Chapter 26 The Refraction of Light

26.1 The Index of Refraction

Light travels through a vacuum at a speed, $c = 3.00 \times 10^{8}\ \text{m/s}$

Light travels through materials at a speed, $v < c$.

DEFINITION OF THE INDEX OF REFRACTION

The index of refraction of a material is the ratio of the speed of light in a vacuum to the speed of light in the material:

$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in the material}} = \frac{c}{v}$$

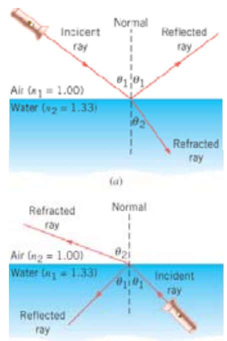
26.1 The Index of Refraction

Table 26.1 Index of Refraction* for Various Substances

Substance	Index of Refraction, n
Solids at 20 °C	
Diamond	2.419
Glass, crown	1.523
Ice (0 °C)	1.309
Sodium chloride	1.544
Quartz	
Crystalline	1.544
Fused	1.458
Liquids at 20 °C	
Benzene	1.501
Carbon disulfide	1.632
Carbon tetrachloride	1.461
Ethyl alcohol	1.362
Water	1.333
Gases at 0 °C, 1 atm	
Air	1.000 293
Carbon dioxide	1.000 45
Oxygen, O_2	1.000 271
Hydrogen, H_2	1.000 139

* Measured with light whose wavelength in a vacuum is 589 nm.

26.2 Snell's Law and the Refraction of Light



Snell'S Law

SNELL'S LAW OF REFRACTION

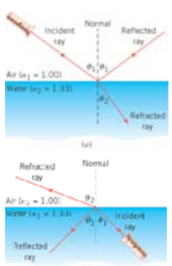
When light travels from a material with one index of refraction to a material with a different index of refraction, the angle of incidence is related to the angle of refraction by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

26.2 Snell's Law and the Refraction of Light

Example 1 **Determining the Angle of Refraction**

A light ray strikes an air/water surface at an angle of 46° with respect to the normal. Find the angle of refraction when the direction of the ray is (a) from air to water and (b) from water to air.



Use Snell's law $n_1 \sin\theta_1 = n_2 \sin\theta_2$

(a) (1.00)sin(46°) = (1.33)Sin(θ)
Sin(θ) = 0.54
θ = $\sin^{-1}$(0.54)= 33°.

(b) (1.33)sin(46°) = (1.00)sin(θ)
sin(θ) = 0.96
θ = $\sin^{-1}$(0.96) = 74°.

26.2 Snell's Law and the Refraction of Light

Example 2 **Finding a Sunken Chest**

The searchlight on a yacht is being used to illuminate a sunken chest. At what angle of incidence should the light be aimed?

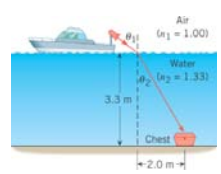
Find θ_2 first, then apply Snell's law

$$\theta_2 = \tan^{-1}(2.0/3.3) = 31^\circ$$

$$\sin\theta_1 = \frac{n_2 \sin\theta_2}{n_1} = \frac{(1.33)\sin 31^\circ}{1.00} = 0.69$$

$$\theta_1 = 44^\circ$$

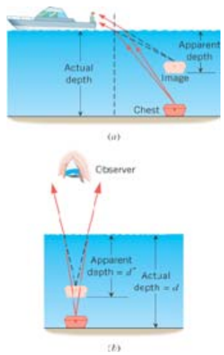
APPARENT DEPTH



26.2 Snell's Law and the Refraction of Light

Apparent depth, observer directly above object

$$d' = d\left(\frac{n_2}{n_1}\right)$$



26.2 Snell's Law and the Refraction of Light

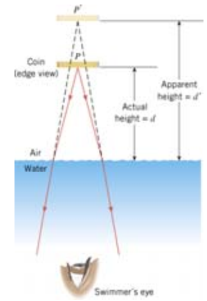
Example 4 **On the Inside Looking Out**

A swimmer is under water and looking up at the surface. Someone holds a coin in the air, directly above the swimmer's eyes. To the swimmer, the coin appears to be at a certain height above the water. Is the apparent height of the coin greater, less than, or the same as its actual height?

Since $n_{water} > n_{air}$, $\theta_{air} > \theta_{water}$,

>>> $\tan\theta_{air} > \tan\theta_{water}$

>>> $d' > d$

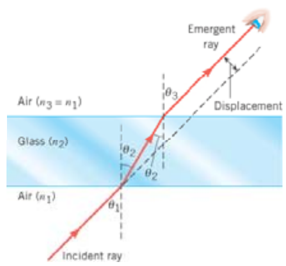


26.2 Snell's Law and the Refraction of Light

A ray of light passes through a pane of glass

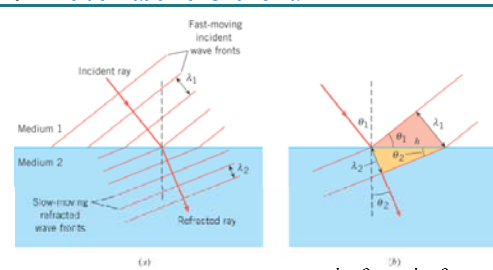
Apply Snell's law at the two interfaces

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_1 \sin\theta_3$$
$$\theta_1 = \theta_3$$



The emerging ray is parallel to the incident ray.

26.2 The derivation of Snell's Law

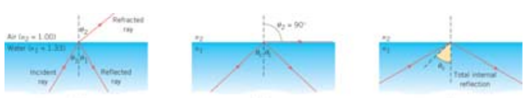


$$\sin\theta_1 = \frac{\lambda_1}{h} = \frac{v_1 t}{h}, \quad \sin\theta_2 = \frac{\lambda_2}{h} = \frac{v_2 t}{h} \qquad \frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

$$c/v = n, \textit{ therefore } \quad n_1 \sin\theta_1 = n_2 \sin\theta_2$$

26.3 Total Internal Reflection

When light passes from a medium of larger refractive index into one of smaller refractive index, the refracted ray bends away from the normal.



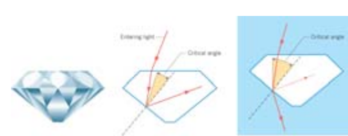
From Snell's law $\Rightarrow$ $\sin\theta_c = \frac{n_2}{n_1}$ $\quad n_1 > n_2$

The Critical angle $\Rightarrow$ $\theta_c = \sin^{-1}(\frac{n_2}{n_1})$ $\quad n_1 > n_2$

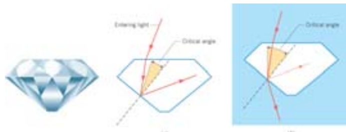
26.3 Total Internal Reflection

Example 5 **Total Internal Reflection**

A beam of light is propagating through diamond and strikes the diamond-air interface at an angle of incidence of 28 degrees. (a) Will part of the beam enter the air or will there be total internal reflection? (b) Repeat part (a) assuming that the diamond is surrounded by water.



26.3 Total Internal Reflection



(a) At diamond-air interface $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.00}{2.42}\right) = 24.4^\circ$

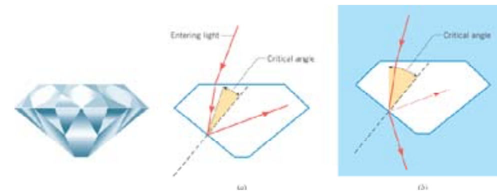
(b) At diamond-water interface $\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right) = \sin^{-1}\left(\frac{1.33}{2.42}\right) = 33.3^\circ$

So in case (a), there is no refraction, the light is totally reflected back into diamond. In case (b), the light refracted into water, no internal reflection.

26.3 Total Internal Reflection

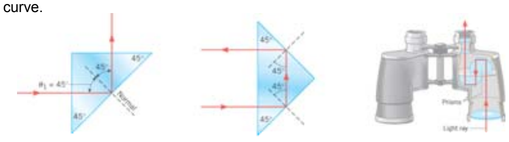
Conceptual Example 6 **The Sparkle of a Diamond**

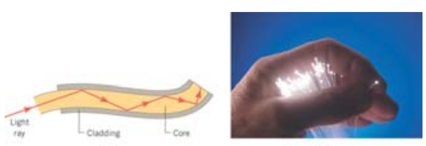
The diamond is famous for its sparkle because the light coming from it glitters as the diamond is moved about. Why does a diamond exhibit such brilliance? Why does it lose much of its brilliance when placed under water?



26.3 Total Internal Reflection

Many optical instruments and optical devices use total internal reflection to turn a beam of light through 90° or to send a beam of light through an arbitrary curve.

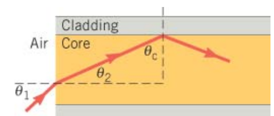




26.3 Total Internal Reflection

Example & An optical Fiber

The figure shows an optical fiber that consists of a core made of flint glass (n_f = 1.667) surrounded by a cladding made of crown glass (n_c = 1.523), what is θ1 if the light will be internally reflected



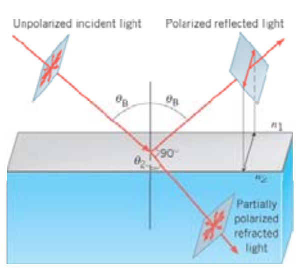
$$\theta_c = \sin^{-1}\left(\frac{1.523}{1.667}\right) = 66.01^\circ, \quad \theta_2 = 90^\circ - \theta_c = 23.99^\circ$$

$$\theta_1 = \sin^{-1}\left(\frac{1.667 \cdot \sin(23.99)}{1.00}\right) = 42.67^\circ$$

26.4 Polarization and the Reflection and Refraction of Light

Reflection from a nonmetallic surface, an unpolarized light becomes partially polarized.

At Brewster angle, the reflected light is completely polarized.



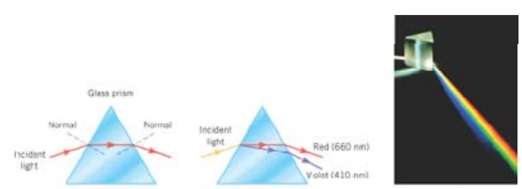
$$n_1 \sin\theta_B = n_2 \sin\theta_2$$
$$n_1 \sin\theta_B = n_2 \cos\theta_B$$
$$\frac{\sin\theta_B}{\cos\theta_B} = \frac{n_2}{n_1}$$

Brewster's law $\tan\theta_B = \frac{n_2}{n_1}$

26.5 The Dispersion of Light: Prisms and Rainbows

The net effect of a prism is to change the direction of a light ray.

Light rays corresponding to different colors bend by different amounts.



26.5 The Dispersion of Light: Prisms and Rainbows

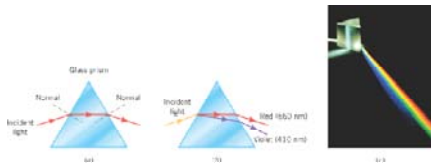


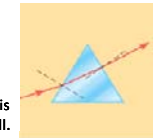
Table 26.2 Indices of Refraction *n* of Crown Glass at Various Wavelengths

Approximate Color	Wavelength in Vacuum (nm)	Index of Refraction, *n*
Red	660	1.520
Orange	610	1.522
Yellow	580	1.523
Green	550	1.526
Blue	470	1.531
Violet	410	1.538

26.5 The Dispersion of Light: Prisms and Rainbows

Example 8 **The Refraction of Light Depends on Two Refractive Indices**

A ray of light passes through identical prisms on the right Each surrounded by a different medium. The ray of light is refracted upward in (a) while in (b) it is not refracted at all. Is it possible?

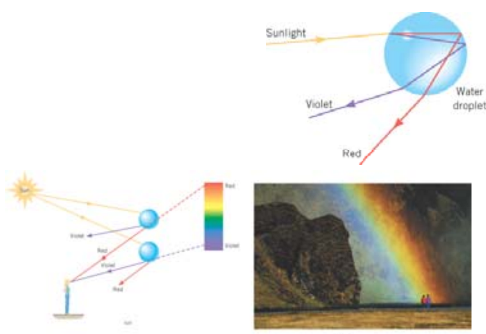


(a)

(b)

Ans.
Yes, It is possible for a prism to bend light upward, downward, or not at all. The refraction of light depends on Two index of refraction, the prism and the surrounding medium. In case (b), the surrounding medium has the Exact Index as in the glass.

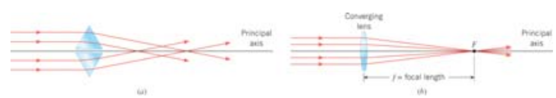
26.5 The Dispersion of Light: Prisms and Rainbows



26.6 Lenses

Lenses refract light in such a way that an image of the light source is formed.

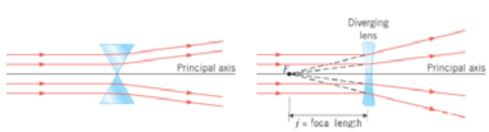
With a converging lens, paraxial rays that are near and parallel to the principal axis converge to **the focal point, F**.

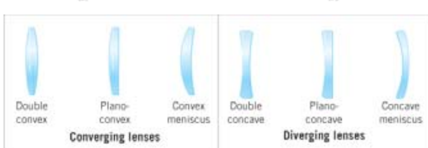


The distance from the center of the mirrpow to focal point is called focal length.

26.6 Lenses

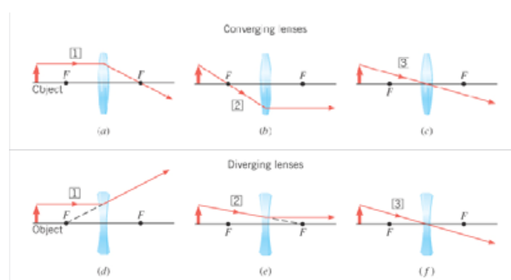
With a diverging lens, paraxial rays that are parallel to the principal axis appear to originate from the focal point.





26.7 The Formation of Images by Lenses

RAY TRACING

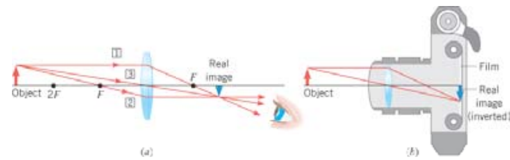


Ray Tracing for Converging and Diverging Lenses

Converging lens	Diverging lens
This ray initially travels parallel to the principal axis. In passing through a converging lens, the ray is refracted toward the axis and travels through the focal point on the right side of the lens, as Figure 26.25a shows.	This ray initially travels parallel to the principal axis. In passing through a diverging lens, the ray is refracted away from the axis, and appears to have originated from the focal point on the left of the lens. The dashed line in Figure 26.25d represents the apparent path of the ray.
This ray first passes through the focal point on the left and then is refracted by the lens in such a way that it leaves traveling parallel to the axis, as in Figure 26.25b.	This ray leaves the object and moves toward the focal point on the right of the lens. Before reaching the focal point, however, the ray is refracted by the lens so as to exit parallel to the axis. See Figure 26.25e, where the dashed line indicates the ray's path in the absence of the lens.
This ray travels directly through the center of the thin lens without any appreciable bending, as in Figure 26.25c.	This ray travels directly through the center of the thin lens without any appreciable bending, as in Figure 26.25f.

26.7 The Formation of Images by Lenses

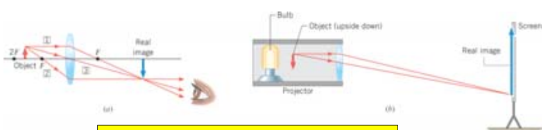
IMAGE FORMATION BY A CONVERGING LENS



In this example, when the object is placed further than twice the focal length from the lens, the real image is inverted and smaller than the object.

Object location, x $\infty > x > 2F$

26.7 The Formation of Images by Lenses



Object location, x $2F > x > F$

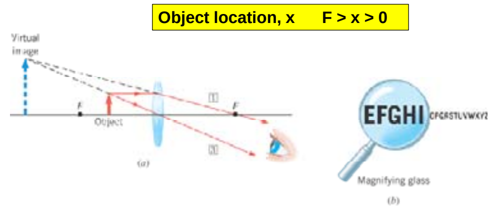
When the object is placed between F and 2F, the real image is inverted and larger than the object.

(1) The ray that is parallel to the principal axis is refracted by the lens and will pass through the focal point on the other side of the lens.
(2) The ray that passes through the focal point on the same side of the object is refracted by the lens and will travel parallel to the principal axis.
(3) The ray travels through the center of the lens will continue its straight line path without bending.

The above three rays will meet at one point.

26.7 The Formation of Images by Lenses

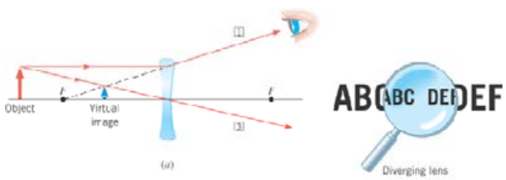
Object location, x $F > x > 0$



When the object is placed between F and the lens, the virtual image is upright and larger than the object.

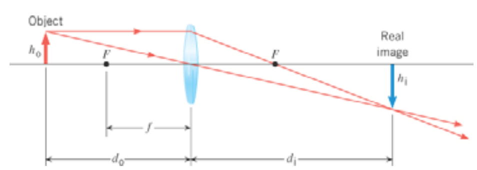
26.7 The Formation of Images by Lenses

IMAGE FORMATION BY A DIVERGING LENS



A diverging lens always forms an upright, virtual, diminished image.

26.8 The Thin-Lens Equation and the Magnification Equation



$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

26.8 The Thin-Lens Equation and the Magnification Equation

Summary of Sign Conventions for Lenses

Focal length

f is + for a converging lens.
f is – for a diverging lens.

Object distance

d_o is + if the object is to the left of the lens.
d_o is – if the object is to the right of the lens.

Image distance

d_i is + for an image formed to the right of the lens (real image).
d_i is – for an image formed to the left of the lens (virtual image).

Magnification

m is + for an upright image.
m is – for an inverted image.

26.8 The Thin-Lens Equation and the Magnification Equation

Example 9 **The Real Image Formed by a Camera Lens**

A 1.70-m tall person is standing 2.50 m in front of a camera. The camera uses a converging lens whose focal length is 0.0500 m. (a) Find the image distance and determine whether the image is real or virtual. (b) Find the magnification and height of the image on the film.

(a) $$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{0.0500\text{ m}}-\frac{1}{2.50\text{ m}}=19.6\text{ m}^{-1}$$

$$d_i = 0.0510\text{ m}$$ real image

(b) $$m=-\frac{d_i}{d_o}=-\frac{0.0510\text{ m}}{2.50\text{ m}}=-0.0204$$

$$h_i = mh_o = (-0.0204)(2.50\text{ m}) = -0.0347\text{ m}$$

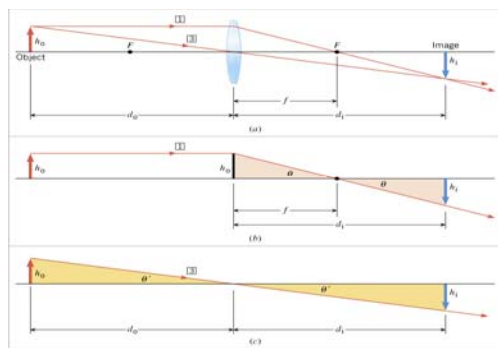
Example 10 The virtual image formed by a diverging lens

An object is placed 7.10 cm to the left of a diverging lens whose focal length is f = -5.08 cm. (a) Find the image distance and determine whether the image is real or virtual. (b) Find the magnification.

(a) D_o =7.1 cm, f =-5.08 cm

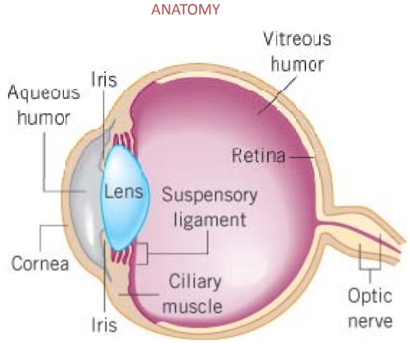
$$\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f}\Rightarrow\frac{1}{d_i}=\frac{1}{-5.08}-\frac{1}{7.10}$$ $$d_i=-2.96\ cm$$

(b) $$m=-\frac{d_i}{d_o}=-\frac{-2.96}{7.10}=0.417$$



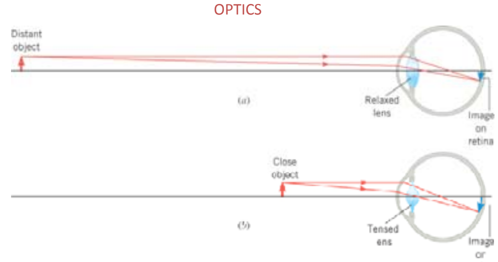
26.10 The Human Eye

ANATOMY



26.10 The Human Eye

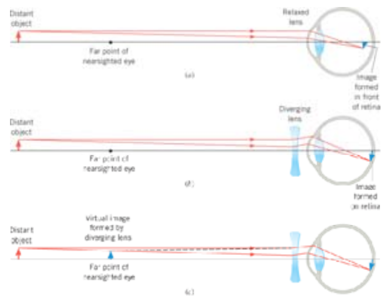
OPTICS



The lens only contributes about 20-25% of the refraction, but its function is important.

26.10 The Human Eye

NEARSIGHTEDNESS

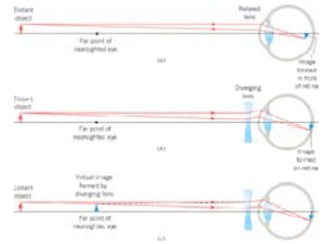


The lens creates an image of the distance object at the far point of the nearsighted eye.

26.10 The Human Eye

Example 12 Eyeglasses for the Nearsighted Person

A nearsighted person has a far point located only 521 cm from the eye. Assuming that eyeglasses are to be worn 2 cm in front of the eye, find the focal length needed for the diverging lens of the glasses so the person can see distant objects.

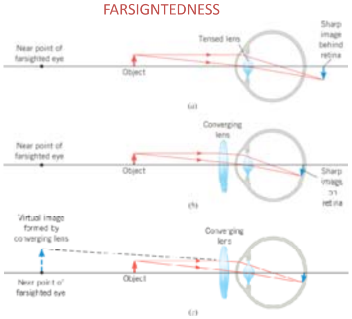


$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{\infty}-\frac{1}{519\text{ cm}}$$

$$f=-519\text{ cm}$$

26.10 The Human Eye

FARSIGNTEDNESS



The lens creates an image of the close object at the near point of the farsighted eye.

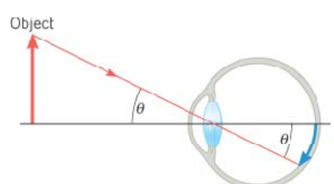
26.10 The Human Eye

THE REFRACTIVE POWER OF A LENS – THE DIOPTER

Optometrists who prescribe correctional lenses and the opticians who make the lenses do not specify the focal length. Instead they use the concept of ***refractive power.***

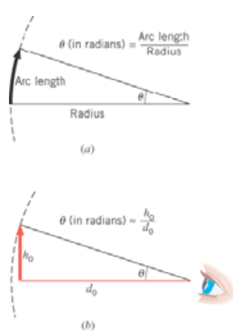
$$\text{Refractive power (in diopters)}=\frac{1}{f\text{ (in meters)}}$$

26.11 Angular Magnification and the Magnifying Glass



The size of the image on the retina determines how large an object appears to be.

26.11 Angular Magnification and the Magnifying Glass



$$\theta\text{ (in radians)}=\text{Angular size}\approx\frac{h_o}{d_o}$$

26.11 Angular Magnification and the Magnifying Glass

***Example 14* A Penny and the Moon**

Compare the angular size of a penny held at arms length with that of the moon.

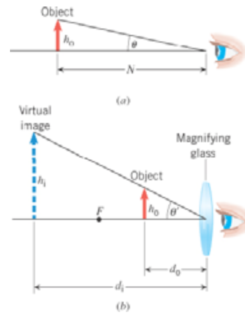
Penny

$$\theta \approx \frac{h_o}{d_o} = \frac{1.9\,\text{cm}}{71\,\text{cm}} = 0.027\,\text{rad}$$

Moon

$$\theta \approx \frac{h_o}{d_o} = \frac{3.5\times10^6\ \text{m}}{3.9\times10^8\ \text{m}} = 0.0090\,\text{rad}$$

26.11 Angular Magnification and the Magnifying Glass



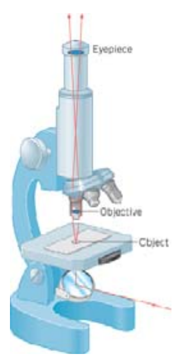
Angular magnification

$$M = \frac{\theta'}{\theta}$$

Angular magnification of a magnifying glass

$$M \approx \left(\frac{1}{f} - \frac{1}{d_i}\right)N$$

26.12 The Compound Microscope

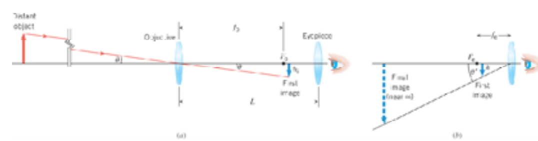


To increase the angular magnification beyond that possible with a magnifying glass, an additional converging lens can be included to "premagnify" the object.

Angular magnification of a compound microscope

$$M \approx -\frac{(L - f_e)N}{f_o f_e}$$

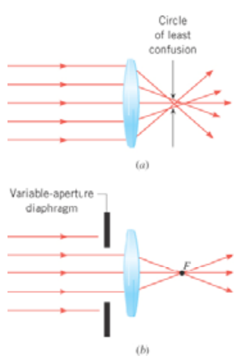
26.13 The Telescope



Angular magnification of an astronomical telescope

$$M \approx -\frac{f_o}{f_e}$$

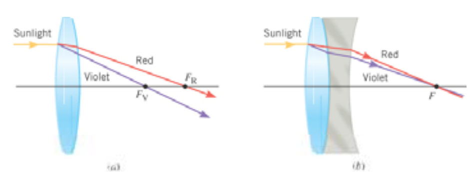
26.14 Lens Aberrations



In a converging lens, spherical aberration prevents light rays parallel to the principal axis from converging at a single point.

Spherical aberration can be reduced by using a variable-aperture diaphragm.

26.14 Lens Aberrations



Chromatic aberration arises when different colors are focused at different points along the principal axis.