

Review for Exam 1

Practice Problem Solving !
Homework Problems
Recitation Problems
Examples from Class

- **Ch 1 Mathematical Concepts**
 - Dimensional Analysis
 - Trigonometry
 - **Vectors**, Addition and Subtraction, Components
 - Addition of Vectors by Means of Components
- **Ch 2 Kinematics in One Dimension**
 - Displacement, Speed, Velocity, Acceleration
 - **Equations of Kinematics for Constant Acceleration**
 - Applications; Freely Falling Bodies
- **Ch 3 Kinematics in Two Dimensions**
 - Displacement, Velocity, Acceleration (Vectors !)
 - Equations of Kinematics in Two Dimensions
 - Projectile Motion
 - Relative Velocity

Adding Vectors

To add vectors graphically, arrange them tip to tail in any order.

$$(\vec{A} - \vec{B}) = (\vec{A} + (-\vec{B})) = (\vec{A} + (-\vec{B}))$$

Subtracting \vec{B} from \vec{A} is equivalent to adding $-\vec{B}$ to \vec{A} .

Components of Vectors

$$A_x = A \cos \theta; \quad A_y = A \sin \theta; \quad A = \sqrt{A_x^2 + A_y^2}; \quad \theta = \tan^{-1} \frac{A_y}{A_x}$$

Example: Displacement of a cross-country skier

On a cross-country ski trip you travel 1.00 km north and then 2.00 km east.

a) How far and in what direction are you from your starting point ?



$$d = \sqrt{(1.00 \text{ km})^2 + (2.00 \text{ km})^2} = 2.24 \text{ km}$$

(magnitude of the displacement vector)

$$\tan \phi = \frac{\text{opposite side}}{\text{adjacent side}} = \frac{2.00 \text{ km}}{1.00 \text{ km}} \quad \phi = 63.4^\circ$$

(direction of the displacement vector)

Equations of Kinematics in One Dimension

Position



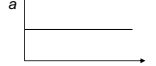
Velocity

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{x}}{\Delta t}$$



Acceleration

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t}$$



For **constant** acceleration we find:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2$$

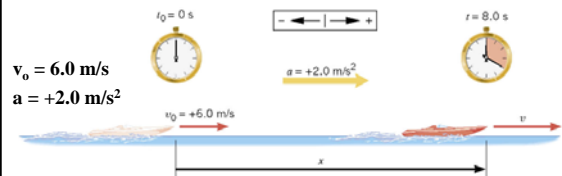
$$v = v_0 + a t$$

$$a = \text{const}$$

$$v^2 - v_0^2 = 2a(x - x_0)$$

$$g = 9.8 \text{ m/s}^2$$

Equations of kinematics for constant acceleration



Find the position of the accelerating speedboat at $t = 8.0 \text{ s}$.

$$\begin{aligned} x &= v_0 t + \frac{1}{2} a t^2 \\ &= (6.0 \text{ m/s})(8.0 \text{ s}) + \frac{1}{2} (2.0 \text{ m/s}^2)(8.0 \text{ s})^2 \\ &= +110 \text{ m} \end{aligned}$$

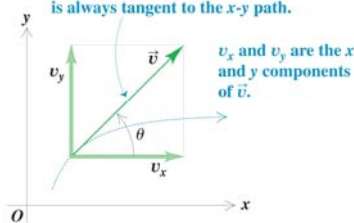
Velocity in two dimensions

Definition of average velocity $\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta \vec{r}}{\Delta t}$

Definition of instantaneous velocity

$$\vec{v} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{r}}{\Delta t}$$

The instantaneous velocity vector \vec{v} is always tangent to the x-y path.



Acceleration in two dimensions

Definition of average acceleration

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

Definition of instantaneous acceleration

$$\vec{a} = \lim_{\Delta t \rightarrow 0} \frac{\Delta \vec{v}}{\Delta t}$$

Most important concept in two-dimensional motion

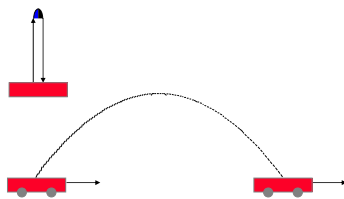
- two-dimensional motion can be decomposed into motion in x-direction and motion in y-direction

“x” and “y” components of motion are independent

- Person on a cart throws ball vertically upwards
 - Seen from two different frames of reference

Reference frame on moving cart

Reference frame on the ground



Constant-Acceleration Equations of Motion in Two-Dimensions

Equations for Projectile Motion (assuming that $a_x = 0$, $a_y = -g$)

$$v_x = v_{0x} + a_x t$$

$$v_x = v_{0x} \quad (1)$$

$$x = x_0 + v_{0x} t + (1/2) a_x t^2$$

$$x = x_0 + v_{0x} t \quad (2)$$

$$v_y = v_{0y} + a_y t$$

$$v_y = v_{0y} - g t \quad (3)$$

$$y = y_0 + v_{0y} t + (1/2) a_y t^2$$

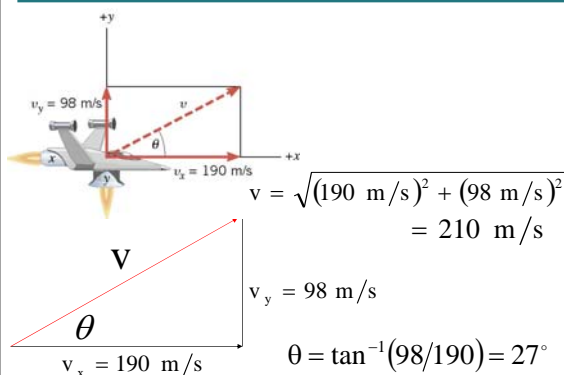
$$y = y_0 + v_{0y} t - (1/2) g t^2 \quad (4)$$

$$v_x^2 - v_{0x}^2 = 2a_x (x - x_0)$$

$$v_y^2 - v_{0y}^2 = -2g (y - y_0)$$

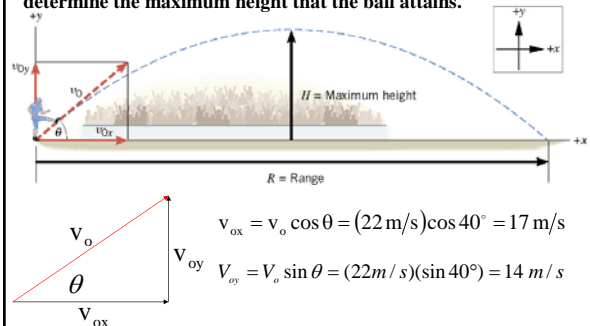
$$v_y^2 - v_{0y}^2 = 2a_y (y - y_0)$$

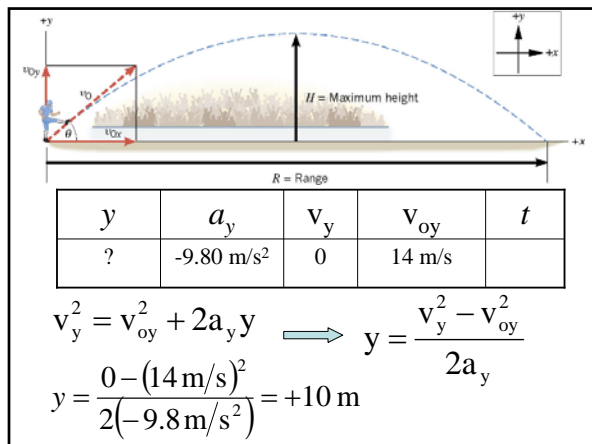
Example: A moving spacecraft



Example: The height of a kickoff

A placekicker kicks a football at an angle of 40.0 degrees and the initial speed of the ball is 22 m/s. Ignoring air resistance, determine the maximum height that the ball attains.





Relative velocity in 2-dim

The concept of relative velocity can be extended from 1-dim to 2-dim

(a)

(b) Relative velocities (seen from above)

Velocities can carry multiple values depending on the position and motion of the object and observer