ELSEVIER Journal of Luminescence 76&77 (1998) 677–680

JOURNAL OF LUMINESCENCE

Trapping and photon number states in a two-photon micromaser

M. Alexanian[a], S. Bose[b,*], L. Chow[b]

[a] *Department of Physics, University of North Carolina at Wilmington, Wilmington NC 28403, USA*
[b] *Department of Physics, University of Central Florida, Orlando, FL 32816, USA*

Abstract

We present a theoretical analysis of a two-photon micromaser and investigate the statistical properties of the radiation. We analyze both vacuum as well as non-vacuum trapped states that follow from the theory. Non-vaccum trapped states have not been found in previous theories of the two-photon micromaser. We explore how photon number states can be generated in the limit of large flux of atoms in the cavity.

Keywords: Two-photon micromaser; Trapped photon states; Photon number state

Single Rydberg atoms in high-Q microcavities are known to produce one as well as two-photon maser action [1,2]. There is considerable current interest in determining the statistical properties of the maser radiation. The one-photon micromaser is characterized by sub-Poissonian photon number distribution and trapped photon states [3]. There is also the interesting possibility of a trapped state developing into a photon number state [4] or a Fock state, one containing a definite number of photons in a mode. Number states are expected to be of considerable importance in quantum information theory. While such states may not yet be realizable, there is a great deal of interest in determining if they can be generated in some limits. We present here a theory that demonstrates the existance of trapped states and how photon number states can be generated in a two-photon micromaser.

A two-photon micromaser is a device in which suitable three-level Rydberg atoms of fixed velocity enter a cavity and undergo atom-cavity interaction emitting two photons. The flux of atoms is such that only one atom is in the cavity at any time. Consider a three-level atom whose states, denoted by $|1\rangle$, $|2\rangle$, and $|3\rangle$, interacts with a single-mode cavity radiation of frequency ω. The "elimination" of the middle level leads to an effective two-level atom interacting via two-photon processes. There are a number of ways of bringing this about. One method to obtain the desired effective two-level atom is to use an unitary transformation [5,6]. In this paper, we consider the atom-field dynamics governed by the unitarily transformed Hamiltonian to analyze the statistics of the radiation of the two-photon maser.

The effective two-level Hamiltonian, using the method of the unitary transformation is

$$H_r = \hbar\omega N + E_0 + \hbar\mu\sigma_{33} + \hbar\eta\sigma_{11} + \hbar\lambda(\sigma_{31}a^2 + \sigma_{13}a^{\dagger 2}), \quad (1)$$

where σ_{ij} $(i, j = 1,3)$ are spin-type opearators and a, and $a^\dagger$, respectively, are annihilation and creation operators of the radiation mode. Expressions for μ,

* Corresponding author. E-mail: skb@physics.ucf.edu.

0022-2313/98/$19.00
PII S0022-2313(97)00266-4

η, λ, E_0, and N can be found by the methods of Ref. [5,6]. The Hamiltonian describes atom-field interactions within the cavity. Following the method of Filipowicz et al. [7], we obtain the following recursion relation for the steady-state photon number probability:

$$P_n = \left[A + \frac{B(n+1)}{(2n+1)^2} \times \sin^2[D\{\sqrt{C^2+2n+1} - C\}] \right] P_{n-1} + B\frac{(n-1)}{(2n-1)^2}\sin^2[D\{\sqrt{C^2+2n-1} - C\}]\, P_{n-2}. \tag{2}$$

The various parameters are given by

$$A = \frac{n_b}{n_b+1},\quad B = \frac{4R}{\gamma}\frac{1}{n_b+1},$$
$$C = \frac{\Delta}{2\hbar g},\quad D = \frac{\tau g}{2}, \tag{3}$$

where, n_b is the average number of thermal photons, γ is the cavity damping rate, Δ the detuning, and g the coupling constant. Also, R is the flux rate of atoms into the cavity and τ is the time the atom spends in the cavity. The statistics of the radiation is given by the average number of photons, $\langle n \rangle$, and the normalized variance, σ. For $\sigma < 1$, the distribution is sub-Poissonian while it is Poissonian if $\sigma = 1$. If $\sigma > 1$, the distribution is said to be super-Poissonian. $\langle n \rangle$ and σ are numerically evaluated from Eq. (2). Our analysis shows sub-Poissonian behavior as well as trapping states in the photon distribution.

Trapping states occur at sufficiently low temperatures, and at particular values of τg and $\Delta/\hbar g$, such that $\langle n \rangle$ stabilizes and additional photons cannot be added to the mode. This means that if the system is trapped at a photon number k, then $P_n = 0$ for $n \geqslant k$, but $P_n \neq 0$ for $n < k$. The simplest possibility is trapping in the vaccum or $n = 0$ state, already discussed for the two-photon micromaser [8]. From Eq. (2) it follows that $n = 0$ is a trapping state provided one satisfies the condition

$$\{\sqrt{C^2+3} - C\}\, D = j\pi, \tag{4}$$

where, j is an integer. The above-condition becomes identical to that obtained in Ref. [8] if we set the detuning $\Delta = 0$, i.e., $C = 0$. Fig. 1 is a plot of $\langle n \rangle$ vs. D/π for $B = 80/(1 + n_b)$, and $C = 1$. We consider three different cavity temperatures represented by $n_b = 0, 10^{-3}$, and 10^{-1}. Vacuum trapping states are indicated by the dips in $\langle n \rangle$ which occur at integer

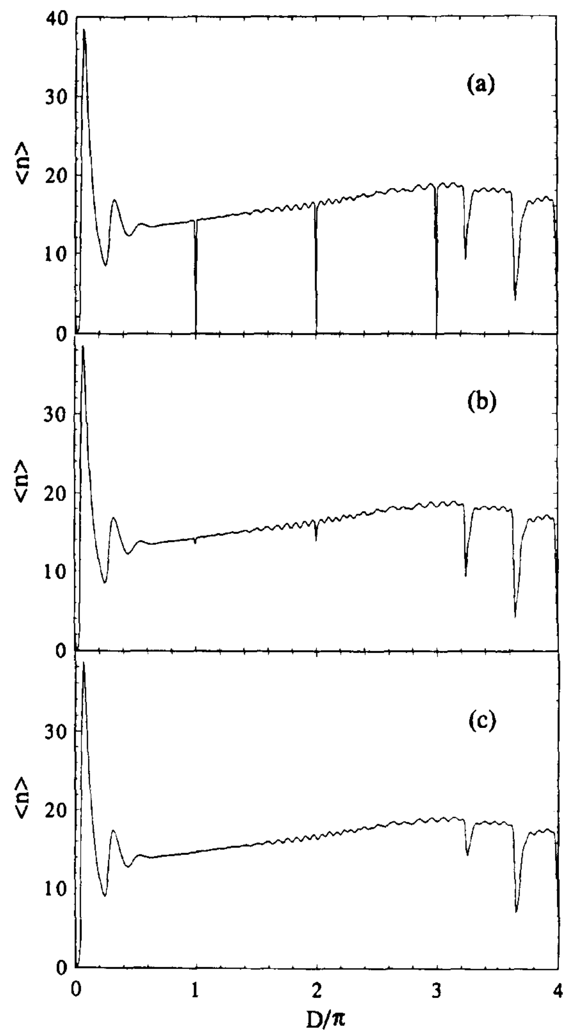

Fig. 1. Plot of $\langle n \rangle$ versus D/π for $B = 80/(1 + n_b)$ and $C = 1$. The minima, denoting vacuum trapping states, occur for integer values of D/π as indicated by Eq. (4). The two minima when $3 < D/\pi < 4$ are not known trapping states. The three graphs, corresponding to the values $n_b = 0$, 10^{-3}, and 10^{-1}, indicate cavity temperature dependence of $\langle n \rangle$.

values of D/π when condition given by Eq. (4) is satisfied. As the cavity temperature, given by n_b, is increased the trapping states disappear as expected. It is interesting that the minima for $3 < D/\pi < 4$, which do not represent known trapped states, are rather insensitive to such temperature changes, except when the value of n_b is made quite large, of the order of 1.

We now discuss the existance of non-vaccum trapping states, a possibility which is non-existant in the theory of the two-photon micromaser considered in Ref. [8]. A photon state $n = k \neq 0$ will be a trapping state if we simultaneously satisfy the conditions:

$$\{\sqrt{C^2 + 2k + 1} - C\}\, D = j\pi\,, \tag{5a}$$

$$\{\sqrt{C^2 + 2k - 1} - C\}\, D = m\pi, \tag{5b}$$

where, $j > m$ (m and j are positive integers) and $k \geqslant 3$. There are several possibilities by which trapping states can be realised. We make a particular choice, i.e., $j = k + 1$ and $m = k$. In this case we can easily obtain $C = (2k^2 - 1)/2\sqrt{(k^2 + k)}$ and $D = \pi\sqrt{(k^2 + k)}$. Hence, k would denote a parametrization for τg and $\Delta/\hbar g$. Fig. 2 is a plot of $\langle n \rangle$ vs. k for $B = 200/(1 + n_b)$ and three values of $n_b = 0$, 10^{-6} and, 10^{-3}. The non-vaccum trapping states are indicated by dips in $\langle n \rangle$ at integer values of k. We notice an interesting feature of the theory, a phase-transition like behavior at $k \approx 6$. Below the transition the system admits trapping as well as non-trapping states but above it the distinction between the two disappears, similar to a first-order phase transition. This behavior is maintained for higher values of the atomic flux, B, although the transition occurs at higher values of k. We also note that the trapping states disappear with increasing temperature. The average photon number $\langle n \rangle$, for higher values of k, approaches saturation obeying the inequality $\langle n \rangle \leqslant B/2$. Also, below and above the transition the photon statistics is sub-Poissonian while at the transition the statistics is super-Poissonian as is found by evaluating the normalized variance σ. To our knowledge, a transition of the kind just discussed has not been previously seen in micromasers.

At higher values of B, the trapping state can become a photon number state. In the limit of large

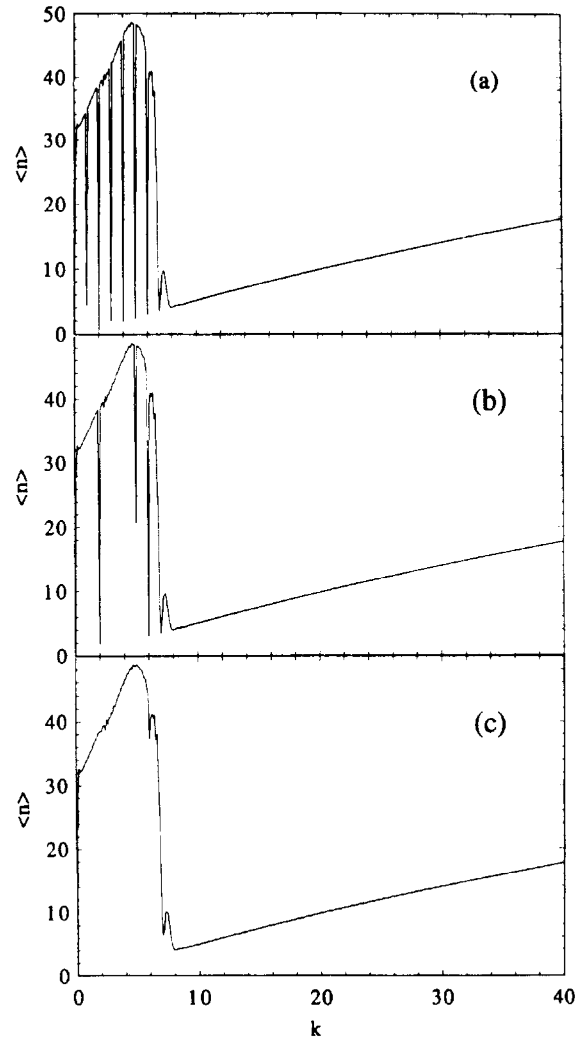

Fig. 2. Plot of $\langle n \rangle$ indicating, for $B = 200/(1 + n_b)$, $C = (2k^2 - 1)/2\sqrt{(k^2 + k)}$, and $D = \pi\sqrt{(k^2 + k)}$, non-vaccum trapping states and phase transition-like behavior. The three graphs correspond to $n_b = 0$, 10^{-6}, and 10^{-3}, respectively. Phase transition-like behavior disappears as temperature is gradually increased.

B, we derive,

$$P_s \to P_0 \prod_{n=1}^{s} \frac{B(n+1)}{(2n+1)^2} \sin^2 \times D\{\sqrt{C^2 + 2n + 1} - C\}\,, \tag{6}$$

where, $s = 0,1,2, \ldots, k - 2$. The normalization condition of P_n implies that for $B \to \infty$, $P_0 \to 1/$

$O(B^{k-2})$. Consequently, only two probabilities, P_{k-1} and P_{k-2} survive, while all others go to zero as $B \to \infty$. The dispersion in the photon number also vanishes, if, in addition, k is large. The reduction in the number of states to one and the limit $\sigma \to 0$ are indications of the approach to a photon number state.

References

[1] D. Meschede, H. Walther, G. Müller, Phys. Rev. Lett. 54 (1985) 551.
[2] M. Brune, J.M. Raimond, P. Goy, L. Davidovitch, S. Haroche, Phy. Rev. Lett. 59 (1987) 1899.
[3] P. Meystre, G. Rempe, H. Walther, Opt. Lett. 13 (1988) 1078.
[4] G. Harel, G. Kurizki, Phys. Rev. A 54 (1996) 5410.
[5] M. Alexanian, S. Bose, Phys. Rev. A 52 (1995) 2218.
[6] Y. Wu, Phys. Rev. A 54 (1996) 1586.
[7] P. Filipowicz, J. Javanainen, P. Meystre, Phy. Rev. A 34 (1986) 3077.
[8] I. Ashraf, J. Gea-Banacloche, M.S. Zubairy, Phys. Rev. A 42 (1990) 6704.