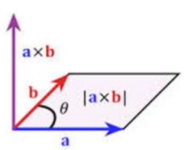
1. Vector dot product

$$C = \vec{A}\cdot\vec{B} = A\cdot B\cdot\cos\theta$$

2. Vector cross product

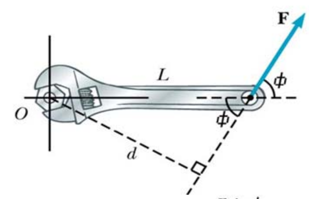
$$\vec{C} = \vec{A}\times\vec{B} = A\cdot B\cdot\sin\theta\hat{k}$$

$\hat{k}$ is a unit vector, $\hat{k}\perp\vec{A}$ and $\hat{k}\perp\vec{B}$



An Alternative Look at Torque

- The force could also be resolved into its x- and y-components
  - The x-component, F cos Φ, produces 0 torque
  - The y-component, F sin Φ, produces a non-zero torque

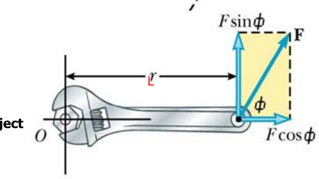


$$\tau = FL\sin\phi$$

F is the force

L is the distance along the object

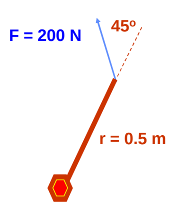
Φ is the angle between force and object



Example: Torque

- To loosen a stuck nut, a man pulls at an angle of 45° on the end of a *50 cm* wrench with a force of *200 N*.

What is the magnitude of the torque on the nut?

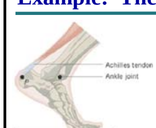


$$\tau = F\,r\sin\phi$$

τ = (200 N) (0.5 m) sin45°

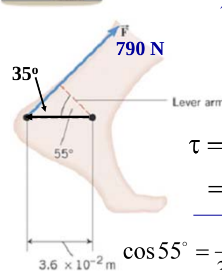
= 70.7 Nm

Example: The Achilles Tendon



The tendon exerts a force of magnitude 790 N. Determine the torque (magnitude and direction) of this force about the ankle joint, which is located 3.6 cm away from the joint.

$$\tau = F\ell$$



$$\sin 35^\circ = \frac{\ell}{3.6\times10^{-2}\,\text{m}}$$

$$\tau = (720\ \text{N})(3.6\times10^{-2}\,\text{m})\sin 35^\circ$$

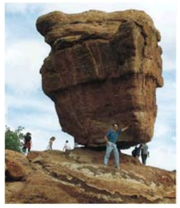
$$= 15\ \text{N}\cdot\text{m}$$

Clockwise, $\tau < 0$

$$\cos 55^\circ = \frac{\ell}{3.6\times10^{-2}\,\text{m}}$$

9.2 Rigid Objects in Equilibrium

If a rigid body is in equilibrium, neither its linear motion nor its rotational motion changes.



no acceleration !

$$a_x = a_y = 0$$

$$\boxed{\begin{array}{c}\Sigma \vec{F} = 0 \\ \Sigma F_x = 0 \text{ and } \Sigma F_y = 0\end{array}}$$

The net external force must be zero

- This is a necessary, but not sufficient, condition to ensure that an object is in complete mechanical equilibrium
- This is a statement of translational equilibrium

no angular acceleration !

$$\alpha = 0 \qquad \boxed{\Sigma\tau = 0}$$

- The net external torque must be zero
- This is a statement of rotational equilibrium

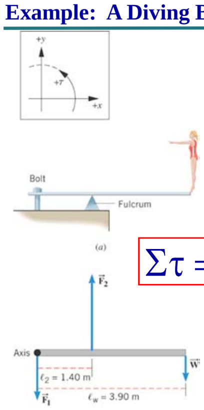
Rigid Objects in Equilibrium

Reasoning Strategy

1. Select the object to which the equations for equilibrium are to be applied.
2. Draw a free-body diagram that shows all of the external forces acting on the object.
3. Choose a convenient set of *x*, *y* axes and resolve all forces into components that lie along these axes.
4. Apply the equations that specify the balance of forces at equilibrium. (Set the net force in the *x* and *y* directions equal to zero.)
5. Select a convenient axis of rotation. Set the sum of the torques about this axis equal to zero
6. Solve the equations for the desired unknown quantities.

Example: A Diving Board

A woman whose weight is 530 N is poised at the right end of a diving board with length 3.90 m. The board has negligible weight and is supported by a fulcrum 1.40 m away from the left end. Find the forces that the bolt and the fulcrum exert on the board.



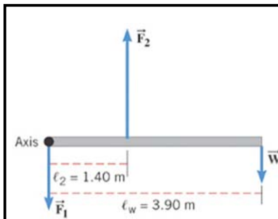
(b) Free-body diagram of the diving board

$$\boxed{\Sigma\tau = 0}$$

$$\sum \tau = F_2 \ell_2 - W \ell_W = 0$$

$$F_2 = \frac{W \ell_W}{\ell_2}$$

$$F_2 = \frac{(530\text{ N})(3.90\text{ m})}{1.40\text{ m}} = 1480\text{ N}$$



from torque equation $\quad F_2 = 1480\text{ N}$

$$\boxed{\sum F_x = 0}$$

Here: no forces in x

$$\boxed{\sum F_y = 0}$$

$$\sum F_y = -F_1 + F_2 - W = 0$$

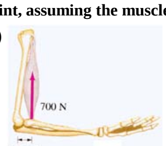
$$-F_1 + 1480\text{ N} - 530\text{ N} = 0$$

$$F_1 = 950\text{ N}$$

Example: Biceps torque

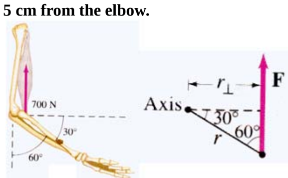
The biceps muscle exerts a vertical force on the lower arm. Two lower arm positions are shown corresponding to different angles relative to the horizontal. For each case, calculate the torque about the axis of rotation through the elbow joint, assuming the muscle is attached 5 cm from the elbow.

a)



$$\tau = r_\perp F = 0.05m \cdot 700N$$
$$= 35\,Nm$$

b)

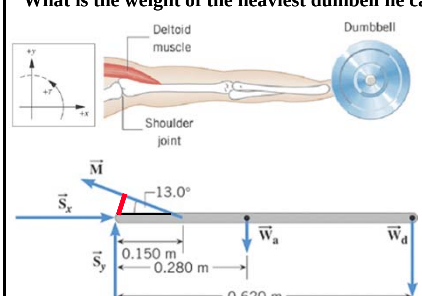


$$r_\perp = 0.05m \cdot \sin 60°$$
$$\tau = 0.05m \cdot \sin 60°(700N)$$
$$= 30\,Nm$$

Note: The attachment point of the biceps muscle to the forearm is much further away from the elbow in chimps than in humans.

Example: Bodybuilding

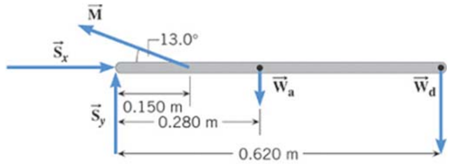
The arm is horizontal and weighs 31.0 N. The deltoid muscle can supply 1840 N of force. Distances are indicated in the figure. What is the weight of the heaviest dumbell he can hold?



$$\boxed{\sum F_x = 0}$$

$$\boxed{\sum F_y = 0}$$

$$\boxed{\Sigma\tau = 0}$$



$$\sum F_x = S_x - M\cos 13^o = 0$$

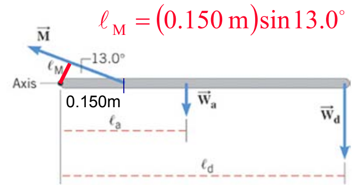
$$S_x = M\cos 13^o = 1840\,N\ \cos 13^o = 1790\,N$$

$$\sum F_y = S_y + M\sin 13^o - W_a - W_d = 0$$

At this point two unknowns: S_y and W_d

$$\ell_M = (0.150\text{ m})\sin 13.0^\circ$$

$$\boxed{\Sigma\tau = 0}$$

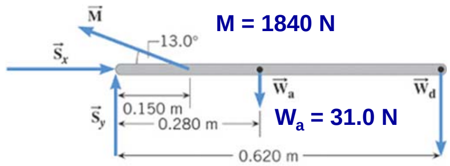


$$\sum\tau = -W_a\ell_a - W_d\ell_d + M\ell_M = 0$$

$$W_d = \frac{-W_a\ell_a + M\ell_M}{\ell_d}$$

$$= \frac{-(31.0\text{ N})(0.280\text{ m}) + (1840\text{ N})(0.150\text{ m})\sin 13.0^\circ}{0.620\text{ m}} = 86.1\text{ N}$$

M = 1840 N



W_a = 31.0 N

S_x = 1790 N

$\boxed{W_d = 86.1\text{ N}}$

Back to second force equ.

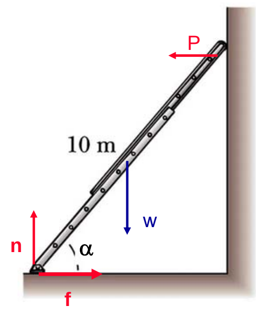
$$S_y - M\sin 13^o - W_a - W_d = 0$$

$$S_y = 1840N\sin 13^o - 31.0N - 86.1N$$

$$S_y = -297\,N$$

Example: Equilibrium of a ladder

Suppose that you placed a 10 m ladder (which weights 100 N) against the wall at the angle of 30°. What are the forces acting on it and when would it be in equilibrium?

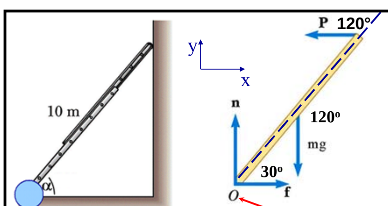


Given:

weights: w_1 = 100 N
length: l = 10 m
angle: α = 30°
Στ = 0

Find:

***f* = ?**
n = ?
P = ?



Forces:

$$\sum F_x = f - P = 0$$

$$f = P$$

$$\sum F_y = n - mg = 0$$

$$n = mg = 100\,N$$

1. Draw all applicable forces

Torques:

2. Choose axis of rotation at bottom corner (τ of *f* and *n* are 0!)

$$\sum\tau = mg\frac{L}{2}\sin(-120^\circ) + PL\sin(120^\circ) = 0$$

Note: f = μ_s n, so

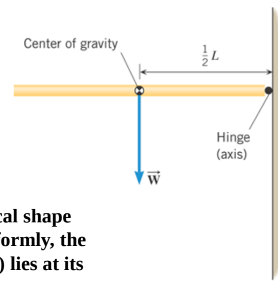
$$0 = -100\,N\cdot\frac{1}{2}\cdot 0.866 + P\cdot 1\cdot\frac{1}{2}$$

$$\mu_s = \frac{f}{n} = \frac{86.6\,N}{100\,N} = 0.866$$

$$P = 86.6\,N$$

9.3 Center of Gravity

The center of gravity of a rigid body is the point at which its weight can be considered to act when the torque due to the weight is being calculated.



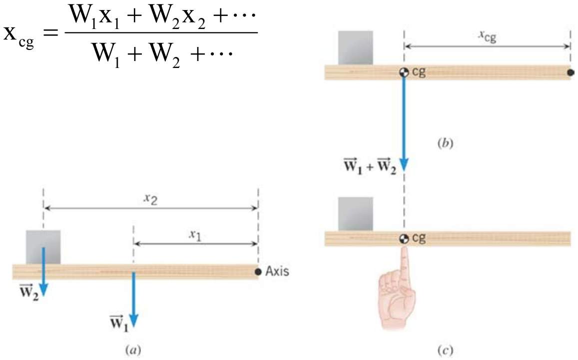
When L << R_E

≡ Center of Mass

When an object has a symmetrical shape and its weight is distributed uniformly, the center of gravity (center of mass) lies at its geometrical center.

Center of Gravity

$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 + \cdots}$$

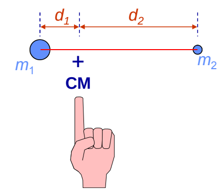


Center of Gravity (Mass) & Statics

- The center of mass is at the point where the system balances!
- Sum of all gravitational torques about an axis through the center of gravity (mass) = 0!

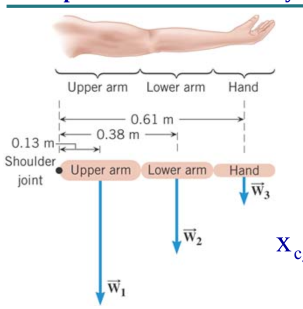
$$m_1 d_1 - m_2 d_2 = 0$$

$$\boxed{\frac{d_1}{d_2} = \frac{m_2}{m_1}}$$



Example 6: Center of Gravity of an Arm

The horizontal arm is composed of three parts: the upper arm (17 N), the lower arm (11 N), and the hand (4.2 N). Find the center of gravity of the arm relative to the shoulder joint.



$$x_{cg} = \frac{W_1 x_1 + W_2 x_2 + \cdots}{W_1 + W_2 + \cdots}$$

$$x_{cg} = \frac{(17\,\mathrm{N})(0.13\,\mathrm{m}) + (11\,\mathrm{N})(0.38\,\mathrm{m}) + (4.2\,\mathrm{N})(0.61\,\mathrm{m})}{17\,\mathrm{N} + 11\,\mathrm{N} + 4.2\,\mathrm{N}}$$
$$= 0.28\,\mathrm{m}$$

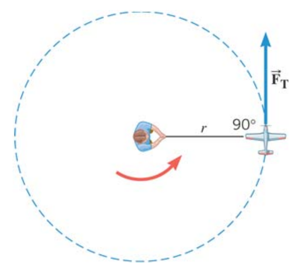
9.4 Newton's Second Law for Rotational Motion

$$F_T = m a_T$$

$$a_T = r\alpha$$

$$\tau = F_T r = m\, a_T\, r$$
$$= m\, r\, \alpha\, r$$

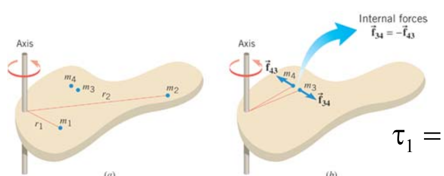
$$\tau = \left(m r^2\right)\alpha$$



Moment of Inertia, I

Newton's Second Law for Rotational Motion (fixed axis)

Rigid Body: Large number of 'fixed' particles



$$\tau_1 = \left(m_1 r_1^2\right)\alpha$$
$$\tau_2 = \left(m_2 r_2^2\right)\alpha$$
$$\vdots$$
$$\tau_N = \left(m_N r_N^2\right)\alpha$$

$$\sum \tau_i = \sum \left(m_i r_i^2\right)\alpha$$

Net external torque **Moment of inertia** **Angular acceleration**

Rotational Analog of Newton's Second Law

FOR A RIGID BODY ROTATING ABOUT A FIXED AXIS

$$\text{Net external torque} = \left(\begin{array}{l}\text{Moment of} \\ \text{inertia}\end{array}\right) \times \left(\begin{array}{l}\text{Angular} \\ \text{acceleration}\end{array}\right)$$

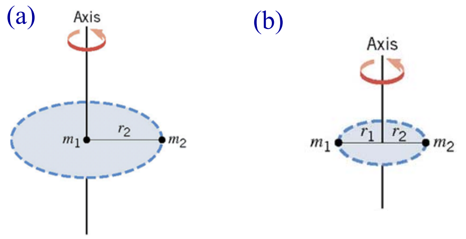
$$\boxed{\sum \tau = I\,\alpha}$$

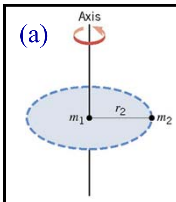
$$\boxed{I = \sum \left(m_i r_i^2\right)}$$

Requirement: **Angular acceleration must be expressed in radians/s².**

Example 9: Moment of Inertia - Depending on Where Axis Is

Two particles each have mass and are fixed at the ends of a thin rigid rod. The length of the rod is *L*. Find the moment of inertia when this object rotates relative to an axis that is perpendicular to the rod at (a) one end and (b) the center.

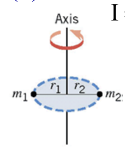




$$I = \sum (mr^2) = m_1 r_1^2 + m_2 r_2^2 = m(0)^2 + m(L)^2$$

$$m_1 = m_2 = m \qquad r_1 = 0 \quad r_2 = L$$

$$\boxed{I = mL^2}$$



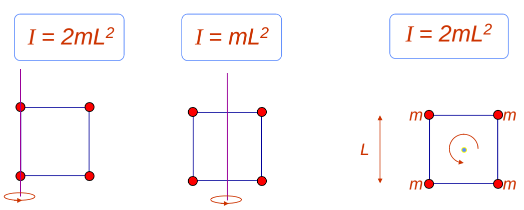
$$I = \sum (mr^2) = m_1 r_1^2 + m_2 r_2^2 = m(L/2)^2 + m(L/2)^2$$

$$m_1 = m_2 = m \qquad r_1 = L/2 \quad r_2 = L/2$$

$$\boxed{I = \tfrac{1}{2} mL^2}$$

Calculating Moment of Inertia

- **For multiple objects, *I* clearly depends on the rotation axis!!**

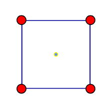


For a rigid body, moment of inertia, *I* also depends on the rotational axis.

Calculating Moment of Inertia

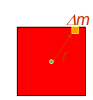
- For a discrete collection of point masses we found:

$$I = \sum_{i=1}^{N} m_i r_i^2$$



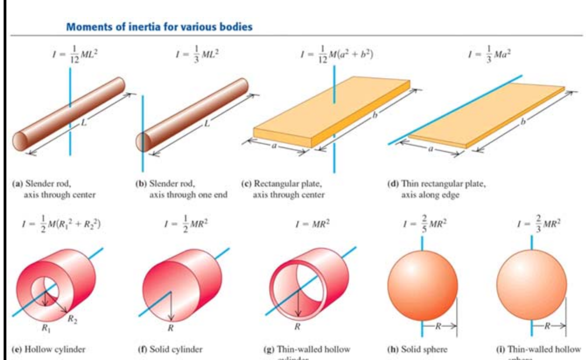
- For a continuous solid object we have to add up the $m\,r^2$ contribution for every infinitesimal mass element Δm.

$$I = \sum r_i^2 \, \Delta m_i$$



will become an integral !

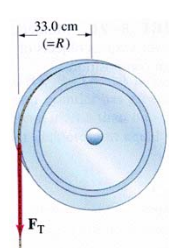
Some Examples of Moment of Inertia for Solid Objects



Example: A Heavy Pulley

A 15 N force is applied to a cord wrapped around a pulley of mass M = 4 kg and radius R = 33 cm. The pulley is observed to accelerate uniformly from rest to reach an angular speed of 30 rad / s in 3 s. Determine the moment of inertia of the pulley (rotates about center).

$$\boxed{\tau_{NET} = I\,\alpha}$$



$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30-0}{3} = 10\ rad/s^2$$

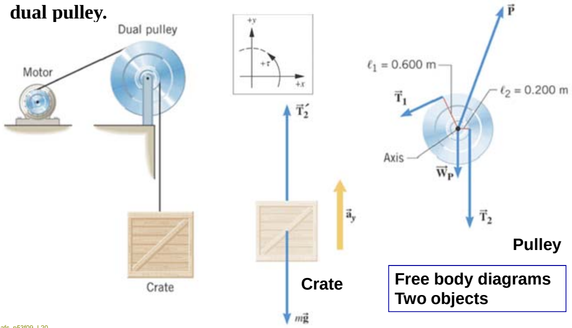
$$\tau_{net} = RF \sin 90° = 0.33m \cdot 15N$$

$$\tau_{net} = 4.95Nm$$

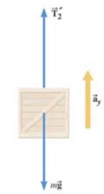
$$I = \tau_{NET}/\alpha = \frac{4.95\,\text{Nm}}{10\ \frac{\text{rad}}{\text{s}^2}} = 0.50\,\text{kg m}^2$$

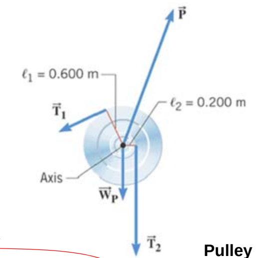
Example 12: Hoisting a Crate

The combined moment of inertia of the dual pulley is 50.0 kg·m². The crate weighs 4420 N. A tension of 2150 N is maintained in the cable attached to the motor. Find the angular acceleration of the dual pulley.



Free body diagrams
Two objects





equal

$$\sum F_y = T_2 - mg = ma_y \qquad \sum \tau = T_1\ell_1 - T_2\ell_2 = I\alpha$$

$$\boxed{T_2 = mg + ma_y}$$

$$T_1\ell_1 - \left(mg + ma_y\right)\ell_2 = I\alpha \qquad \boxed{T_2 = mg + ma_y}$$

$$a_y = \ell_2\alpha$$

$$T_1\ell_1 - \left(mg + m\ell_2\alpha\right)\ell_2 = I\alpha$$

$$T_1\ell_1 - mg\,\ell_2 - m\,\ell_2{}^2\alpha = I\alpha$$

$$\boxed{\alpha = \frac{T_1\ell_1 - mg\ell_2}{I + m\ell_2^2}}$$

$$\alpha = \frac{(2150\text{ N})(0.600\text{ m}) - (451\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m})}{46.0\text{ kg}\cdot\text{m}^2 + (451\text{ kg})(0.200\text{ m})^2}$$

$$= 6.3\text{ rad/s}^2$$

9.5 Rotational Work and Energy

Definition of Rotational Work

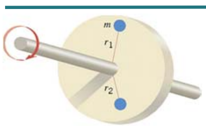
The rotational work W_R done by a constant torque τ in turning an object through an angle θ is

$$W_R = \tau\theta \qquad \textit{Rotational work}$$

$$W_T = F \cdot D \qquad \textit{Linear work}$$

Requirement: θ must be expressed in radians.
SI unit of Rotational Work: joule (J)

9.5 Rotational Work and Energy



$$KE = \tfrac{1}{2}mv_T^2 = \tfrac{1}{2}mr^2\omega^2$$

$$v_T = r\omega$$

$$KE = \sum\left(\tfrac{1}{2}mr^2\omega^2\right) = \tfrac{1}{2}\left(\sum mr^2\right)\omega^2 = \tfrac{1}{2}I\omega^2$$

DEFINITION OF ROTATIONAL KINETIC ENERGY

The rotational kinetic energy of a rigid rotating object is

$$\boxed{K_R = \tfrac{1}{2}I\omega^2}$$

Requirement: **The angular speed must be expressed in rad/s.**

SI Unit of Rotational Kinetic Energy: **joule (J)**

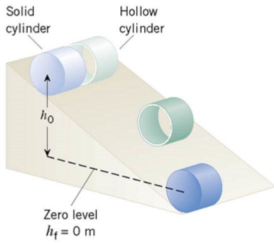
Analogy: Linear and Rotational Motion

linear		angular
x	**Position**	θ
$v = \dfrac{\Delta x}{\Delta t}$	Velocity	$\omega = \dfrac{\Delta\theta}{\Delta t}$
$a = \dfrac{\Delta v}{\Delta t}$	Acceleration	$\alpha = \dfrac{\Delta\omega}{\Delta t}$
$F = m\,a$	Force / Torque	$\tau = I\,\alpha$
m	Mass / Moment of Inertia	$I = \sum_i m_i R_i^2$
$K = \dfrac{m}{2}v^2$	Kinetic Energy	$K = \dfrac{I}{2}\omega^2$

Example: Rolling Cylinders

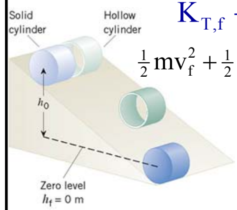
A thin-walled hollow cylinder (mass = m_h, radius = r_h) and a solid cylinder (mass = m_s, radius = r_s) start from rest at the top of an incline.

Determine which cylinder has the greatest translational speed upon reaching the bottom.



$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgh$$

ENERGY CONSERVATION



$$K_{T,f} + K_{R,f} + U_f = K_{T,i} + K_{R,i} + U_i$$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f = \tfrac{1}{2}mv_i^2 + \tfrac{1}{2}I\omega_i^2 + mgh_i$$

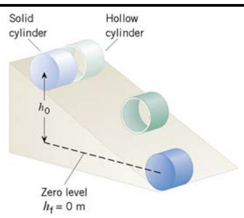
$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 = mgh_i$$

$$\omega_f = v_f / r$$

$$\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\,v_f^2/r^2 = mgh_i$$

$$v_f = \sqrt{\frac{2mgh_o}{m + I/r^2}}$$

The cylinder with the smaller moment of inertia I will have a greater final translational speed.



$$v_f = \sqrt{\frac{2mgh_o}{m + I/r^2}}$$

The cylinder with the <u>smaller moment of inertia I</u> will have a greater final translational speed.

Hollow cylinder

$$I_H = m r^2 \qquad v_H = \sqrt{\frac{2mgh_o}{m + m r^2/r^2}} = \sqrt{gh_o}$$

Solid cylinder

$$I_S = \frac{m}{2} r^2 \qquad v_S = \sqrt{\frac{2mgh_o}{m + \frac{m}{2} r^2 / r^2}} = \sqrt{\frac{4}{3}gh_o}$$

faster

9.6 Angular Momentum

DEFINITION OF ANGULAR MOMENTUM

The angular momentum *L* of a body rotating about a fixed axis is the product of the body's moment of inertia and its angular velocity with respect to that axis:

$$\boxed{L = I\omega}$$

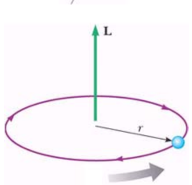
Requirement: **The angular speed must be expressed in rad/s.**

SI Unit of Angular Momentum: **kg·m²/s**

Angular Momentum

$$\boxed{L = I\,\omega}$$

- The gymnast or the diver change rotational velocities by changing body shape.



Analogy: Linear and Rotational Motion

linear		angular
x	**Position**	θ
$v = \dfrac{\Delta x}{\Delta t}$	Velocity	$\omega = \dfrac{\Delta\theta}{\Delta t}$
$a = \dfrac{\Delta v}{\Delta t}$	Acceleration	$\alpha = \dfrac{\Delta\omega}{\Delta t}$
m	Mass / Moment of Inertia	$I = \sum_i m_i R_i^2$
$K = \dfrac{m}{2}v^2$	Kinetic Energy	$K = \dfrac{I}{2}\omega^2$
$F = m\,a$	Force / Torque	$\boxed{\tau = I\,\alpha}$
$p = m\,v$	Momentum	$L = I\,\omega$

Torque and the rate of change of Angular Momentum

Analogue of Newton's second law for rotational motion $\boxed{\sum \tau = I\alpha}$

If the axis doesn't move within the rotating object, then **I** is constant. In this case

$$\tau = I\,\alpha \quad = I\,\frac{\Delta\omega}{\Delta t} \quad = \frac{\Delta L}{\Delta t}$$

$$\boxed{\sum \tau = \frac{\Delta L}{\Delta t}}$$

Analogous to: $F = \dfrac{\Delta p}{\Delta t}$

Prinicple of Conservation of Angular Momentum

$\boxed{\sum \tau = \dfrac{\Delta L}{\Delta t}}$ If $\sum \tau = 0$, then $0 = \dfrac{\Delta L}{\Delta t} \Leftrightarrow L = \text{const.}$

The angular momentum of a system remains constant (is conserved) if the net external torque acting on the system is zero.

$\vec{F} = \dfrac{\Delta \vec{P}}{\Delta t}$ $\vec{F} = 0$

Linear momentum is conserved in absence of an applied force.

(translational invariance of physical laws)

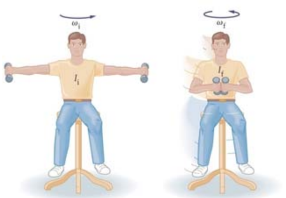
$\vec{\tau} = \dfrac{\Delta \vec{L}}{\Delta t}$ $\vec{\tau} = 0$

Angular momentum is conserved in absence of an applied torque.

(rotational invariance of physical laws)

Example: Conservation of Angular Momentum

A person sits on a piano stool holding a sizable mass in each hand. Initially, he holds his arms outstretched and spins about the axis of the stool with an angular speed of 3.74 rad/s. The moment of inertia in this case is 5.33 kg.m². While still spinning, the person pulls his arms in to his chest, reducing the moment of inertia to 1.60 kg.m². What is the person's angular speed now?



$$L_f = L_i$$

$$I_f\omega_f = I_i\omega_i$$

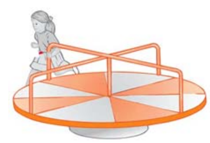
$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i$$

$$\omega_f = \left(\frac{5.33\,\text{kg.m}^2}{1.60\,\text{kg.m}^2}\right)3.74\,\text{rad/s}$$

$$\omega_f = 12.5\,\text{rad/s}$$

Another Example: Conservation of Angular Momentum

A 3.0-m-diameter merry-go-around with rotational inertia 120 kg m² is spinning freely at 1.57 rad / s. Four 25-kg children sit suddenly on the edge of the merry-go-around. Find the new angular speed.



$$L = const. = I_1\omega_1 = I_2\omega_2$$

$$I_1 = 120 kg \cdot m^2$$

$$I_2 = I_1 + (4 \cdot 25kg)(1.5m)^2$$

$$I_2 = (120 + 225) kg \cdot m^2$$

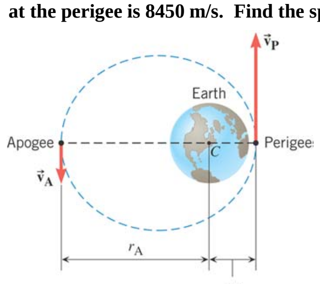
$$I_2 = 345 kg \cdot m^2$$

$$\omega_2 = \frac{I_1}{I_2}\omega_1 = \frac{120 kg \cdot m^2}{345 kg \cdot m^2} \cdot 1.57\ rad/s$$

$$\omega_2 = 0.546\ rad/s.$$

Example: A Satellite in an Elliptical Orbit

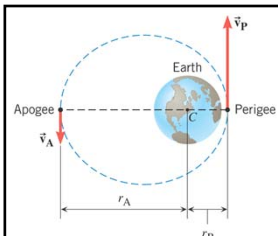
An artificial satellite is placed in an elliptical orbit about the earth. Its point of closest approach is 8.37x10⁶m from the center of the earth, and its point of greatest distance is 25.1x10⁶m from the center of the earth. The speed of the satellite at the perigee is 8450 m/s. Find the speed at the apogee.



Angular momentum of satellite in orbit is conserved

$$L = I\,\omega$$

$$= mr^2\,\frac{v}{r} = mrv$$



angular momentum conservation

$$I_A\omega_A = I_P\omega_P$$

$$I = mr^2 \qquad \omega = v/r$$

$$mr_A^2\,\frac{v_A}{r_A} = mr_P^2\,\frac{v_P}{r_P}$$

$$r_A v_A = r_P v_P$$

$$v_A = \frac{r_P v_P}{r_A} = \frac{(8.37\times10^6\ \text{m})(8450\,\text{m/s})}{25.1\times10^6\ \text{m}} = 2820\,\text{m/s}$$

Satellite does not have constant speed in an elliptical orbit.
Planets: Kepler's second law

Equal areas in equal time intervals