Chapter 22

Electromagnetic Induction

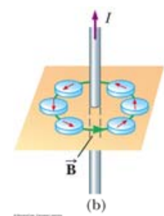
Goals for Chapter 21

- To understand induced emf.
- To study motional emf.
- To understand the concept of magnetic flux.
- To study Faraday's Law.
- To study Lenz's Law.
- To study and calculate mutual and self inductance.
- To study the basic principle of a generator.
- To study the basic principle of a transformer

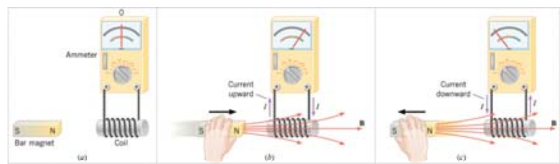
22.1 Induced emf and induced current

From last chapter, we know that a current will generate a magnetic field.

Here we want to show that magnetic field can also generate induced current.



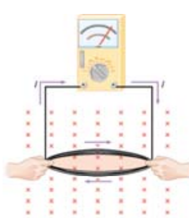
22.1 Induced Emf and Induced Current



The current in the coil is called the ***induced current*** because it is brought about by a changing magnetic field. Since a source emf is always needed to produce a current, the coil behaves as if it were a source of emf. This emf is known as the ***induced emf.***

22.1 Induced emf and induced current

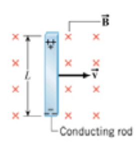
There are more than one way to generate induced emf/current. An emf can be induced by changing the area of a coil in a constant magnetic field as shown in the figure on the right.



In each example, both an emf and a current are induced because the coil is part of a complete circuit. If the circuit were open, there would be no induced current, but there would be an induced emf.

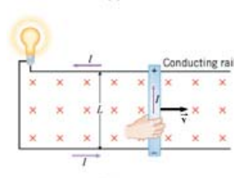
22.2 Motional Emf

THE EMF INDUCED IN A MOVING CONDUCTOR



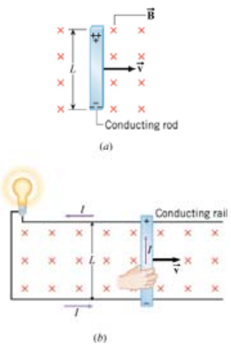
Each positive (negative) charge within the conductor is moving and experiences a magnetic force pointing up (down).

$$F = qvB$$



The separated charges on the ends of the conductor give rise to an induced emf, called a motional emf.

22.2 Motional Emf



Motional emf when v, B, and L are mutually perpendicular

From Lorentz law, F=qvB and definition of E field, F=qE, so

E = vB

From eq. 19.7,

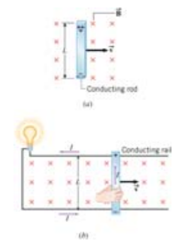
E = ε/L

$$\varepsilon = vBL$$

22.2 Motional Emf

Example 1 **Operating a Light Bulb with Motional Emf**

Suppose the rod is moving with a speed of 5.0m/s perpendicular to a 0.80-T magnetic field. The rod has a length of 1.6 m and a negligible electrical resistance. The rails also have a negligible electrical resistance. The light bulb has a resistance of 96 ohms. Find (a) the emf produced by the rod and (b) the current induced in the circuit. (c) electric power deliver to the bulb, (d) the energy used in 60.0 s.

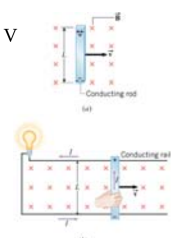


V = 5.0 m/s, B = 0.80 T, L = 1.6 m, R = 96 Ω.

22.2 Motional Emf

(a) $\varepsilon = vBL = (5.0\,\text{m/s})(0.80\,\text{T})(1.6\,\text{m}) = 6.4\,\text{V}$

(b) $I = \dfrac{\varepsilon}{R} = \dfrac{6.4\,\text{V}}{96\Omega} = 0.067\,\text{A}$



(c) The power delivered is

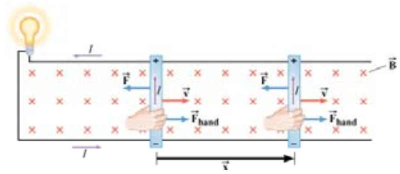
P = Iε=(0.067A)(6.4V) = 0.43 W

(d) Power is energy per unit time

E = Pt = (0.43W)(60.0 s) = 26 J.

22.2 Motional Emf

MOTIONAL EMF AND ELECTRICAL ENERGY

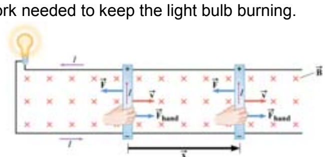


In order to keep the rod moving at constant velocity, the force the hand exerts on the rod must balance the magnetic force on the current:

$$F_{\text{hand}} = F = ILB$$

22.2 Motional Emf

Example 2. The work needed to keep the light bulb burning.



Work done by the hand in 60 seconds

W = F·x = ILBx = (0.067)(1.6)(0.8)(5.0)(60) = 26 J.

Energy consumed by the light bulb

E = Pt = I·ε·t = (0.067A)(6.4V)(60s) = 26 J

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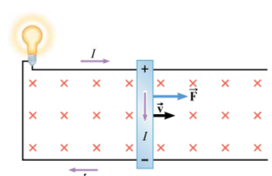
22.2 Motional Emf

The direction of the force in this figure would violate the principle of conservation of energy.

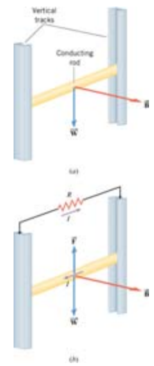


22.2 Motional Emf

Conceptual Example 3 **Conservation of Energy**

A conducting rod is free to slide down between two vertical copper tracks. There is no kinetic friction between the rod and the tracks. Because the only force on the rod is its weight, it falls with an acceleration equal to the acceleration of gravity.

Suppose that a resistance connected between the tops of the tracks. (a) Does the rod now fall with the acceleration of gravity? (b) How does the principle of conservation of energy apply?



22.2 Motional Emf

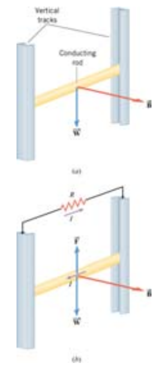
Conceptual Example 3 **Conservation of Energy**

(a) It does not fall with acceleration g anymore.

Net Force = W – F = mg – ILB = ma

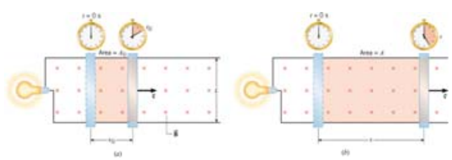
a = g – (ILB/m)

(b) The rod falls and generates a current, which causes a Force F that opposes the weight W. Eventually F = W and the rod moving at a constant speed. The loss of potential energy of the rod converts into heat energy at the resistor, IR^2.



22.3 Magnetic Flux

MOTIONAL EMF AND MAGNETIC FLUX



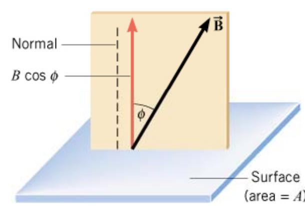
$$\varepsilon = vBL = \left(\frac{x-x_o}{t-t_o}\right)BL = \left(\frac{xL-x_oL}{t-t_o}\right)B = \left(\frac{A-A_o}{t-t_o}\right)B = \frac{(BA)-(BA)_o}{t-t_o}$$

magnetic flux $\Phi = BA$

$$\varepsilon = \frac{\Phi - \Phi_o}{t-t_o} = \frac{\Delta\Phi}{\Delta t}$$

22.3 Magnetic Flux

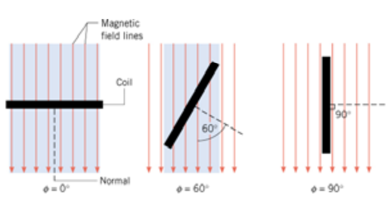
GENERAL EXPRESSION FOR MAGNETIC FLUX



$$\Phi = BA\cos\phi$$

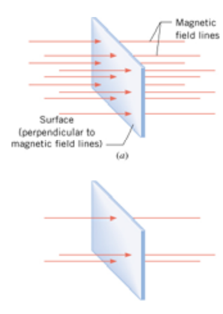
22.3 Magnetic Flux

$$\Phi = BA\cos\phi$$



22.3 Magnetic Flux

GRAPHICAL INTERPRETATION OF MAGNETIC FLUX



The magnetic flux is proportional to the number of field lines that pass through a surface.

Magnetic Flux

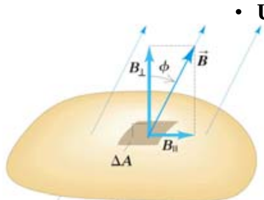
- **Units : 1 tesla × m² = 1 Weber**

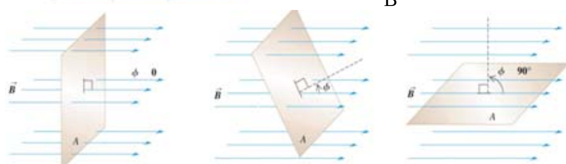
Flux through Surface

$$\Phi_B = B_\perp A = BA\cos\phi$$

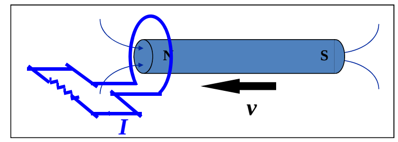
If B and Area normal are at at right angles

$$\Phi_B = BA$$



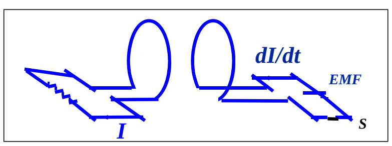


22.4 Faraday's Law of Electromagnetic Induction



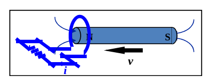
- **Michael Faraday discovered induction in 1831.**
- **Moving the magnet induces a current I.**
- **Reversing the direction reverses the current.**
- **Reversing the magnet reverses the currents.**
- **The induced current is set up by an *induced EMF.***

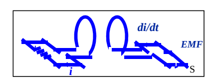
Faraday's Experiments: Changing Current in Coil



- **Changing the current in the right-hand coil induces a current in the left-hand coil.**
- **The induced current does not depend on the size of the current in the right-hand coil;**
- **It depends on *dI/dt* .**

Faraday's Law





- Moving the magnet changes the flux Φ_B .
- Changing the current changes the flux Φ_B .
- ***Faraday:* changing the flux induces an emf:**

$$\varepsilon = -N\frac{\Delta\Phi_B}{\Delta t}$$

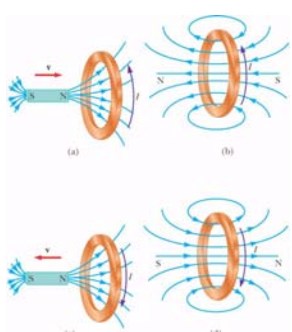
Faraday's law

The emf induced around a loop

equals the rate of change of the flux through that loop

Minus Sign ? Lenz's Law

Induced EMF is in direction that opposes the change in flux which caused it



22.4 Faraday's Law of Electromagnetic Induction

Example 5 **The Emf Induced by a Changing Magnetic Field**

A coil of wire consists of 20 turns each of which has an area of 0.0015 m². A magnetic field is perpendicular to the surface. Initially, the magnitude of the magnetic field is 0.050 T and 0.10s later, it has increased to 0.060 T. Find the average emf induced in the coil during this time.

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{BA\cos\phi - B_o A\cos\phi}{\Delta t}$$

$$= -NA\cos\phi\left(\frac{B - B_o}{\Delta t}\right) = -(20)\left(0.0015\,\text{m}^2\right)\cos(0)\frac{0.060\,\text{T} - 0.050\,\text{T}}{0.10\,\text{s}}$$

$$= -3.0\times10^{-3}\ \text{V}$$

22.4 Faraday's Law of Electromagnetic Induction

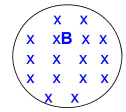
Conceptual Example 7 **An Induction Stove**

Two pots of water are placed on an induction stove at the same time. The stove itself is cool to the touch. The water in the ferromagnetic metal pot is boiling while that in the glass pot is not. How can such a cool stove boil water, and why isn't the water in the glass pot boiling?

Example: A circular UHF TV antena has a diameter of 11.2 cm. The magnetic field of a TV signal is normal to the plane of the loop, and at any instant in time its magnitude is changing at the rate of 157 mT/s. What is the EMF ?

Magnetic flux:

$$\Phi_B = BA\cos\phi \quad = BA$$

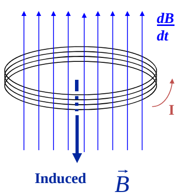


Induced EMF:

$$\mathcal{E} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta(BA)}{\Delta t} = -A\frac{\Delta B}{\Delta t}$$

$$= -[\pi(0.112\,m/2)^2](0.157\,T/s)$$

$$= -1.55\,mV$$



Consider a coil of radius 5 cm with N = 250 turns. A magnetic field through it changes at the rate of $\Delta B / \Delta t = 0.6$ T/s. The total resistance of the coil is 8 Ω. What is the induced current ?

The induced EMF is $\mathcal{E} = -\Delta\Phi_B / \Delta t$

Magnetic Flux (B): $\Phi_B = N(BA) = NB(\pi r^2)$

Therefore $\mathcal{E} = -N(\pi r^2)\,\Delta B/\Delta t$

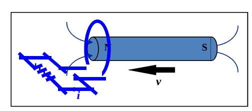
$\mathcal{E} = -(250)(\pi\, 0.05^2)(0.6\,T/s) = -1.18\,V$ (1V=1Tm²/s)

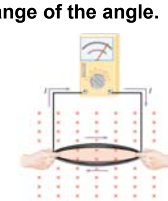
Current $I = \mathcal{E}/R = (-1.18\,V)/(8\,\Omega) = -0.147\,A$

Faraday's Law

$$\varepsilon = -\frac{\Delta\Phi_B}{\Delta t}; \quad \Phi_B = \vec{B}\cdot\vec{A} = BA\cos\theta$$

From the above equation, we can see that change of the flux can be due to (a) change of the B field, (b) change of the area, and (c) change of the angle.





22.5 Lenz's Law

LENZ'S LAW

The induced emf resulting from a changing magnetic flux has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change.

22.5 Lenz's Law

LENZ'S LAW

Reasoning Strategy

1. Determine whether the magnetic flux that penetrates the coil is increasing or decreasing.

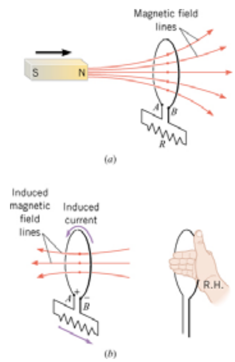
2. Find what the direction of the induced magnetic field must be so that it can oppose the change influx by adding or subtracting from the original field.

3. Use **Right-Hand Rule** to determine the direction of the induced current.

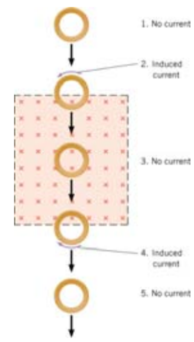
22.5 Lenz's Law

Conceptual Example 8 **The Emf Produced by a Moving Magnet**

A permanent magnet is approaching a loop of wire. The external circuit consists of a resistance. Find the direction of the induced current and the polarity of the induced emf.



22.5 Lenz's Law



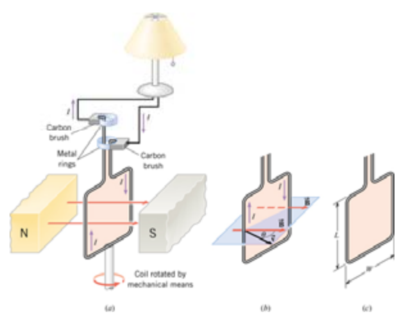
Conceptual Example 9 **The Emf Produced by a Moving Copper Ring.**

There is a constant magnetic field directed into the page in the shaded region. The field is zero outside the shaded region. A copper ring slides through the region.

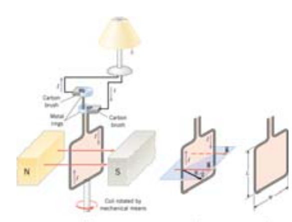
For each of the five positions, determine whether an induced current exists and, if so, find its direction.

22.7 The Electric Generator

HOW A GENERATOR PRODUCES AND EMF



22.7 The Electric Generator



$$\varepsilon = v_{\perp} BL = BLv \sin\theta$$

For each side

$$v = r\omega = \frac{W}{2}\omega$$

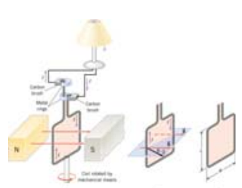
22.7 The Electric Generator

For the complete loop, $\varepsilon = 2BLv\sin\theta = 2BL(r\omega)\sin\theta$

$$r = \frac{w}{2} \text{ and } Lw = A, \Rightarrow \varepsilon = AB\omega\sin\theta$$

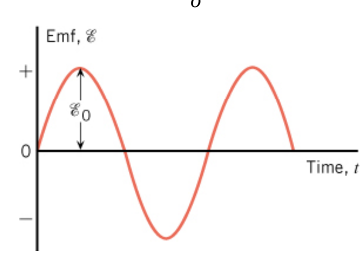
$$\varepsilon = NAB\omega\sin\omega t = \varepsilon_o \sin\omega t$$

$$\omega = 2\pi f$$



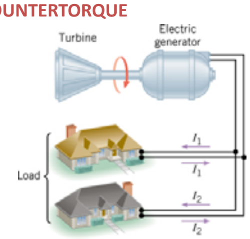
22.7 The Electric Generator

$$\varepsilon = \varepsilon_o \sin\omega t$$



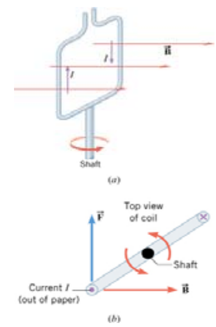
22.7 The Electric Generator

THE ELECTRICAL ENERGY DELIVERED BY A GENERATOR AND THE COUNTERTORQUE



When the generator is delivering current, there is a magnetic force acting on the coils.

22.7 The Electric Generator

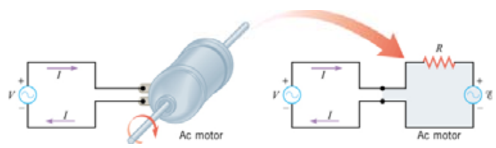


The magnetic force gives rise to a countertorque that opposes the rotational motion.

22.7 The Electric Generator

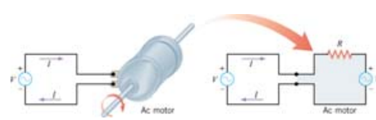
THE BACK EMF GENERATED BY AN ELECTRIC MOTOR

When a motor is operating, two sources of emf are present: (1) the applied emf *V* that provides current to drive the motor, and (2) the emf induced by the generator-like action of the rotating coil.



22.7 The Electric Generator

Consistent with Lenz's law, the induced emf acts to oppose the applied emf and is called ***back emf***.



$$I = \frac{V - \mathrm{E}}{R}$$

Example 11 A bike Generator

A bicyclist is traveling at night, and a generator mounted on the bike powers a headlight. A small rubber wheel on the shaft of the generator presses against the bike tire and turns the coil of the generator at an angular speed that is 44 times as great as the angular speed of the tire itself. The tire has a radius of 0.33 m. The coil consists of 75 turns, has an area of 2.6x10^{-3} m^2, and rotates in a 0.10 T magnetic field. When the peak emf being generated is 6.0 V, what is the linear speed of the bike?

$$v = r(\frac{1}{44})\omega_{coil} = r(\frac{1}{44})(\frac{\varepsilon}{NBA})$$

$$= (0.33)(0.0227)(\frac{6.0}{(75)(0.0026)(0.1)}) = 2.3 m/s$$

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22.7 The Electric Generator

Example 12 **Operating a Motor**

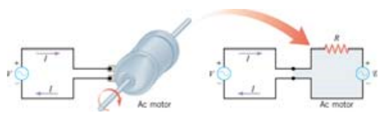
The coil of an ac motor has a resistance of 4.1 ohms. The motor is plugged into an outlet where the voltage is 120.0 volts (rms), and the coil develops a back emf of 118.0 volts (rms) when rotating at normal speed. The motor is turning a wheel. Find (a) the current when the motor first starts up and (b) the current when the motor is operating at normal speed.

(a) $$I = \frac{V - \mathrm{E}}{R} = \frac{120\text{ V} - 0\text{ V}}{4.1\Omega} = 29\text{ A}$$

(b) $$I = \frac{V - \mathrm{E}}{R} = \frac{120\text{ V} - 118.0\text{ V}}{4.1\Omega} = 0.49\text{ A}$$

The back emf generated by an electrc motor

Consistent with Lenz's law, the induced emf acts to oppose the applied emf and is called ***back emf.***



$$I = \frac{V - \varepsilon}{R}$$

Example 12 Operating a motor

The coil of an ac motor has a resistance of R = 4.1 ohm. The motor is plugged into an outlet where V=120 Volt(rms), and the coil develops a back emf of ε = 118.0 volts (rms) when rotating at normal speed. Find the current when the motor is first start up, (b) the current when the motor is operating at normal speed.

(a)

$$I = \frac{V - \varepsilon}{R} = \frac{120 - 0}{4.1} = 29A.$$

(b)

$$I = \frac{V - \varepsilon}{R} = \frac{120 - 118}{4.1} = 0.49A.$$

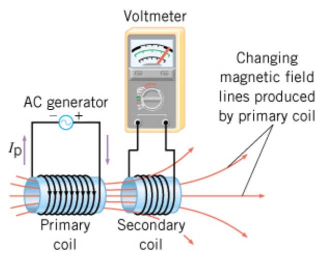
22.8 Mutual Inductance and Self Inductance

MUTUAL INDUCTANCE

The changing current in the primary coil creates a changing magnetic flux through the secondary coil, which leads to an induced emf in the secondary coil.

The effect in called *mutual induction.*

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$$



22.8 Mutual Inductance and Self Inductance

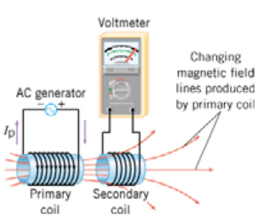
MUTUAL INDUCTANCE

We define the mutual inductance M as $N_S\Phi_S = MI_P$

$$\varepsilon_S = -N_S\frac{\Delta\Phi_S}{\Delta t} = -\frac{\Delta(N_S\Phi_S)}{\Delta t}$$
$$= -\frac{\Delta(MI_P)}{\Delta t} = -M\frac{\Delta I_P}{\Delta t}$$

$$\varepsilon_S = -M\frac{\Delta I_P}{\Delta t}$$

22.8 Mutual Inductance and Self Inductance



Emf due to mutual induction

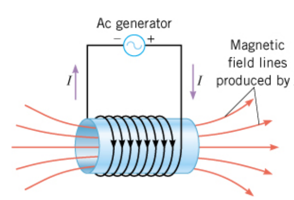
$$\varepsilon_s = -M\frac{\Delta I_p}{\Delta t}$$

mutual inductance

SI Unit of mutual inductance: $1\,\text{V}\cdot\text{s/A} = 1\,\text{H}$ (Henry)

22.8 Mutual Inductance and Self Inductance

SELF INDUCTANCE



The effect in which a changing current in a circuit induces and emf in the same circuit is referred to as *self induction.*

22.8 Mutual Inductance and Self Inductance

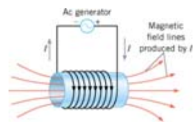
SELF INDUCTANCE

$$N\Phi = LI \quad or \quad L = \frac{N\Phi}{I}$$

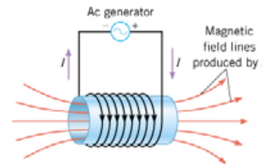
$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -\frac{\Delta(N\Phi)}{\Delta t}$$

$$= -\frac{\Delta(LI)}{\Delta t} = -L\frac{\Delta I}{\Delta t}$$

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$



22.8 Mutual Inductance and Self Inductance



$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

self inductance

SI Unit of self inductance: $1\,V\cdot s/A = 1\,H$ (Henry)

22.8 Mutual Inductance and Self Inductance

THE ENERGY STORED IN AN INDUCTOR

Energy stored in an inductor $\quad \text{Energy} = \tfrac{1}{2}LI^2$

Energy density $\quad \text{Energy density} = \frac{1}{2\mu_o}B^2$

Example 13 The emf induce in a long solenoid

A long solenoid of length 0.08 m and cross-sectional area 5.0x10^{-5} m^2 contains 6500 turns per meter. Determine the emf induced in the solenoid when the current in the solenoid changes from 0 to 1.5A in 0.20 s.

$$We \quad will \quad use \quad \varepsilon = -L\frac{\Delta I}{\Delta t}$$

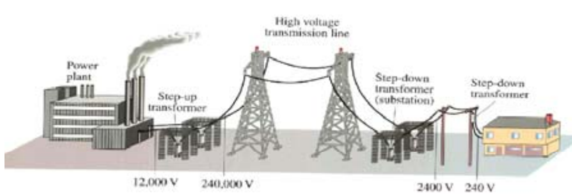
$$L = \frac{N\Phi}{I} = \frac{n\ell\Phi}{I}$$

$$\Phi = BA = (\mu_o nI)A$$

$$L = (4\pi\times10^{-7})(6500)^2(0.08)(5\times10^{-5}) = 2.1\times10^{-4}H$$

$$\varepsilon = -(2.1\times10^{-4})(\frac{1.5}{0.2}) = -1.6\times10^{-3}V$$

Transmission of Electric Power



Power loss can be greatly reduced if transmitted at high voltage

Example: Transmission lines

An average of 120 kW of electric power is sent from a power plant. The transmission lines have a total resistance of 0.40 Ω. Calculate the power loss if the power is sent at (a) 240 V, and (b) 24,000 V

(a) $I = \frac{P}{V} = \frac{1.2\times10^5\,W}{2.4\times10^2\,V} = 500\,A$ **83 % loss !!**

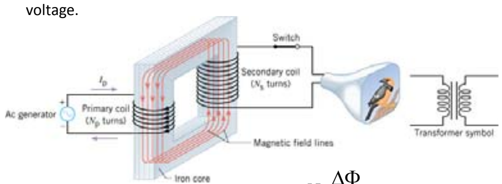
$$P_L = I^2R = (500\,A)^2(0.40\Omega) = 100\,kW$$

(b) $I = \frac{P}{V} = \frac{1.2\times10^5\,W}{2.4\times10^4\,V} = 5.0\,A$ **0.0083 % loss !!**

$$P_L = I^2R = (5.0\,A)^2(0.40\Omega) = 10\,W$$

22.9 Transformers

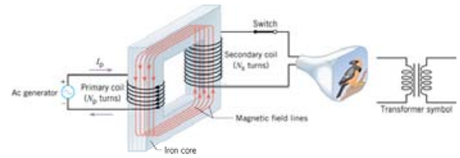
A ***transformer*** is a device for increasing or decreasing an ac voltage.



$$\varepsilon_p = -N_p \frac{\Delta\Phi}{\Delta t} \qquad \varepsilon_s = -N_s \frac{\Delta\Phi}{\Delta t}$$

$$\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p} \qquad \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

22.9 Transformers



$$\frac{I_s}{I_p} = \frac{V_p}{V_s} = \frac{N_p}{N_s}$$

A transformer that steps up the voltage simultaneously steps down the current, and a transformer that steps down the voltage steps up the current.