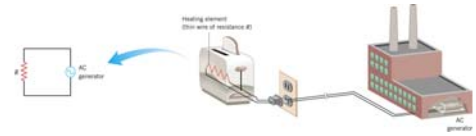
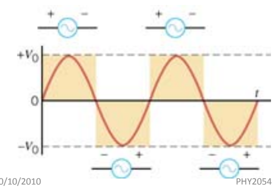
Chapter 23
Alternating Current Circuits

20.5 Alternating Current



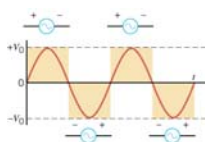
In an AC circuit, the charge flow reverses direction periodically.



$$V = V_o \sin(2\pi\, ft)$$

20.5 Alternating Current

In circuits that contain only resistance, the current reverses direction each time the polarity of the generator reverses.



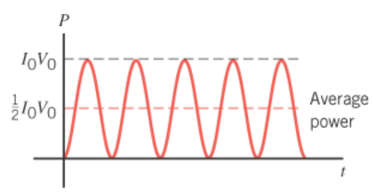
$$I = \frac{V}{R} = \frac{V_o}{R}\sin(2\pi\, ft) = I_o \sin(2\pi\, ft)$$

peak current

20.5 Alternating Current

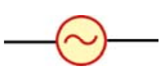
$I = I_o \sin(2\pi\, ft)$ $V = V_o \sin(2\pi\, ft)$

$$P = IV = I_o V_o \sin^2(2\pi\, ft)$$



Alternating-Current Circuit

- **direct current (dc) - current flows one way (battery)**
- **alternating current (ac) - current oscillates**
  - **sinusoidal voltage source**



$$V(t) = V_P \sin(\omega t)$$

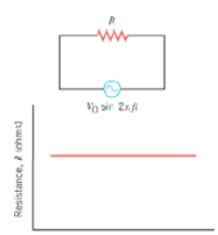
$\omega = 2\pi f$: **angular frequency**

V_P : **voltage amplitude**

23.1 Capacitors and Capacitive Reactance

We will study three basic circuit elements and investigate the current-voltage relationship for different devices:

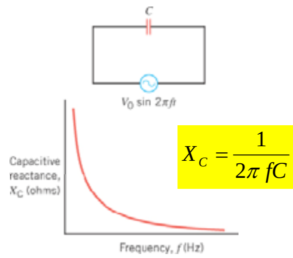
(a) Resistor
(b) Capacitor
(c) inductor



$$V_{rms} = I_{rms} R$$

23.1 Capacitors and Capacitive Reactance

For an ideal capacitor, the rms current varies as a function of the frequency.

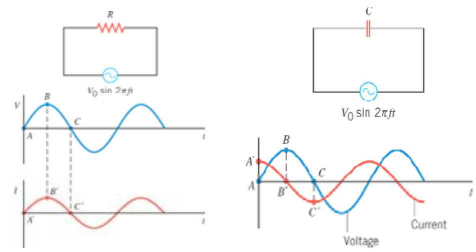


$$X_C = \frac{1}{2\pi fC}$$

X_c is the capacitive reactance

$$V_{rms} = I_{rms} X_C$$

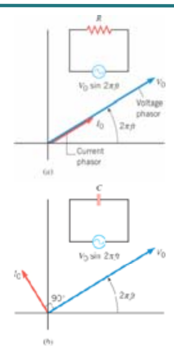
23.1 Capacitors and Capacitive Reactance



For a purely resistive circuit, the current and voltage are ***in phase***.
For a purely capacitive circuit, the current leads the voltage by 90°.

The average power used by a capacitor in an ac circuit is zero.

23.1 Capacitors and Capacitive Reactance



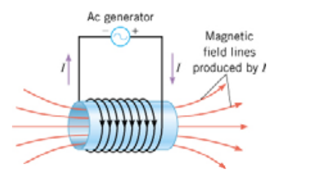
Phasor Model

In the ***phasor*** model, the voltage and current are represented by rotating arrows (called ***phasors***).

These phasors rotate at a frequency *f*.

The vertical component of the phasor is the instantaneous value of the current or voltage.

22.8 Mutual Inductance and Self Inductance

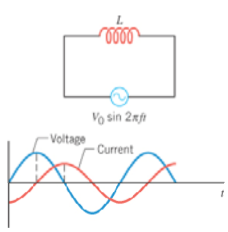


$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

self inductance

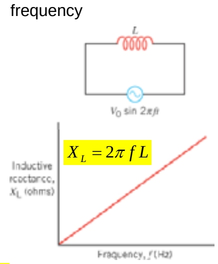
SI Unit of self inductance: $1\,\text{V}\cdot\text{s/A} = 1\,\text{H}$ (Henry)

23.2 Inductors and Inductive Reactance



The current ***lags*** behind the voltage by a phase angle of 90 degrees.

For an inductor, the inductive reactance is proportional to the frequency

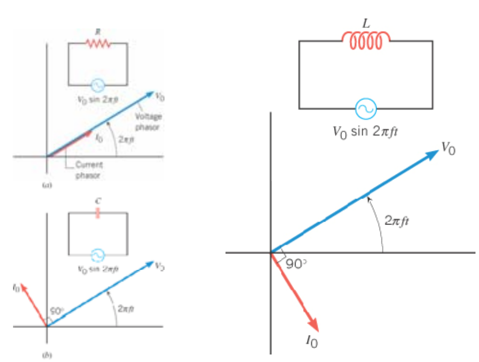


$$X_L = 2\pi f L$$

The average power used by an inductor in an ac circuit is zero.

$$V_{rms} = I_{rms} X_L$$

23.2 Inductors and Inductive Reactance



Example: household voltage

In the U.S., standard wiring supplies ***120 V*** at ***60 Hz***.
Write this in sinusoidal form, assuming ***V(t)=0 at t=0.***

This 120 V is the RMS amplitude: so $V_p = V_{rms}\sqrt{2} = 170$ V.
This 60 Hz is the frequency f: so $\omega = 2\pi f = 377\ s^{-1}$.

So $V(t) = 170 \sin(377t - \varphi_v)$.
Choose $\varphi_v = 0$ so that $V(t)=0$ at $t=0$: **V(t) = 170 sin (377 t)**.

AC Circuits: Summary

Element	I_0	Current vs. Voltage	Resistance Reactance Impedance
Resistor	$\frac{V_{0R}}{R}$	In Phase	$R = R$
Capacitor	$\omega C V_{0C}$	Leads	$X_C = \frac{1}{\omega C}$
Inductor	$\frac{V_{0L}}{\omega L}$	Lags	$X_L = \omega L$

Although derived from single element circuits, these relationships hold true generally !

What is *reactance*?

You can think of it as a frequency-dependent resistance.

$$X_C = \frac{1}{\omega C}$$

For low ω, $X_C \to \infty$
- Capacitor looks like a break

For high ω, $X_C \to 0$
- Capacitor looks like a wire ("short")

$$X_L = \omega L$$

For low ω, $X_L \to 0$
- Inductor looks like a wire ("short")

For high ω, $X_L \to \infty$
- Inductor looks like a break
(inductors resist change in current)

$$(\text{"}X_R\text{"} = R)$$

Example 2: An inductor in an AC Circuit

The circuit contains a 3.60 mH inductor. The rms voltage is 25.0 V. Find the rms current in the circuit when the generaor frequency is (a) 100 Hz, (b) 5000 Hz.

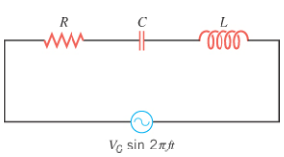
(a) $X_L = 2\pi f L = 2\pi(100)(3.6x10^2) = 2.26\,\Omega$

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{25V}{2.26\Omega} = 11.1\,A$$

(b) $X_L = 2\pi f L = 2\pi(5000)(0.0036) = 113\Omega$

$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{25}{113} = 0.221\,A$$

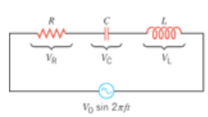
23.3 Circuits Containing R, C, and L



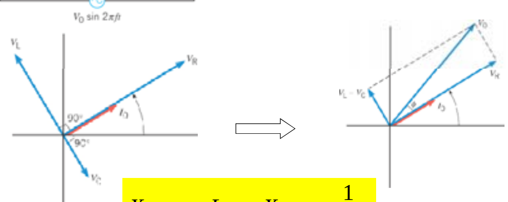
In a series RLC circuit, the total opposition to the flow is called the ***impedance***.

$$V_{rms} = I_{rms} Z \qquad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

23.3 Circuits Containing R, C, and L

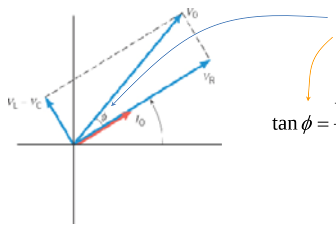


$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$



$$X_L = \omega L, \quad X_C = \frac{1}{\omega C}$$

23.3 Circuits Containing R, C, and L



$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

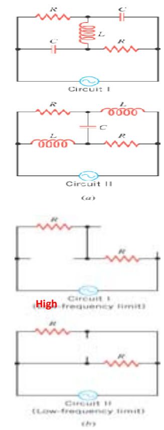
$$\overline{P} = I_{\text{rms}}^2 Z\cos\phi = I_{\text{rms}}V_{\text{rms}}\cos\phi$$

23.3 Circuits Containing R, C, and L

Conceptual Example 5 **The Limiting Behavior of Capacitors and Inductors**

The rms voltage of the generator is the same in each case. The values of the resistance, capacitance, and inductance are the same. The frequency of the ac generator is very near zero.

In which circuit does the generator supply more rms current?



Suppose:
$\mathrm{I} = \mathrm{I}_0 \sin\omega t$

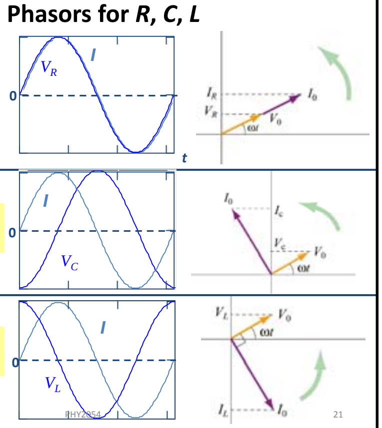
⇓

$\mathrm{V_R} = \mathrm{RI_0}\sin\omega t$

Phasors for *R, C, L*

$$\mathrm{V_C} = \frac{1}{\omega C}\mathrm{I_0}\sin(\omega t - \frac{\pi}{2})$$

$$\mathrm{V_L} = \omega \mathrm{LI_0}\sin(\omega t + \frac{\pi}{2})$$



Impedance Values and Phase Angles for Various Circuit-Element Combinations[a]

Circuit Elements	Impedance Z	Phase Angle ϕ
R	R	0°
C	X_C	−90°
L	X_L	+90°
R C	$\sqrt{R^2 + X_C^2}$	Negative, between −90° and 0°
R L	$\sqrt{R^2 + X_L^2}$	Positive, between 0° and 90°
R L C	$\sqrt{R^2 + (X_L - X_C)^2}$	Negative if $X_C > X_L$ Positive if $X_C < X_L$

[a] In each case, an AC voltage (not shown) is applied across the elements.

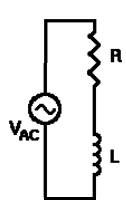
Clicker Question Check your understanding

5. An air-core inductor is connected in series with a light bulb, and this circuit is plugged into an ac outlet. When a piece of iron is inserted inside the inductor, does the brightness of the bulb

(a) Increase
(b) Decrease
(c) Remain the same ?

$$Z = \sqrt{R^2 + X_L^2}$$

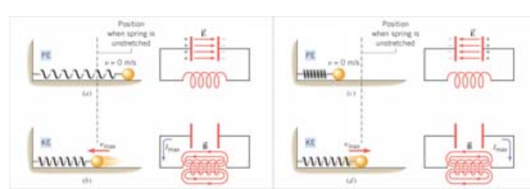
$$V_{rms} = I_{rms}Z$$



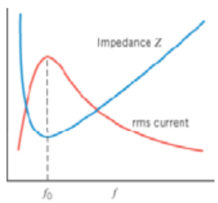
23.4 Resonance in Electric Circuits

Resonance occurs when the frequency of a vibrating force exactly matches a natural (resonant) frequency of the object to which the force is applied.

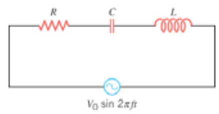
The oscillation of a mass on a spring is analogous to the oscillation of the electric and magnetic fields that occur, respectively, in a capacitor and an inductor.



23.4 Resonance in Electric Circuits



For an RLC circuit



$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}}$$

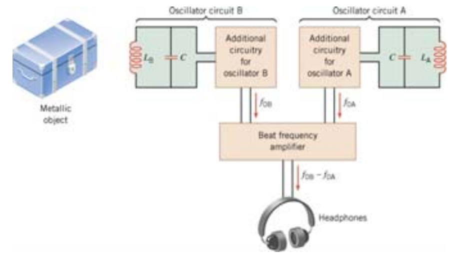
Resonant frequency

$$f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$2\pi f_o = \omega_o = \frac{1}{\sqrt{LC}}$$

Example 6 A heterodyne metal detector

Figure below is a heterodyne metal detector. This device utilizes capacitor/inductor oscillator circuits, A and B. Each produces its own resonant frequency. Any difference between these frequencies is detected as a beat frequency. Initially each oscillator has a resonant frequency of 855.5 kHz. Assume the inductance of B decreases by 1%, Find the beat frequency.



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Example 6 A heterodyne metal detector

Initially the resonant freqencies of the two circuits are the same

$$f_{oA} = 855.5\,kHz = f_{oB}$$

Now the metal object causes the L_B to decrease 1.00%

$$f'_{oB} = \frac{1}{2\pi\sqrt{0.99\,L_B C}} = \frac{855.5}{\sqrt{0.99}} = 859.82\ kHz$$

$$Beat\ \ frequency = \left|f'_{oB} - f_{oA}\right|$$
$$= |859.8 - 855.5| = 4.3\,kHz$$

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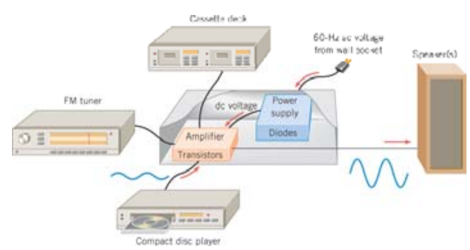
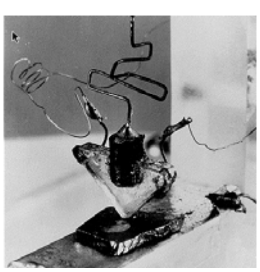
23.5 Semiconductor Devices

Semiconductor devices such as diodes and transistors are widely used in modern electronics.



In 1947, Bardeen, Branttain, and Shockley invented the first transistor.



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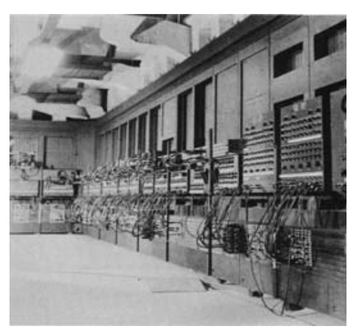
The first digital computer, ENIAC was built in 1947. It consisted of 17 thousand vacuum tubes.



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The first IBM PC was introduced in 1981.

The IBM PC Spec.

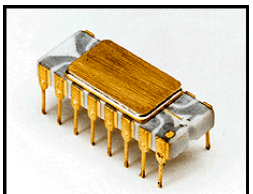
Micro-process----- 4.77 MHz Intel 8088

Memory----------- 16 kB

Storage------------ 160 kB of floppy disk

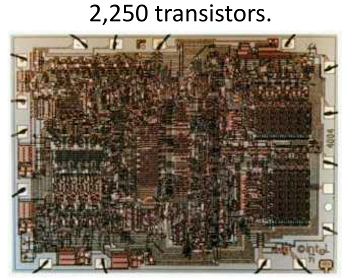
Price -------------- $1,565 (1981)

The Intel 4004 micro-processor, running at 108 kHz and 0.6 MIPS.

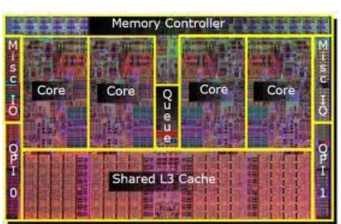


The Intel 4004, it was supposed to be the brains of a calculator. Instead, it turned into a general-purpose micro-processor as powerful as ENIAC.

Inside the Intel 4004 micro-processor. It has 2,250 transistors.

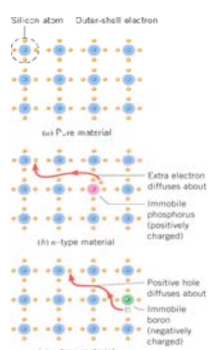


The new Intel Core i7. It has 4 different processors and contains 731 millions of transistors.



All the advance in computer technology comes from the developments of simple semiconductor devices such as diodes and transistors.

23.5 Semiconductor Devices

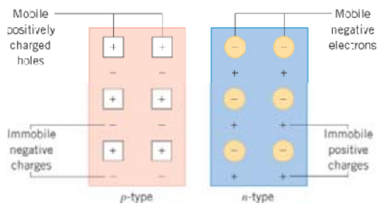


n-TYPE AND p-TYPE SEMICONDUCTORS

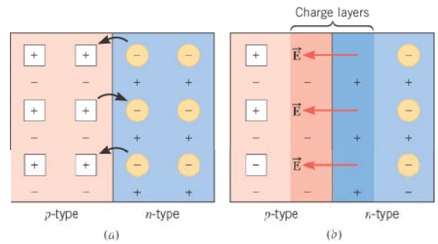
The semiconducting materials (silicon and germanium) used to make diodes and transistors are ***doped*** by adding small amounts of an impurity element.

23.5 Semiconductor Devices

THE SEMICONDUCTOR DIODE

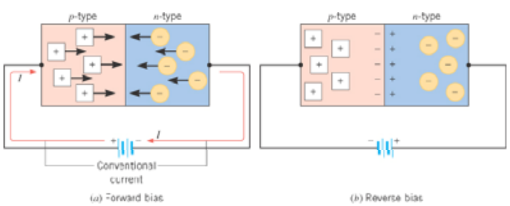


23.5 Semiconductor Devices



At the junction between the n and p materials, mobile electrons and holes combine and create positive and negative charge layers.

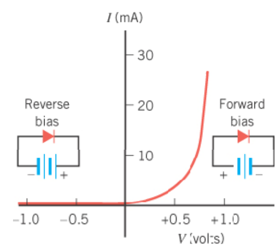
23.5 Semiconductor Devices



There is an appreciable current through the diode when the diode is forward biased.

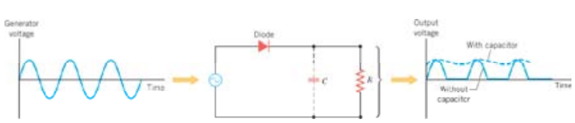
Under a reverse bias, there is almost no current through the diode.

23.5 Semiconductor Devices



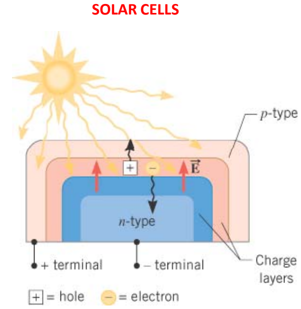
23.5 Semiconductor Devices

A diode can be used in A half-wave rectifier.

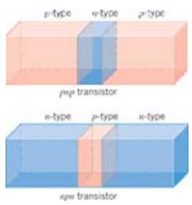


23.5 Semiconductor Devices

SOLAR CELLS



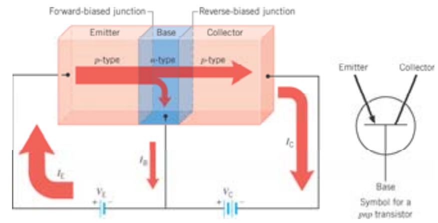
23.5 Semiconductor Devices



TRANSISTORS

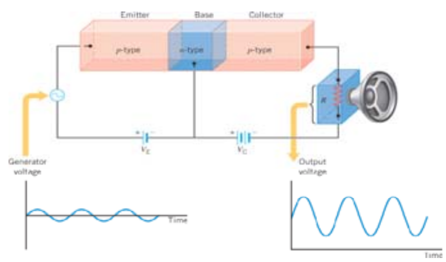
A bipolar junction transistor can be used to amplify a smaller voltage into a larger one.

23.5 Semiconductor Devices



The voltages are applied in such a way that the p-n junction on the left has a forward bias, while the p-n junction on the right has a reverse bias.

23.5 Semiconductor Devices



A small change in the emitter voltage input will cause a large voltage change at the collector output. This is the main function of a transistor, i.e. the amplification of a small signal.