Chapter 23
Alternating Current Circuits

20.5 Alternating Current

In circuits that contain only resistance, the current reverses direction each time the polarity of the generator reverses.

\[ I = \frac{V}{R} = \frac{V_0}{R} \sin(2\pi ft) = I_0 \sin(2\pi ft) \]

23.1 Capacitors and Capacitive Reactance

We will study three basic circuit elements and investigate the current-voltage relationship for different devices:

- (a) Resistor
- (b) Capacitor
- (c) Inductor

\[ V_{\text{rms}} = I_{\text{rms}} R \]

Alternating-Current Circuit

- direct current (dc) - current flows one way (battery)
- alternating current (ac) - current oscillates
- sinusoidal voltage source

\[ V(t) = V_p \sin (\omega t) \]

\[ \omega = 2\pi f \quad \text{angular frequency} \]

\[ V_p \quad \text{voltage amplitude} \]
23.1 Capacitors and Capacitive Reactance

For an ideal capacitor, the rms current varies as a function of the frequency.

\[ X_c = \frac{1}{2\pi fC} \]

\( X_c \) is the capacitive reactance

For a purely resistive circuit, the current and voltage are in phase. For a purely capacitive circuit, the current leads the voltage by 90°.

The average power used by a capacitor in an ac circuit is zero.

23.2 Inductors and Inductive Reactance

For an inductor, the inductive reactance is proportional to the frequency.

\[ X_l = \frac{2\pi fL}{L} \]

The current lags behind the voltage by a phase angle of 90 degrees.

The average power used by an inductor in an ac circuit is zero.
Example: household voltage

In the U.S., standard wiring supplies **120 V** at **60 Hz**. Write this in sinusoidal form, assuming **V(t)=0 at t=0**.

This 120 V is the RMS amplitude: so \( V_p = \frac{V_{rms}}{\sqrt{2}} = 170 \text{ V} \). This 60 Hz is the frequency \( \omega \) so \( \omega = 2\pi f = 377 \text{ s}^{-1} \).

So \( V(t) = 170 \sin(377t - \phi_v) \). Choose \( \phi_v = 0 \) so that \( V(t)=0 \) at \( t=0 \): \( V(t) = 170 \sin (377 t) \).

AC Circuits: Summary

<table>
<thead>
<tr>
<th>Element</th>
<th>( I_0 )</th>
<th>Current vs. Voltage</th>
<th>Resistance</th>
<th>Reactance</th>
<th>Impedance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>( \frac{V}{R} )</td>
<td>In Phase</td>
<td>( R )</td>
<td>( R )</td>
<td>( R )</td>
</tr>
<tr>
<td>Capacitor</td>
<td>( \alpha CV_{ac} )</td>
<td>Leads</td>
<td>( X_C = \frac{1}{\omega C} )</td>
<td>( X_C )</td>
<td>( Z )</td>
</tr>
<tr>
<td>Inductor</td>
<td>( \frac{V_{em}}{\omega L} )</td>
<td>Lags</td>
<td>( X_L = \omega L )</td>
<td>( X_L )</td>
<td>( Z )</td>
</tr>
</tbody>
</table>

Although derived from single element circuits, these relationships hold true generally!

What is reactance?

You can think of it as a frequency-dependent resistance.

\[
X_C = \frac{1}{\omega C} \quad \text{Capacitor looks like a break for low } \omega, X_C \to \infty \]

\[
X_L = \omega L \quad \text{Inductor looks like a short for high } \omega, X_L \to 0
\]

\[
\Rightarrow X_C \text{ is a break for low } \omega, X_L \text{ is a wire for high } \omega.
\]

Example 2: An Inductor in an AC Circuit

The circuit contains a 3.60 mH inductor. The rms voltage is 25.0 V. Find the rms current in the circuit when the generator frequency is (a) 100 Hz, (b) 5000 Hz.

\[
\begin{align*}
\omega &= 2\pi (100) = 209.4 \text{ rad/s} \\
X_L &= 2\pi L = 2\pi (3.60 \text{ mH})(1000) = 2.26 \Omega \\
I_{rms} &= \frac{V_{rms}}{X_L} = \frac{25}{2.26} = 11.1 \text{ A}
\end{align*}
\]

\[
\begin{align*}
\omega &= 2\pi (5000) = 31415.9 \text{ rad/s} \\
X_L &= 2\pi L = 2\pi (3.60 \text{ mH})(5000) = 113 \Omega \\
I_{rms} &= \frac{V_{rms}}{X_L} = \frac{25}{113} = 0.221 \text{ A}
\end{align*}
\]

23.3 Circuits Containing R, C, and L

In a series RLC circuit, the total opposition to the flow is called the **impedance**.

\[
V_{rms} = I_{rms} Z \quad Z = \sqrt{R^2 + (X_L - X_C)^2}
\]

23.3 Circuits Containing R, C, and L

\[
X_L = \omega L, \quad X_C = \frac{1}{\omega C}
\]
23.3 Circuits Containing R, C, and L

Circuits Containing R, C, and L

Conceptual Example 5 The Limiting Behavior of Capacitors and Inductors

The rms voltage of the generator is the same in each case. The values of the resistance, capacitance, and inductance are the same. The frequency of the ac generator is very near zero.

In which circuit does the generator supply more rms current?

Phasors for R, C, L

Suppose:

- \( I = I_0 \sin \omega t \)
- \( V_R = R I_0 \sin \omega t \)
- \( V_C = \frac{1}{\omega C} I_0 \sin(\omega t - \frac{\pi}{2}) \)
- \( V_L = \omega L I_0 \sin(\omega t + \frac{\pi}{2}) \)

Clicker Question Check your understanding

5. An air-core inductor is connected in series with a light bulb, and this circuit is plugged into an ac outlet. When a piece of iron is inserted inside the inductor, does the brightness of the bulb

(a) Increase
(b) Decrease
(c) Remain the same?

23.4 Resonance in Electric Circuits

Resonance occurs when the frequency of a vibrating force exactly matches a natural (resonant) frequency of the object to which the force is applied.

The oscillation of a mass on a spring is analogous to the oscillation of the electric and magnetic fields that occur, respectively, in a capacitor and an inductor.
23.4 Resonance in Electric Circuits

For an RLC circuit

\[ Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2} \]

\[ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \]

Resonant frequency

\[ f_r = \frac{1}{2\pi\sqrt{LC}} \]

\[ 2\pi f_r = \omega_r = \frac{1}{\sqrt{LC}} \]

Example 6 A heterodyne metal detector

Figure below is a heterodyne metal detector. This device utilizes capacitor/inductor oscillator circuits, A and B. Each produces its own resonant frequency. Any difference between these frequencies is detected as a beat frequency. Initially each oscillator has a resonant frequency of 855.5 kHz. Assume the inductance of B decreases by 1%. Find the beat frequency.

\[ f_{oa} = 855.5 \text{ kHz} \]

Now the metal object causes the \( L_B \) to decrease 1.00%

\[ f_{ob} = \frac{1}{2\pi \sqrt{0.99 \cdot L_B \cdot C}} = \frac{855.5}{\sqrt{0.99}} = 859.82 \text{ kHz} \]

Beat frequency:

\[ |f_{ob} - f_{oa}| = |859.8 - 855.5| = 4.3 \text{ kHz} \]

Example 6 A heterodyne metal detector

Initially the resonant frequencies of the two circuits are the same

\[ f_{oa} = 855.5 \text{ kHz} = f_{ob} \]

The first digital computer, ENIAC was built in 1947. It consisted of 17 thousand vacuum tubes.

23.5 Semiconductor Devices

Semiconductor devices such as diodes and transistors are widely used in modern electronics.

In 1947, Bardeen, Brattain, and Shockley invented the first transistor.
The first IBM PC was introduced in 1981.

The IBM PC Spec.
- Micro-process --- 4.77 MHz Intel 8088
- Memory --------- 16 kB
- Storage -------- 160 kB of floppy disk
- Price ---------- $1,565 (1981)

The Intel 4004 micro-processor, running at 108 kHz and 0.6 MIPS.

Inside the Intel 4004 micro-processor. It has 2,250 transistors.

The new Intel Core i7. It has 4 different processors and contains 731 millions of transistors.

All the advance in computer technology comes from the developments of simple semiconductor devices such as diodes and transistors.

23.5 Semiconductor Devices

n-TYPE AND p-TYPE SEMICONDUCTORS

The semiconducting materials (silicon and germanium) used to make diodes and transistors are doped by adding small amounts of an impurity element.
At the junction between the n and p materials, mobile electrons and holes combine and create positive and negative charge layers.

There is an appreciable current through the diode when the diode is forward biased. Under a reverse bias, there is almost no current through the diode.

A diode can be used in a half-wave rectifier.
23.5 Semiconductor Devices

TRANSISTORS

A bipolar junction transistor can be used to amplify a smaller voltage into a larger one.

The voltages are applied in such a way that the p-n junction on the left has a forward bias, while the p-n junction on the right has a reverse bias.

A small change in the emitter voltage input will cause a large voltage change at the collector output. This is the main function of a transistor, i.e., the amplification of a small signal.