Chapter 3
Kinematics in two dimensions

Goals for Chapter 3

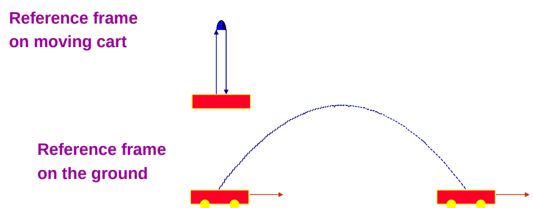
- to study position, velocity, and acceleration vectors in two dimensions
- to understand how displacement, velocity, and acceleration are applied in two dimensional motion
- to study two-dimensional motion as it occurs in the motion of projectiles
- to study the concept of relative motion

Most important concept in two-dimensional motion

- two-dimensional motion can be decomposed into motion in x-direction and motion in y-direction

"x" and "y" components of motion are independent

- Person on a cart throws ball vertically upwards
  - Seen from two different frames of reference



3.1 Displacement, velocity, acceleration

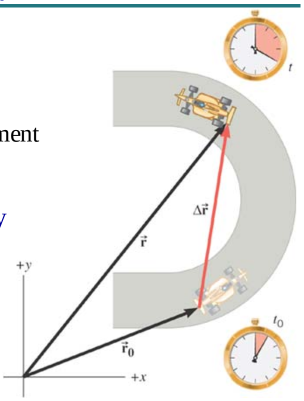
$\vec{\mathbf{r}}_o$ = initial position

$\vec{\mathbf{r}}$ = final position

$\Delta\vec{\mathbf{r}} = \vec{\mathbf{r}} - \vec{\mathbf{r}}_o$ = displacement

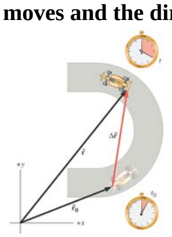
Average velocity is the displacement divided by the elapsed time.

$$\bar{\vec{\mathbf{v}}} = \frac{\vec{\mathbf{r}} - \vec{\mathbf{r}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

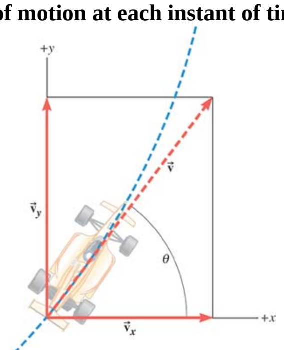


2-d Displacement, velocity, acceleration

The *instantaneous velocity* indicates how fast the car moves and the direction of motion at each instant of time.



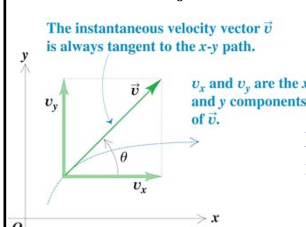
$$\vec{\mathbf{v}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

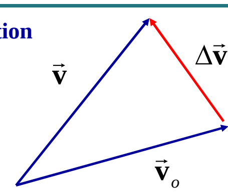


2-d Displacement, velocity, acceleration

Definition of average acceleration

$$\bar{\vec{\mathbf{a}}} = \frac{\vec{\mathbf{v}} - \vec{\mathbf{v}}_o}{t - t_o} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$





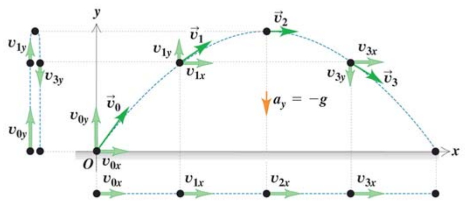
Definition of instantaneous acceleration

$$\vec{\mathbf{a}} = \lim_{\Delta t \to 0} \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

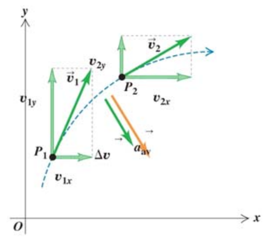
Motion in two dimensions

equations of kinematics in two dimesions

projectile motion



The vertical and horizontal components of a projectile's motion are independent.



Acceleration must now be considered during change in magnitude AND / OR direction

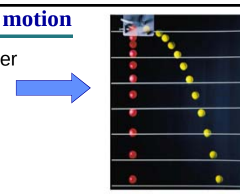
Please note that the above example is not a "projectile" motion near the surface of earth.

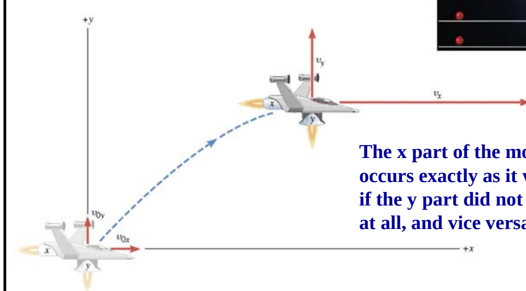
Equations of kinematics for constant acceleration

Variables related	Equation $a = const$
velocity, time, acceleration	$v = v_o + at$
position, velocity, time	$x = \frac{1}{2}(v_o + v)t$
initial position $x_o = 0$	
velocity, position, acceleration	$v^2 = v_o^2 + 2ax$
position, time, acceleration	$x = v_o t + \frac{1}{2}at^2$

The independence of x and y motion

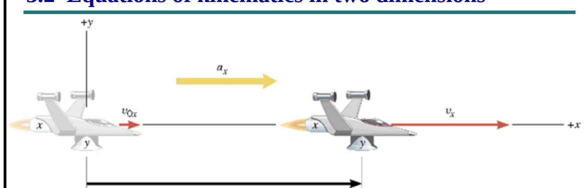
Notice that the vertical motion under free fall spaces out exactly as the vertical motion of the projectile





The x part of the motion occurs exactly as it would if the y part did not occur at all, and vice versa.

3.2 Equations of kinematics in two dimensions



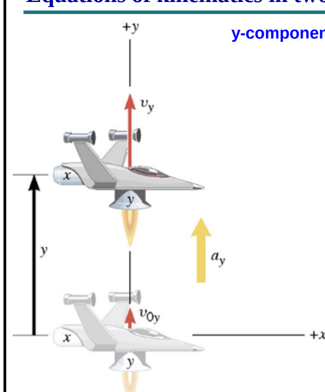
x-component (horizontal)

$$v_x = v_{ox} + a_x t \qquad x = \tfrac{1}{2}(v_{ox} + v_x)t$$

$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 \qquad v_x^2 = v_{ox}^2 + 2a_x x$$

Equations of kinematics in two dimensions

y-component (vertical)



$$v_y = v_{oy} + a_y t$$

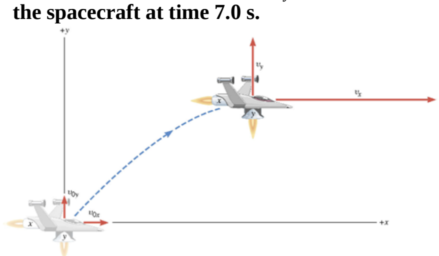
$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

$$y = \tfrac{1}{2}(v_{oy} + v_y)t$$

$$v_y^2 = v_{oy}^2 + 2a_y y$$

Example: A moving spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s². In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s².
Find (a) x and v_x, (b) y and v_y, and (c) the final velocity of the spacecraft at time 7.0 s.



Example: A moving spacecraft

In the *x* direction, the spacecraft has an initial velocity component of +22 m/s and an acceleration of +24 m/s². In the *y* direction, the analogous quantities are +14 m/s and an acceleration of +12 m/s².
Find (a) *x* and v_x, (b) *y* and v_y, and (c) the final velocity of thespacecraft at time 7.0 s.

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s²	?	+22 m/s	7.0 s

y	a_y	v_y	v_{oy}	t
?	+12.0 m/s²	?	+14 m/s	7.0 s

Example: A moving spacecraft

x	a_x	v_x	v_{ox}	t
?	+24.0 m/s²	?	+22 m/s	7.0 s

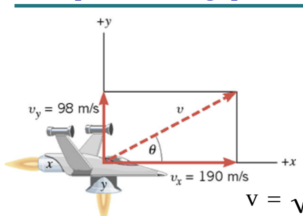
$$x = v_{ox}t + \tfrac{1}{2}a_x t^2$$
$$= (22\,\text{m/s})(7.0\,\text{s}) + \tfrac{1}{2}(24\,\text{m/s}^2)(7.0\,\text{s})^2 = +740\,\text{m}$$
$$v_x = v_{ox} + a_x t$$
$$= (22\,\text{m/s}) + (24\,\text{m/s}^2)(7.0\,\text{s}) = +190\,\text{m/s}$$

Example: A moving spacecraft

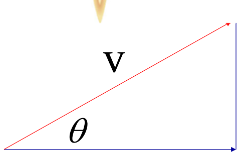
y	a_y	v_y	v_{oy}	t
?	+12.0 m/s²	?	+14 m/s	7.0 s

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$
$$= (14\,\text{m/s})(7.0\,\text{s}) + \tfrac{1}{2}(12\,\text{m/s}^2)(7.0\,\text{s})^2 = +390\,\text{m}$$
$$v_y = v_{oy} + a_y t$$
$$= (14\,\text{m/s}) + (12\,\text{m/s}^2)(7.0\,\text{s}) = +98\,\text{m/s}$$

Example: A moving spacecraft



$$v = \sqrt{(190\ \text{m/s})^2 + (98\ \text{m/s})^2} = 210\ \text{m/s}$$



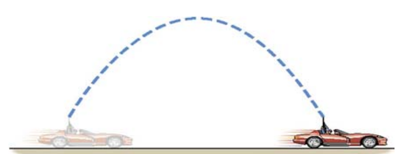
$v_y = 98$ m/s

$v_x = 190$ m/s

$$\theta = \tan^{-1}(98/190) = 27^\circ$$

3.3 Projectile motion: Conceptual example

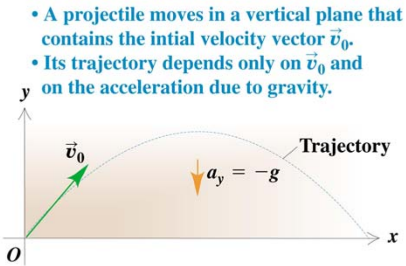
Suppose you are driving a convertible with the top down. The car is moving to the right at constant velocity. You point a rifle straight up into the air and fire it. In the absence of air resistance, where would the bullet land – behind you, aheadof you, or in the barrel of the rifle?



3.3 Projectile Motion

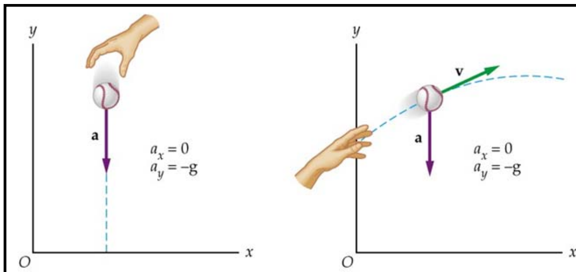
Determined by the initial velocity, gravity,
(air resistance ignored)

Footballs, baseballs, … any projectiles will follow this parabolic path



In studying projectile motion we make the following assumptions:

1. Air resistance is ignored.
2. The acceleration of gravity is constant, downward, and has a magnitude equal to $g = 9.8\ m/s^2$.
3. The Earth's rotation is ignored.
4. The Earth's curvature is ignored.

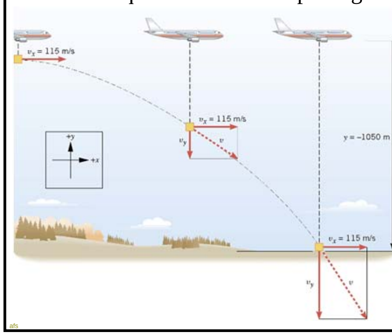


$$a_x = 0$$
$$a_y = -9.80\,m/s^2$$

$$v_x = v_{ox} = \text{constant}$$

Example 3: A falling care package

The airplane is moving horizontally with a constant velocity of +115 m/s at an altitude of 1050m. Determine the time required for the care package to hit the ground.



Given:

y = - 1050m

a_y =- 9.80 m/s²

v_{oy} = 0 m/s

Find t

Example: A falling care package

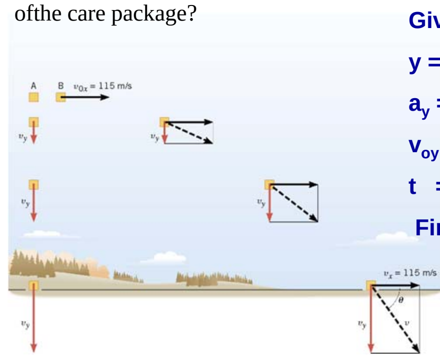
y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s²		0 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2 \quad \Rightarrow \quad y = \tfrac{1}{2}a_y t^2$$

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-1050\,m)}{-9.80\,m/s^2}} = 14.6\,s$$

Example 4: The velocity of the care package

What are the magnitude and direction of the final velocity ofthe care package?



Given:

y = - 1050m

a_y = - 9.80 m/s²

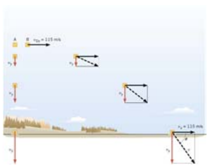
v_{oy} = 0 m/s

t = 14.6 m/s

Find v_y first.

Example: The velocity of the care package

y	a_y	v_y	v_{oy}	t
-1050 m	-9.80 m/s^2	?	0 m/s	14.6 s



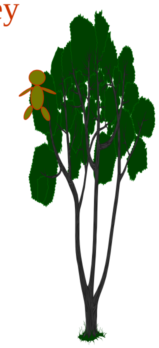
$$v_y = v_{oy} + a_y t$$
$$= 0 + (-9.80\,m/s^2)(14.6\,s)$$
$$= -143\,m/s$$

$$V = \sqrt{v_x^2 + v_y^2} = 183.5\ m/s$$

Shooting the Monkey (tranquilizer gun)

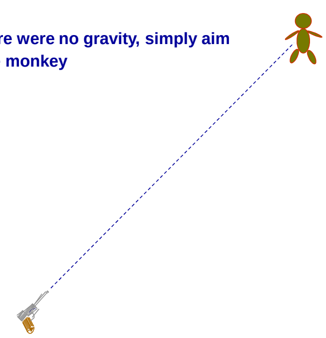
- Where does the zookeeper aim if he wants to hit the monkey?

(He knows the monkey will let go as soon as he shoots !)



Shooting the Monkey...

- If there were no gravity, simply aim at the monkey

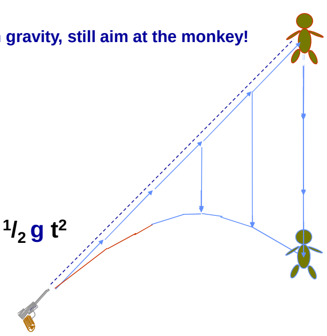


$r = r_0$

$r = v_0 t$

Shooting the Monkey...

- With gravity, still aim at the monkey!

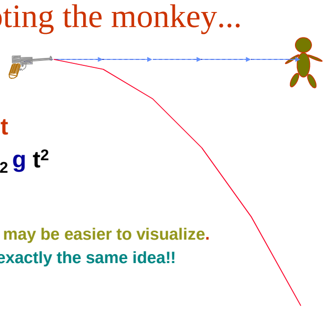


$r = r_0 - {}^1/_2\, g\, t^2$

$= v_0\, t - {}^1/_2\, g\, t^2$

Dart hits the monkey!

Recap: Shooting the monkey...



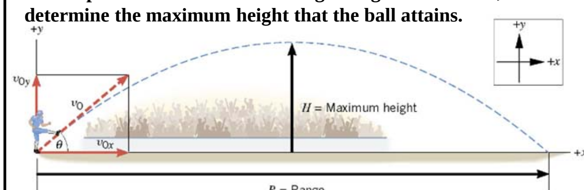
$x = v_0\, t$
$y = -{}^1/_2\, g\, t^2$

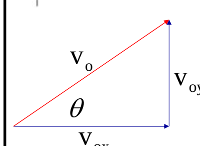
- This may be easier to visualize.
  It's exactly the same idea!!

$x = x_0$
$y = -{}^1/_2\, g\, t^2$

Example 6: The height of a kickoff

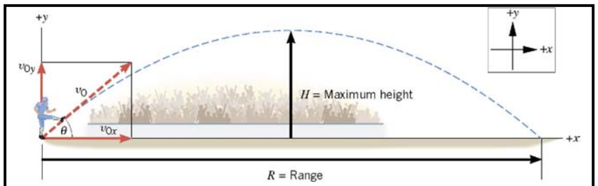
A placekicker kicks a football at and angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.





$$V_{oy} = V_o \sin\theta = (22)(\sin(40^\circ)) = 14 m/s$$

$$v_{ox} = v_o \cos\theta = (22\,m/s)\cos 40^\circ = 17\,m/s$$



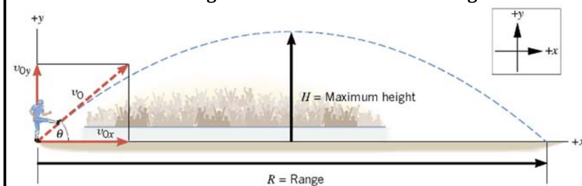
y	a_y	v_y	v_{oy}	t
?	-9.80 m/s²	0	14 m/s	

$$v_y^2 = v_{oy}^2 + 2a_y y \quad \Rightarrow \quad y = \frac{v_y^2 - v_{oy}^2}{2a_y}$$

$$y = \frac{0-(14\,\mathrm{m/s})^2}{2(-9.8\,\mathrm{m/s^2})} = +10\,\mathrm{m}$$

Example 7: The time of flight of a kickoff

What is the time of flight between kickoff and landing?



y	a_y	v_y	v_{oy}	t
0	-9.80 m/s²		14 m/s	?

y	a_y	v_y	v_{oy}	t
0	-9.80 m/s²		14 m/s	?

$$y = v_{oy}t + \tfrac{1}{2}a_y t^2$$

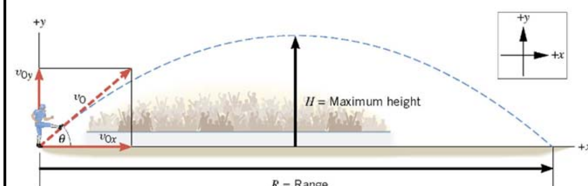
$$0 = (14\,\mathrm{m/s})t + \tfrac{1}{2}(-9.80\,\mathrm{m/s^2})t^2$$

$$0 = 2(14\,\mathrm{m/s}) + (-9.80\,\mathrm{m/s^2})t$$

$$t = 2.9\,\mathrm{s}$$

Example 8: The range of a kickoff

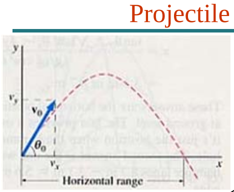
Calculate the range R of the projectile.

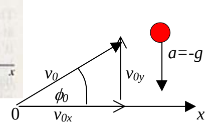


$$x = v_{ox}t + \tfrac{1}{2}a_x t^2 = v_{ox}t$$

$$= (17\,\mathrm{m/s})(2.9\,\mathrm{s}) = +49\,\mathrm{m}$$

Projectile motion - trajectory





$\tan\phi_o = \frac{v_{0y}}{v_{0x}}$

$v_{ox} = v_o \cos\phi_o$

$v_{oy} = v_o \sin\phi_o$

components:

1′: $a_x = 0$; $a_y = -g$

2′: $v_x = v_{0x}$; $v_y = -g \cdot t + v_{0y}$

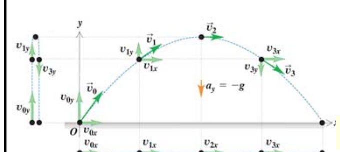
3′; $x = v_{0x}\,t + x_0$ $\Rightarrow y = y_0 + v_{0y}\,t - \frac{1}{2}\,g \cdot t^2$

$\Rightarrow t = (x - x_o)/v_{ox}$ **Eliminate t**

Projectile motion - trajectory

$$y = y_0 + \frac{v_{0y}}{v_{0x}}(x - x_0) - \frac{1}{2}g\frac{(x-x_0)^2}{v_{0x}^2}$$

Parabola in x-y plane



The vertical and horizontal components of a projectile's motion are independent.

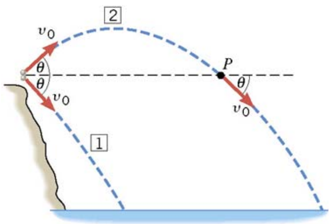
Initial values: $x_o = 0$, $y_o = 0$

$$y = \frac{v_{0y}}{v_{0x}} \cdot x - \frac{1}{2}g\frac{x^2}{v_{0x}^2};$$

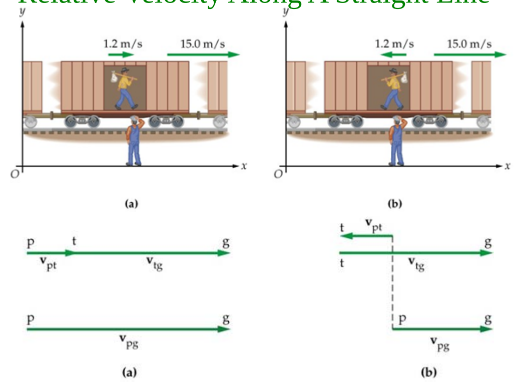
trajectory $y = \tan\phi_o\, x - \frac{1}{2}g\frac{x^2}{v_0^2\cos^2\phi_0}$

Conceptual Example 10 Two Ways to Throw a Stone

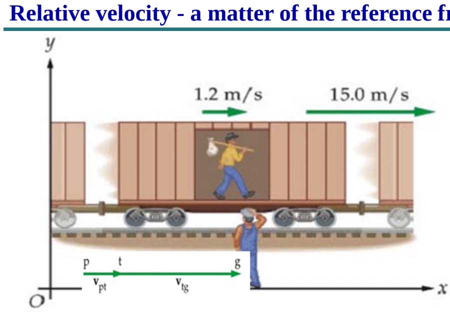
From the top of a cliff, a person throws two stones. The stones have identical initial speeds, but stone 1 is thrown downward at some angle above the horizontal and stone 2 is thrown at the same angle below the horizontal. Neglecting air resistance, which stone, if either, strikes the water with greater velocity?



Relative Velocity Along A Straight Line



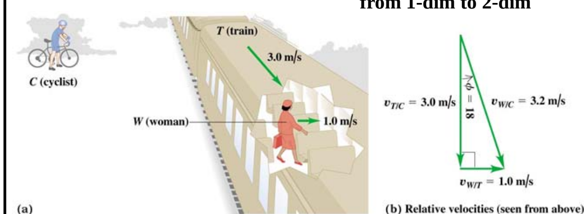
Relative velocity - a matter of the reference frame



$$\overrightarrow{V}_{pg} = \overrightarrow{V}_{pt} + \overrightarrow{V}_{tg}$$

Relative velocity in 2-dim

The concept of relative velocity can be extended from 1-dim to 2-dim



Velocities can carry multiple values depending on the position and motion of the object and observer

Relative velocity

$$\vec{V}_{AC} = \vec{V}_{AB} + \vec{V}_{BC}$$

$\vec{V}_{AC}$ Velocity of A relative to C

$\vec{V}_{AB}$ Velocity of A relative to B

$\vec{V}_{BC}$ Velocity of B relative to C

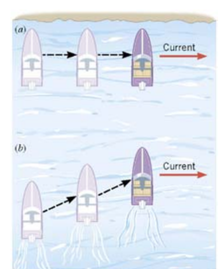
$$\vec{V}_{BC} = -\vec{V}_{CB}$$

Example: Crossing a river

A boat is crossing a river that is 1800m wide. The velocity of the boat relative to the water is 4.0m/s directed perpendicular to the current. The velocity of the water relative to the shore is 2.0m/s.

(a) What is the velocity of the boat relative to the shore?

(b) How long does it take for the boat to cross the river?



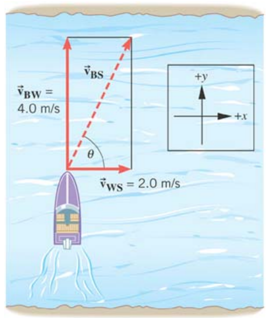
$$\vec{\mathbf{v}}_{\mathbf{BS}} = \vec{\mathbf{v}}_{\mathbf{BW}} + \vec{\mathbf{v}}_{\mathbf{WS}}$$

$$\theta = \tan^{-1}\left(\frac{4.0}{2.0}\right) = 63^\circ$$

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2}$$

$$= \sqrt{(4.0\,\text{m/s})^2 + (2.0\,\text{m/s})^2}$$

$$= 4.5\,\text{m/s}$$



$$t = \frac{1800\,\text{m}}{4.0\,\text{m/s}} = 450\,\text{s}$$

Example: An airplane in a crosswind

The compass of an airplane indicates that it is headed due north, and the airspeed indicator shows that the plane is moving through the air at 240 km / h. If there is a wind of 100 km / h from west to east, what is the velocity of the aircraft relative to the earth ?

$$\vec{V}_{PE} = \vec{V}_{PA} + \vec{V}_{AE}$$

$$V_{PE} = \left|\vec{V}_{PE}\right| = \sqrt{(240)^2 + (100)^2}$$

$$= 260 km/h$$

$$\alpha = \tan^{-1}\left(\frac{100}{240}\right) = 23^\circ$$