Review for Exam 2

Practice Problem Solving!

Homework Problems
Recitation Problems
Examples from Class

Newton's Laws

Forces add as vectors

Inertial reference frame

If the vector sum of the forces acting on an object is zero, the object remains at rest or (a here) in motion with constant velocity.

An action-reaction pair: \( F_{12} = -F_{21} \)

The two forces represent a mutual interaction of two objects, and each acts on a different object.

\[
\begin{align*}
F_{12} &= -F_{21} \\
F_{\text{NET}} &= m \ddot{a}
\end{align*}
\]

Newton's 2nd Law

\[ F_{\text{NET}} = m \ddot{a} \]

Weight

An object's energy depends on its mass and the net force acting on it.

An action-reaction pair: \( F_{\text{net}} = -F_{\text{on ball}} \)

The two forces represent a mutual interaction of two objects, and each acts on a different object.

\[
\begin{align*}
F_{\text{net}} &= -F_{\text{on ball}} \\
F_{\text{NET}} &= m \ddot{a}
\end{align*}
\]

Application of Newton's laws

Equilibrium

A free-body diagram of a man dragging a crate.

The diagram shows all the forces acting on the man, and only forces acting on the man.

Static and Kinetic Friction

Friction keeps box motionless.

Friction opposes motion.

The static-friction force remains equal in magnitude to the tension force until its maximum value \( f_{\text{max}} \) is exceeded.
Example: Moving a crate with constant velocity

You are moving a 500 N crate by pulling upward on the rope at an angle of 30° above the horizontal. The coefficient of kinetic friction between the crate and the floor is μ_k = 0.40. How hard do you have to pull to keep the crate moving with constant velocity? Is this easier or harder than pulling horizontally?

![Free-body diagram for moving crate](image)

Example: Tethered bodies on two inclined planes

From the free body diagrams for each body, and the chosen coordinate system for each block, we can apply Newton's Second Law:

1) \( T_1 - m_1 g \sin \theta_1 = m_1 a_{1X} \)
2) \( T_2 - m_2 g \sin \theta_2 = m_2 a_{2X} \)

But \( T_1 = T_2 = T \) and \( a_{1X} = -a_{2X} = a \)

Using the constraints, solve the equations.

\[
T - m_1 g \sin \theta_1 = -m_1 a \\
T - m_2 g \sin \theta_2 = m_2 a
\]

Subtracting (a) from (b) gives:

\[
a = \frac{m_1 \sin \theta_1 - m_2 \sin \theta_2}{m_1 + m_2} g
\]

Summary Chapter 5

Circular Motion Dynamics

- Satellite in circular (earth) orbit
- vertical circular motion normal force also

\[
\begin{align*}
F_N \pm mg &= m\frac{v^2}{r} \\
\end{align*}
\]
Example: Rounding a flat curve
A car rounds a flat, unbanked curve with radius R. a) If the coefficient of static friction between tires and road is \( \mu_s \), derive an expression for the maximum speed \( v_{max} \) at which the driver can take the curve without sliding. b) What is \( v_{max} \) for \( R = 250 \) m and \( \mu_s = 0.90 \)?

\[
\begin{align*}
n + (-mg) &= 0 \\
\Rightarrow \quad f_n &= mg \\
n &= \left( \frac{m v^2}{R} \right) \\
f_s &= \mu_s n \\
v_{max} &= \sqrt{\frac{\mu_s g R}{R}} \\
v_{max} &= \sqrt{0.90 \cdot 9.8 \, \text{m/s}^2 \cdot 250 \, \text{m}} = 47 \, \text{m/s}
\end{align*}
\]

Rounding a banked curve - Daytona speedway
The turns at the Daytona International Speedway have a maximum radius of 316 m and are steeply banked at 31 degrees. Suppose these turns were frictionless. As what speed would the cars have to travel around them?

\[
\begin{align*}
\tan \theta &= \frac{v^2}{R g} \\
v &= \sqrt{931 \, \text{m} \cdot (9.8 \, \text{m/s}^2 \cdot \tan 31^\circ)} \\
v &= 43 \, \text{m/s} \quad (96 \, \text{mph})
\end{align*}
\]

Example 6.2 - A tetherball problem
A tetherball is attached in the ceiling by a light cord of length L. The ball swings in a horizontal circle with constant speed \( v \), and the cord makes a constant angle \( \beta \) with the vertical direction. The ball goes through one revolution in time \( T \). Assuming that \( T \), the mass \( m \), and the length \( L \) of rope are known, derive algebraic expressions for the tension \( F_t \) in the cord and the angle \( \beta \).

\[
\begin{align*}
\Sigma F &= m a_v = ma \cos \beta \\
F_t \sin \beta &= \frac{m v^2}{L} \\
F_t \cos \beta &= \frac{m v^2}{L} \\
F_t &= \frac{m v^2}{L} \\
\beta &= \sin \left( \frac{4 m v^2}{L T^2} \right)
\end{align*}
\]

Work - Energy Theorem
\[
\begin{align*}
W_{total} &= K_f - K_i = \Delta K \\
W_{total} &= W_c + W_{nc} \\
W_c &= U_{ini} - U_{fin} \\
&= -\Delta U \\
PE &= mgy \\
W_{nc} &= \Delta KE + \Delta PE \\
\bar{P} &= \frac{\Delta W}{\Delta t} = FV
\end{align*}
\]

Conservation of mechanical Energy
\[
\begin{align*}
K_i + U_i &= K_f + U_f \\
\rho_y &= 0 \\
\rho_z &= \rho_x + \rho_y + \rho_z \\
\rho_x &= \rho_y + \rho_z \\
\rho_{xy} &= \rho_{xz} \\
\rho_{y} &= \rho_{z} \\
\rho_{x} &= \rho_{x}
\end{align*}
\]

Example:
Tarzan swings on a 30.0-m-long vine initially inclined at an angle of 37.0° with the vertical. What is his speed at the bottom of the swing (a) if he starts from rest? (b) if he pushes off with a speed of 4.00 m/s?

\[
\begin{align*}
\text{PE} &= mgy \\
\Delta KE &= \frac{1}{2} m v^2
\end{align*}
\]
A skier descends along the hill as indicated in the figure. The skier starts from rest at the point A at a height of 40 m above the bottom point B, and jumps off at the point C which is 10 m above B. What is his speed at point C?

\[ v_C = \sqrt{2g(l_A - l_C)} \]

\[ = \sqrt{2 \times 9.8 \times 30} \approx 24 \text{ m/s} \]

A cross-country skier moving at 4.8 m/s on level ground encounters an approximately frictionless downward slope 6.1 m high. On the level ground below, the snow has been worn thin, and consequently, the coefficient of friction is 0.27.

After coasting down the hill, how far will the skier glide across the level stretch?

\[ \Delta x = \frac{v_i^2}{2 \mu g} + \frac{h}{\mu} \]

\[ = \frac{4.8^2}{2 \times 0.27 \times 9.8} + \frac{6.1}{0.27} \]

\[ = 29 \text{ m} \]

Formulas

\[ F = ma \]

\[ w = mg \]

\[ f_r = \mu N \]

\[ f_t = \mu_s N \]

\[ G = 6.674 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]

\[ \alpha = \frac{v_f^2}{2x} \]

\[ v = \frac{\sqrt{2xm}}{t} \]

\[ t = \frac{v - \sqrt{2xm}}{\sqrt{2mg}} \]

Chapter 6

Work = W = F \cdot s \cdot \cos \theta

\[ K = \frac{1}{2}mv^2 \]

\[ W = K_f - K_i \]

\[ U_{pot} = mgx \]

\[ W_{pot} = -\Delta U_{pot} \]

\[ E = K + U \]

\[ W = -\Delta U \]

\[ K_f + U_f = K_i + U_i + W \]