Chapter 5

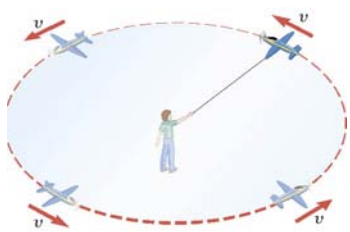
Dynamics of Uniform Circular Motion

Goals for Chapter 5

- **To understand the dynamics of circular motion.**
- **To study the motion of satellites in orbit as a special case of circular motion where the centripetal force is due to gravitational pull of the earth.**

5.1 Uniform Circular Motion

Uniform circular motion is the motion of an object traveling at a constant speed on a circular path.



speed:

Let T be the time it takes for the object to travel once around the circle.

$$v = \frac{2\pi r}{T}$$

Example: A tire-balancing machine

The wheel of a car has a radius of 0.29m and it being rotated at 830 revolutions per minute (rpm) on a tire-balancing machine. Determine the speed at which the outer edge of the wheel is moving.

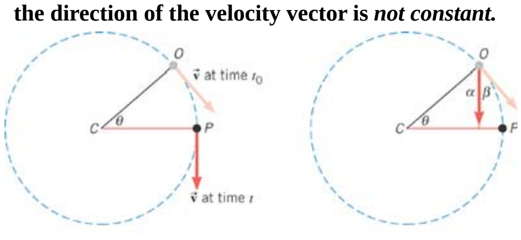
$$\frac{1}{830 \text{ revolutions/min}} = 1.2\times10^{-3} \text{ min/revolution}$$

$$T = 1.2\times10^{-3} \text{ min} = 0.072 \text{ s}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi\,(0.29 \text{ m})}{0.072 \text{ s}} = 25 \text{ m/s}$$

5.2 Centripetal Acceleration

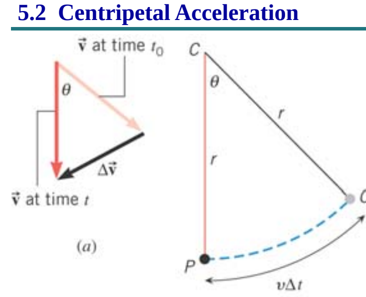
In uniform circular motion, the speed is constant, but the direction of the velocity vector is *not constant.*



(a) (b)

$$\alpha + \beta = 90^\circ \quad and \quad \alpha + \theta = 90^\circ \Rightarrow \beta = \theta$$

5.2 Centripetal Acceleration



(b)

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

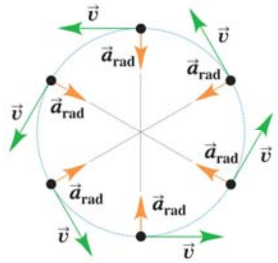
$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$a_c = \frac{v^2}{r}$$

The direction of the centripetal acceleration is towards the center of the circle; in the same direction as the change in velocity.

Acceleration in uniform circular motion

- **Uniform circular motion results in acceleration:**
  - **magnitude:** $a = v^2 / R$
  - **direction:** **- R (towards center of circle)**



The velocity vector changes direction, not magnitude

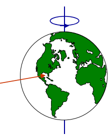
travels the circumference of the circle in the time T for one revolution

speed:

$$v = \frac{2\pi R}{T}$$

Is Orlando a good inertial reference frame ?

- Is Orlando accelerating ?
- YES! Orlando is on earth.
  - The earth is rotating.
- What is the centripetal acceleration of Orlando ?



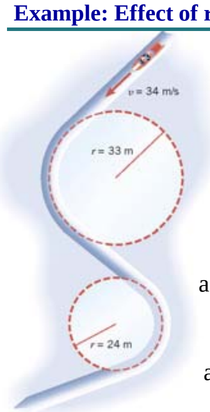
T = 1 day = 8.64 x 10⁴ sec,

R ~ R_E = 6.4 x 10⁶ Meter .

$$v = \frac{2\pi R}{T} \qquad a = \frac{v^2}{R} = \frac{1}{R}\left(\frac{2\pi R}{T}\right)^2 = \left(\frac{2\pi}{T}\right)^2 R$$

- **Plug this in: a = .034 m/s² (~ 1/300 g)**
- **Close enough to 0 that we will ignore it.**
- **Orlando is a pretty good inertial reference frame.**

Example: Effect of radius of curvature



The bobsled track contains turns with radii of 33 m and 24 m. Find the centripetal acceleration at each turn for a speed of 34 m/s. Express answers as multiples of $g = 9.8\,\text{m/s}^2$.

$$a_c = v^2/r$$

$$a_c = \frac{(34\,\text{m/s})^2}{33\,\text{m}} = 35\,\text{m/s}^2 = 3.6g$$

$$a_c = \frac{(34\,\text{m/s})^2}{24\,\text{m}} = 48\,\text{m/s}^2 = 4.9g$$

5.3 Centripetal Force

Recall Newton's Second Law

When a net external force acts on an object of mass *m*, the acceleration that results is directly proportional to the net force and has a magnitude that is inversely proportional to the mass. The direction of the acceleration is the same as the direction of the net force.

$$\vec{a} = \frac{\sum \vec{F}}{m} \qquad \sum \vec{F} = m\vec{a}$$

5.3 Centripetal Force

Thus, in uniform circular motion there must be a net force to produce the centripetal acceleration.

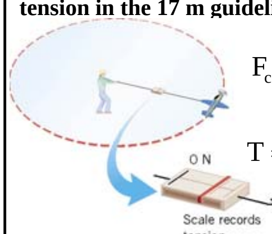
The centripetal force is the name given to the net force required to keep an object moving on a circular path.

The direction of the centripetal force always points toward the center of the circle and continually changes direction as the object moves.

$$F_c = ma_c = m\frac{v^2}{r}$$

Example: The effect of speed on centripetal force

The model airplane has a mass of 0.90 kg and moves at constant speed on a circle that is parallel to the ground. The path of the airplane and the guideline lie in the same horizontal plane because the weight of the plane is balanced by the lift generated by its wings. Find the tension in the 17 m guideline for a speed of 19 m/s.

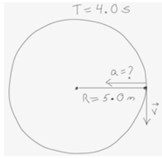


$$F_c = T = m\frac{v^2}{r}$$

$$T = (0.90\,\text{kg})\frac{(19\,\text{m/s})^2}{17\,\text{m}} = 19\,\text{N}$$

Example: A high-speed carnival ride

The passengers in a carnival ride travel in a circle with radius 5.0 m. They complete one circle in a time T = 4.0 s. What is their acceleration? What is the centripetal force if the mass of a passenger is 90 kg ?



$$v = \frac{2\pi R}{T}$$

$$a = \frac{v^2}{R}$$

$$v = \frac{2\pi (5.0\,m)}{4.0\,s} = 7.9\ m/s$$

$$a = \frac{v^2}{R} = \frac{(7.9)^2}{5.0} = 12\ m/s^2$$

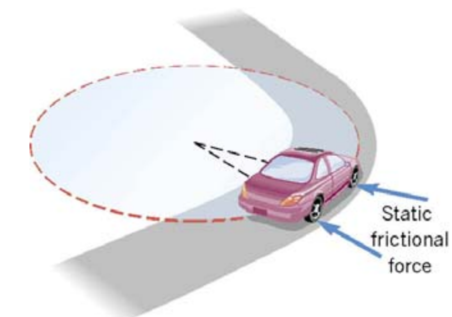
$$F_{rad} = m \cdot a = 90 \cdot 12 = 1080\ N$$

Two cars on a curving road

- A small car with mass m and a large car with mass 2m drive around a highway curve of radius R with the same speed v. As they travel around the curve, what is their acceleration?
- (a) equal.
- (b) along the direction of motion.
- (c) in the ratio of 2 to 1.
- (d) zero.

Flat curves

On an unbanked curve, the static frictional force provides the centripetal force.



Example: Rounding a flat curve

A car rounds a flat, unbanked curve with radius R. a) If the coefficient of static friction between tires and road is μ_s, derive an expression for the maximum speed v_{max} at which the driver can take the curve without sliding. b) What is v_{max} for R = 250 m and μ_s = 0.90 ?

n: normal force

$$n + (-mg) = 0$$

$$f_s = m\frac{v^2}{R}$$

$$f_s = \mu_s n$$

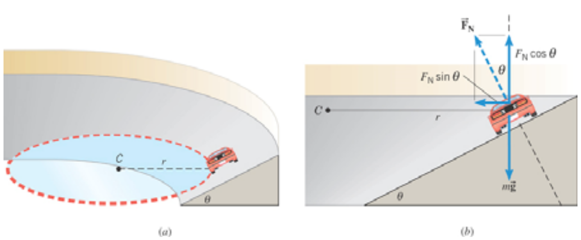
$$\mu_s mg = m\frac{v_{max}^2}{R}$$

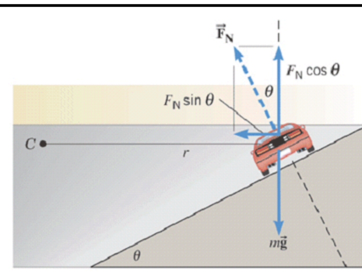
$$v_{max} = \sqrt{\mu_s gR}$$

$$v_{max} = \sqrt{0.90 \cdot 9.8\,m/s^2 \cdot 250m} = 47\ m/s$$

5.4 Banked curves

On a frictionless banked curve, the centripetal force is the horizontal component of the normal force. The vertical component of the normal force balances the car's weight.



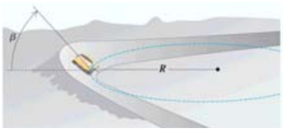


$$F_N \sin\theta = m\frac{v^2}{r}$$

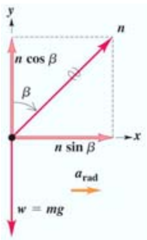
$$F_N \cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg}$$

Rounding a banked curve - Daytona speedway



The turns at the Daytona International Speedway have a maximum radius of 316 m and are steely banked at 31 degrees. Suppose these turns were frictionless. As what speed would the cars have to travel around them?



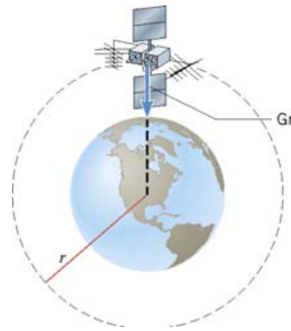
$$\tan\theta = \frac{v^2}{Rg} \Rightarrow v = \sqrt{Rg\tan\theta}$$

$$v = \sqrt{(316\,\mathrm{m})(9.8\,\mathrm{m/s^2})\tan 31^\circ}$$

$$= 43\,\mathrm{m/s}\ \ (96\,\mathrm{mph})$$

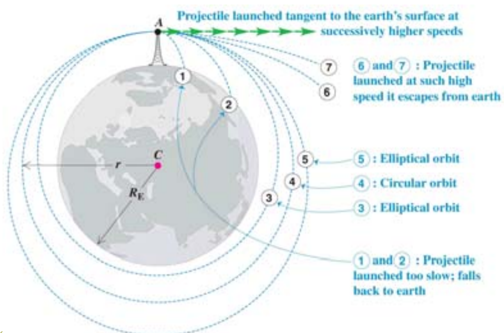
5.5 Satellite in circular orbits

There is only one speed that a satellite can have if the satellite is to remain in an orbit with a fixed radius.



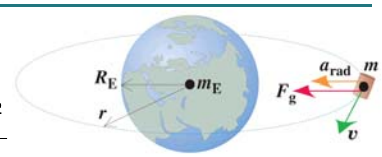
Circular orbits - projectile at the 'right' velocity

- Increase projectile velocity: when $\mathbf{F_g}$ balances ⇒ orbit
- When **v** is large enough, you achieve escape velocity



5.5 Satellite in circular orbits

$$\sum \overline{F} = m\vec{a}$$



$$\frac{Gm_s m_E}{r^2} = \frac{m_s v^2}{r}$$

$$v = \sqrt{\frac{Gm_E}{r}}$$

$$T = \frac{2\pi r}{v} = 2\pi r\sqrt{\frac{r}{G\,m_E}}$$

Larger period T corresponds to larger orbit

$$T = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

Example: Orbital speed of the Hubble space telescope

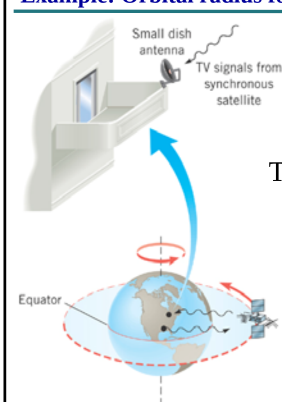
Determine the speed of the Hubble Space Telescope orbiting at a height of 598 km above the earth's surface.

$$v = \sqrt{\frac{Gm_E}{r}}$$

$$v = \sqrt{\frac{(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\,\mathrm{kg})}{6.38\times10^6\,\mathrm{m} + 598\times10^3\,\mathrm{m}}}$$

$$= 7.56\times10^3\,\mathrm{m/s}\quad (16{,}900\,\mathrm{mi/h})$$

Example: Orbital radius for synchronous satellites



What is the height H above the earth's surface at which all synchronous satellites (regardless of mass) must be placed in orbit ?

$$T = 24\,\text{hours}$$

$$T = \frac{2\pi r^{3/2}}{\sqrt{G\,m_E}}$$

$$r^{3/2} = T\frac{\sqrt{G\,m_E}}{2\pi}$$

$$r^{3/2} = T\frac{\sqrt{G\,m_E}}{2\pi} \;\Rightarrow\; r = \left(\frac{T}{2\pi}\right)^{2/3}(G\,m_E)^{1/3}$$

$$T = 24\text{ hours} \quad = 86400\text{ s}$$

$$r = \left(\frac{8.64\times10^4\text{ s}}{2\pi}\right)^{2/3}\left(6.67\times10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}5.98\times10^{24}\text{ kg}\right)^{1/3}$$

$$r = 4.23\times10^7\text{ m}$$

since earth's radius is

$$R_E = 6.38\times10^6\text{ m}$$

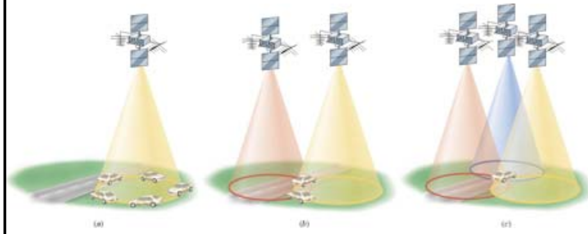
$$H = r - R_E = 4.23\times10^7\text{ m} - 6.38\times10^6\text{ m}$$

$$= 3.59\times10^7\text{ m} = 35900\text{ km}$$

Example: Global Positioning System(24 satellites)

These satellites are all synchronous satellites, having a period of exactly 24 hours.

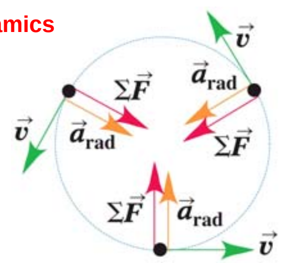
$$T = 24\text{ hours} \qquad T = \frac{2\pi r^{3/2}}{\sqrt{G\,m_E}}$$



Circular Motion Dynamics

$$F_{net} = m\frac{v^2}{r}$$

$$G\frac{m\cdot m_E}{r^2} = m\frac{v^2}{r}$$

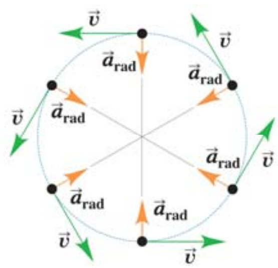


Satellite in circular (earth) orbit

$$v = \sqrt{\frac{Gm_E}{r}}$$

Acceleration in uniform circular motion

- **Uniform circular motion results in acceleration:**
  - **magnitude:** $a_c = v^2 / R$
  - **direction:** $-R$ (towards center of circle)



The velocity vector changes direction, not magnitude

travels the circumference of the circle in the time T for one revolution

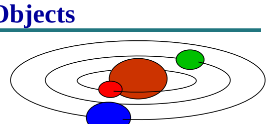
$$a_c = \frac{v^2}{r}$$

speed:

$$v = \frac{2\pi R}{T}$$

Masses of Astronomical Objects

$$|F_g| = G\frac{M\,m}{R^2}$$



Where G = 6.67 x 10 $^{-11}$ m^3 kg^{-1} s^{-2}

M: central mass
m: mass of satellite

- But $|F_g|$ = **m a** $= m\frac{v^2}{R}$

For F_g all objects accelerate with same acceleration, regardless of their mass!

$$G\frac{M}{R^2} = \frac{(2\pi)^2}{T^2}R$$

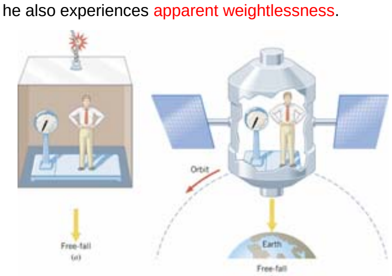
$$T = \frac{2\pi r^{3/2}}{\sqrt{G\,M}}$$

If the orbit radius r and the period T of a center mass can be measured, we can calculate the mass of the object !

$$M = \frac{(2\pi)^2}{T^2\,G}R^3$$

5.6 Apparent weightlessness and artificial gravity

During free-fall, the elevator, the person, and the scale all accelerate downward with the same acceleration due to gravity. The apparent weight of the person is zero. When the person is in a orbiting space station, he also experiences apparent weightlessness.



Example: Artificial Gravity

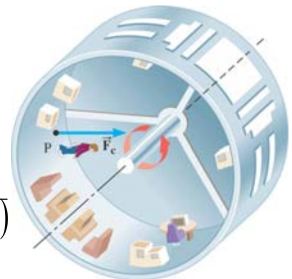
At what speed must the surface of the space station rotate so that the astronaut experiences a push on his feet equal to his weight on earth? The radius is 1700 m.

$$F_c = m\frac{v^2}{r} = mg$$

$$v = \sqrt{rg}$$

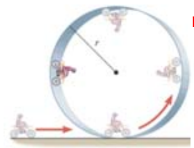
$$= \sqrt{(1700\,\text{m})(9.80\,\text{m/s}^2)}$$

$$= 129\,m/s$$



5.7 Vertical circular motion

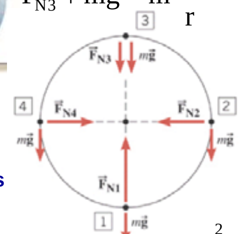
Motor cycle stunt driver going around a vertical circular track



normal force !

as the cycle goes around the normal force changes

$$F_{N3} + mg = m\frac{v_3^2}{r}$$



$$F_{N4} = m\frac{v_4^2}{r}$$

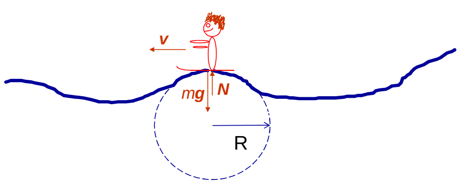
$$F_{N2} = m\frac{v_2^2}{r}$$

centripetal force is the sum of all the force components oriented along the radial direction

$$F_{N1} - mg = m\frac{v_1^2}{r}$$

Example: Skiing over a bump

- **A skier of mass *m* goes over a mogul having a radius of curvature *R*. How fast can she go without leaving the ground?**

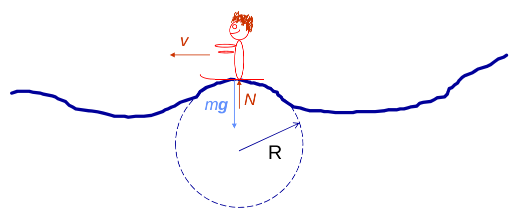


(a) $v = \sqrt{mRg}$ (b) $v = \sqrt{\frac{Rg}{m}}$ (c) $v = \sqrt{Rg}$

Example: Skiing over a bump

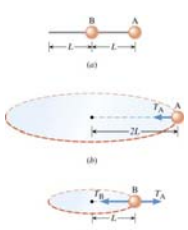
- **$mv^2 / R = mg - N$**

  - For **N = 0**: $v = \sqrt{Rg}$



Example 16, Centripetal force

Ball A is attached to one end of a rigid massless rod, while an identical ball B is attached to the center of the rod, as shown. Each ball has a mass of 0.5 kg, and the length of each half of the rod is L = 0.4 m. Each ball is in uniform circular motion. Ball A travel at a constant speed of 5.0 m/s. Find the tension in each half of the rod.



$$T_A = \frac{m{v_A}^2}{2L} = 15.6N$$

$$T_B - T_A = \frac{m{v_B}^2}{L} = 7.8$$

$$T_B = 23.4N$$