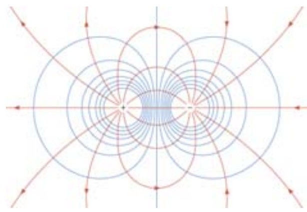
Chapter 19

Electric Potential Energy and the Electric Potential



Goals for Chapter 19

- To understand electrical potential energy.
- To define electricalpotential.
- To study equipotential surfaces.
- To study capacitors and dielectrics.
- To study energy conservation involving electrical potential energy

Gravity / Electricity

	Gravity	Electricity
	Mass *M*	Charge *Q* (±)
Field:	$\vec{\mathbf{g}} = -G\dfrac{M\,\hat{\mathbf{r}}}{r^2}$	$\vec{\mathbf{E}} = k_e\dfrac{Q\,\hat{\mathbf{r}}}{r^2}$
Force:	$\vec{\mathbf{F}}_g = \mathbf{m}\,\vec{\mathbf{g}}$	$\vec{\mathbf{F}}_e = \mathbf{q}\vec{\mathbf{E}}$

This is the easiest way to picture field

19.1 Potential energy

The concepts of electric potential energy and electric potential are exactly the same as the concepts of gravitational potential energy and gravitational potential.

Let's review what we have learned in PHY2053---College Physics I.

Work and Gravitational Potential Energy

- Consider block of mass m at initial height y_i
- Work done by the gravitational force

$$W_{grav} = (F\cos\theta)s = (mg\cos\theta)s$$

but : $s = y_i - y_f$, $\cos\theta = 1$,

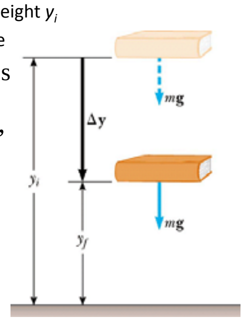
Thus : $W_{grav} = mg(y_i - y_f)$

$$= \underbrace{mgy_i}_{} - \underbrace{mgy_f}_{}$$

$PE = mgy$

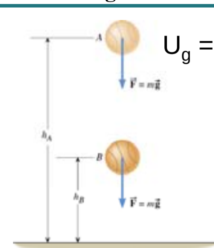
This quantity is called potential energy:

$$W_{gravity} = PE_i - PE_f = -\Delta PE$$

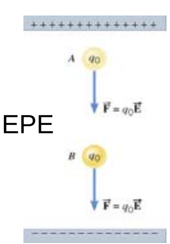


Important: work is related to the difference in PE's!

Electric and gravitational forces are conservative



U_g = GPE



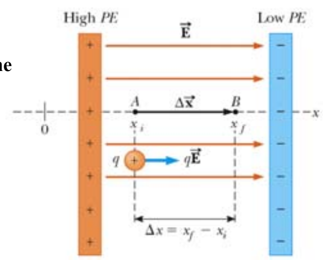
U_e = EPE

$$W_{AB} = mgh_A - mgh_B = GPE_A - GPE_B = -\Delta U_g$$

$$W_{AB} = \vec{F}\cdot\vec{\Delta y} = q_o E(y_A - y_B) = EPE_A - EPE_B = -\Delta U_e$$

Work and Potential Energy

- Assume a uniform field between the two plates
- As the charge moves from A to B, work is done on it
- W = Fd = q $E_x (x_f - x_i)$
- ΔPE = - W
  = - q E_x Δx
  – Only for a uniform field



Potential Energy

Work done by gravity moving m from A to B:

$\Delta U_G = U_B - U_A \quad = - W_G$ (For conservative forces only)

Near the surface of earth (assume U = 0 at y = 0)

(1) $\vec{F}_g = - m g \hat{y} \qquad U_g = m g y + U_0$

In general, at an arbitrary point (assume U = 0, at r = ∞)

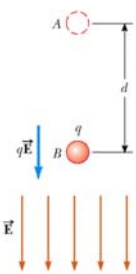
(2) $\vec{F}_g = -\frac{G M m}{r^2}\hat{r} \qquad U_g = -\frac{G M m}{r} + U_0$

U_0 : constant depending on reference point

Only potential energy difference ΔU has physical significance

Energy and Charge Movement

- A positive charge gains electrical potential energy when it is moved in a direction opposite the electric field
- A negative charge loses electrical potential energy when it moves in the direction opposite the electric field
- When the electric field is directed downward, point B is at a lower potential than point A
- A positive test charge that moves from A to B loses electric potential energy
- It will gain the same amount of kinetic energy as it loses in potential energy



19.2 The electric potential difference

Definition of electric potential

The electric potential V at a given point is the electric potential energy U_e of a small test charge q_o at that point divided by the charge itself.

$$V = \frac{U_e}{q_0}$$

SI Unit of electric potential : joule/coulomb = volt (V).

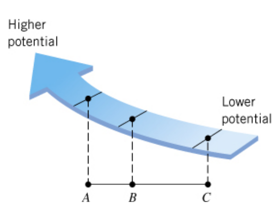
Potential Difference

$$\Delta V = V_B - V_A = \frac{U_B}{q_o} - \frac{U_A}{q_o} = \frac{-W_{AB}}{q_o}$$

Example 2: How positive and negative charges accelerate

Three points A, B, and C are located along a horizontal line as shown. A positive test charge is released at A and accelerates toward C. What will a negative charge do when it is released from point B?

(a) Accelerates toward C,
(b) Remains stationary,
(c) Accelerates toward A.



Direction of electric field is from high potential to low potential.

Example 3: Operating a Headlight

The wattage of the headlight in this drawing is 60 W. Determine the number of charge each carrying 1.6×10^{-19} C, that pass between the terminals of the 12V car battery when the headlight burns for one hour.

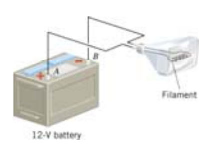
Power = energy/time
Total charge = (number of particles) x (charge of each particle)

So total energy used is 60Wx3600s = 2.16×10^5 J

$\Delta U = U_A - U_B = q(V_A - V_B) = 2.16\times10^5$ J

q = (2.16×10^5 Joule)/(12 Volt) = 1.8×10^4 C

(1.8×10^4 C) / (1.60×10^{-19} C) = 1.12×10^{23}



Now we can include electric potential energy Ue as part of the total energy that an object can have:

$$E = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 + mgh + \tfrac{1}{2}kx^2 + U_e$$

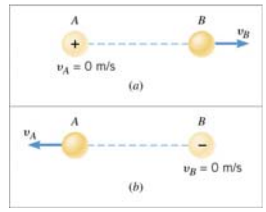
Electron volt (eV) ------- another unit of energy.

The amount of energy that an electron changes when it moves through a potential difference of one volt is called electron volt (eV)

$$1\,\text{eV} = (1.60\times10^{-19}\,C)\times(1V) = 1.60\times10^{-19}\,\text{J}$$

Example 4 The Conservation of Energy

A particle has a mass of $1.8\text{x}10^{-5}$kg and a charge of $+3.0\text{x}10^{-5}$C. It is released from point A and accelerates horizontally until it reaches point B. The only force acting on the particle is the electric force, and the electric potential at A is 25V greater than at B. (a) What is the speed of the particle at point B? (b) If the same particle had a negative charge and were released from point B, what would be its speed at A?



Assume a point particle → No rotational kinetic energy

Ignore gravitational potential energy and elastic potential energy,

We have only kinetic energy and electric potential energy.

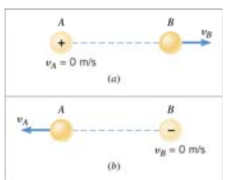
$$\tfrac{1}{2}mv_B^2 + U_B = \tfrac{1}{2}mv_A^2 + U_A$$

$$\Downarrow$$

$$\tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_A^2 + U_A - U_B$$

$$\Downarrow$$

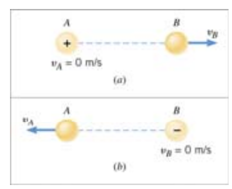
$$\tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_A^2 + q_o(V_A - V_B)$$



(a) $\tfrac{1}{2}mv_B^2 = q_o(V_A - V_B)$

$$v_B = \sqrt{2q_o(V_A - V_B)/m}$$

$$= \sqrt{2(3.0\times10^{-5}\,\text{C})(25\,\text{V})/(1.8\times10^{-5}\,\text{kg})} = 9.1\,\text{m/s}$$



(b) $v_A = \sqrt{-2q_o(V_A - V_B)/m}$

$$= \sqrt{-2(-3.0\times10^{-5}\,\text{C})(25\,\text{V})/(1.8\times10^{-5}\,\text{kg})} = 9.1\,\text{m/s}$$

Check your understanding

(1) An ion, starting from rest, accelerates from point *A* to point *B* due to a potential difference between the two points. Does the electric potential energy of the ion at point *B* depend on

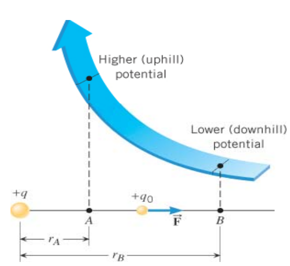
(a) the magnitude of its charge and
(b) its mass? Does the speed of the ion at *B* depend on
(c) the magnitude of its charge and
(d) its mass?

(2) A proton and an electron are released from rest at the midpoint between the plates of a charged parallel plate capacitor. Except for these particles, nothing else is between the plates. Ignore the attraction between the proton and the electron, and decide which particle strikes a capacitor plate first.

(a) Electron will strike the plate first.
(b) The proton will strike the plate first.
(c) Both particles will strike at the same time.

19.3 Electric Potential of Point Charges

A positive point charge +q at the origin will create an electric potential as the figure below indicated. The work done on a test charge q' when it moves in this electric potential caused by the fixed point charge +q is given by equation below.



$$W_{AB} = \frac{kqq_o}{r_A} - \frac{kqq_o}{r_B}$$

$$V_B - V_A = \frac{-W_{AB}}{q_o} = \frac{kq}{r_B} - \frac{kq}{r_A}$$

Potential Energy of Point Charges

From previous page we can see that the electric potential of a point charge can be defined as,

$$V = k\frac{q}{r} \qquad (19.6)$$

V = 0 at r = ∞.

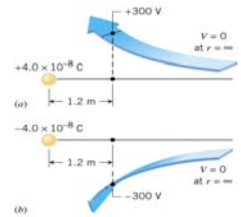
You may recall that electric field due to a point charge is given by,

$$\vec{E} = k\frac{q}{r^2}\hat{r} \qquad (18.3)$$

Compare the above two formula, we can tell that they are related.

Example 5 The potential of a point charge

Using a zero reference potential at infinity, determine the potential due to a point charge of 4.0x10-8 C at a spot 1.2 m away from the charge when the charge is (a) positive and (b) negative.



(a)

$$V = \frac{kq}{r} = \frac{(8.89\times10^{9})\cdot(+4.0\times10^{-8})}{1.2}$$

$$= +300\,Volts$$

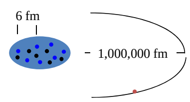
(b)

V = -300 Volts

EXAMPLE: What is the electric potential energy between two protons in the Uranium nucleus ?

The 92 protons in the nucleus of ^{238}U are about 6 fm apart.

$q_1 = q_2 = 1.6 \times 10^{-19}$ C



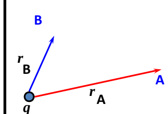
$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$$

$$= \frac{(8.99\times10^{9}\,Nm^2/C^2)(1.6\times10^{-19}C)^2}{6.0\times10^{-15}m}$$

$$= 3.8\times10^{-14}J = 2.4\times10^{5}eV = 240keV$$

This is a huge energy. The atomic binding energy of an electron is only about 1 eV. Why doesn't the Coulomb force split the nucleus apart? Because the two protons are held together in the nucleus by an attractive "strong nuclear force."

Potential Difference



- The **potential difference** between points A and B is defined as the change in the potential energy (final value minus initial value) of a charge q moved from A to B divided by the size of the charge

$$\Delta V = V_B - V_A = \Delta U / q$$

- Potential difference is *not* the same as potential energy
- Another way to relate the energy and the potential difference: $\Delta U = q\,\Delta V$
- **Both electric potential energy and potential difference are *scalar* quantities**

Potential Energy Compared to Potential

- **Electric potential is characteristic of the field only**
  - **Independent of any test charge that may be placed in the field**
- **Electric potential energy is characteristic of the charge-field system**
  - **Due to an interaction between the field and the charge placed in the field**
- **Both, potential and potential energy are defined within a constant (can set constant to zero)**

  e.g. Potential at distance infiniy is zero (rather than some constant value)

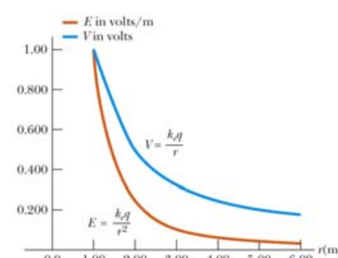
Electric Potential of a Point Charge

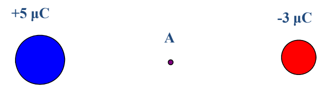
The point of zero electric potential is taken to be at an infinite distance from the charge.
A potential exists at some point in space whether or not there is a test charge at that point.

The potential created by a point charge q at any distance r from the charge is

$$V = k\frac{q}{r}$$

- The electric field is proportional to $1/r^2$
- The electric potential is proportional to 1/r





Two charges $q_1 = +5\ \mu C$, $q_2 = -3\mu C$ are placed at equal distancesfrom the point A.
What is the electric potential at point A?

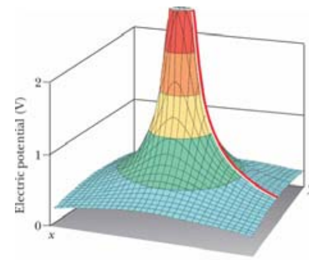
a) $V_A < 0$
b) $V_A = 0$
c) $V_A > 0$

Potential from a point charge

$$V(r) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

where

$$V(\infty) \equiv 0$$

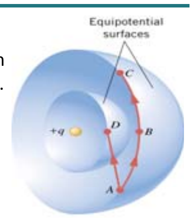


- The electric field of the point charge has spherical symmetry.
- The potential depends only on the distance from the point charge, as is expected from spherical symmetry.

19.4 Equipotential surfaces and their relation to the electric field

An equipotential surface is a surface on which the electric potential is the same everywhere.

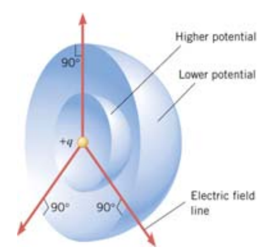
$$V = \frac{kq}{r}$$



The net electric force does no work on a charge as it moves on an equipotential surface.

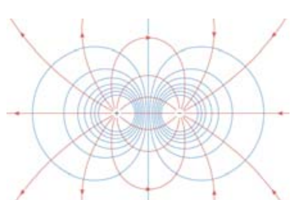
19.4 Equipotential Surfaces and Their Relation to the Electric Field

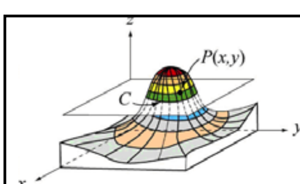
The electric field created by any charge or group of charges is perpendicular to the associated equipotential surfaces and points in the direction of decreasing potential.



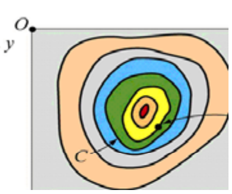
19.4 Equipotential Surfaces and Their Relation to the Electric Field

This is a cross-sectional view of the equipotential surfaces (blue) and the electric field (red) of an electric dipole.

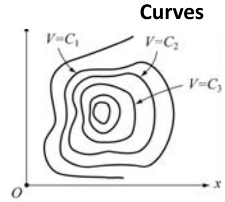




Topographic Maps



Equipotential Curves



All points on an equipotential curve are at the same potential. Each curve represented by V(x,y) = const

Potential difference in a uniform electric field

The relation between electric field and electric potential can be understood through the following example: a parallel plate capacitor.

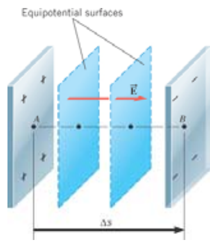
$$\Delta V = V_B - V_A = -\frac{W_{AB}}{q_o}$$

But work done is FΔs, and F is q_oE, so we have

$$\Delta V = -\frac{q_o E \Delta s}{q_o} = -E \cdot \Delta s$$

or

$$E = -\frac{\Delta V}{\Delta s}$$



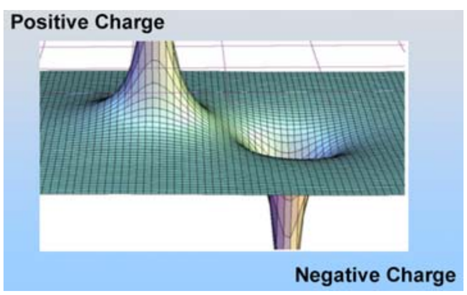
Electric potential due to many point charges

- Just add up the contribution from each charge.
- This Principle of Superposition applies to both the electric field and the electric potential. But it's much easier to apply for a scalar quantity such as the potential.
- This gives a very easy way to calculate the electric field of complicated charge distributions: first find the total potential V(r), and then calculate from V.

one point charge	collection of point charges
$V(r) = k\frac{Q}{r}$	$V(r) = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \ldots\right)$

Superposition theorem

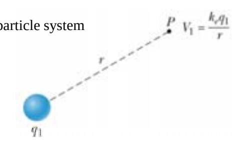
Charges CREATE Potential Landscapes



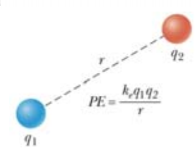
Electric Potential Energy of Two Charges

V_1 is the electric potential due to q_1 at some point P
The work required to bring q_2 from infinity to P without acceleration is q_2V_1
This work is equal to the potential energy of the two particle system

$$U = q_2 V_1 = k_e \frac{q_1 q_2}{r}$$



- If the charges have the *same* sign, U is positive
  - Positive work must be done to force the two charges near one another
  - The like charges would repel
- If the charges have *opposite* signs, U is negative
  - The force would be attractive
  - Work must be done to hold back the unlike charges from accelerating as they are brought close together

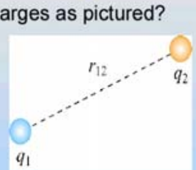


Potential energy of charge q_2 in potential V_1 generated by charge q_1

Configuration Energy

How much energy to put two charges as pictured?

1) First charge is free
2) Second charge sees first:



$$U_{12} = W_2 = q_2 V_1 = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{12}}$$

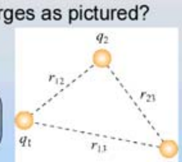
Potential energy of charge q_3 in potential V_1+ V_2 generated by charges q_1 and q_2

Configuration Energy

How much energy to put three charges as pictured?

1) Know how to do first two
2) Bring in third:

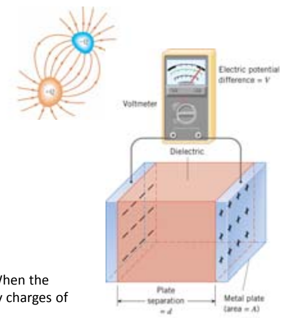
$$W_3 = q_3\left(V_1 + V_2\right) = \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$



Total configuration energy:

$$U = W_2 + W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right) = U_{12} + U_{13} + U_{23}$$

19.5 Capacitors and Dielectrics

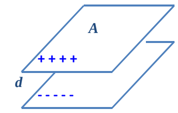


A capacitor consists of two conductors. When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign

Capacitance

- **A capacitor is a device whose purpose is to store electrical energy which can then be released in a controlled manner during a short period of time.**
- **A capacitor consists of 2 spatially separated conductors which can be charged to *+Q* and *-Q* respectively.**
- **The capacitance C is defined as the ratio of the charge on one conductor of the capacitor to the potential difference between the conductors.**

$$Q_+ = \sigma_+ A$$



$$Q_- = \sigma_- A$$

$$Q = |Q_+| = |Q_-|$$

$$C \equiv \frac{Q}{\Delta V}$$

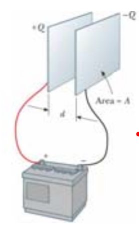
Charge and potential difference of a capacitor

The magnitude q of the charge on each plate of a capacitor is directly proportional to the magnitude V of the potential difference between the two plates.

$$q = CV$$

Capacitance: Parallel plate capacitor

- ***The capacitance of a capacitor is fixed , independent of the charge and voltage.***



$$C \equiv \frac{Q}{\Delta V}$$

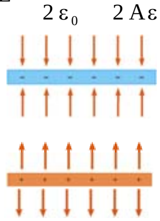
- **Units: Farad (F)**
  - **1 F = 1 C / V**
  - A Farad is very large
    - Often will see µF or pF
- **The capacitance of a capacitor is a measure of how much charge the capacitor can store per unit voltage.**

- **The capacitance of a device depends on the geometric arrangement of the conductors**
- **For a parallel-plate capacitor whose plates are separated by air:**

$$C = \varepsilon_o \frac{A}{d}$$

Field of one charged sheet

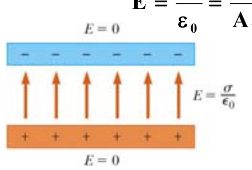
$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$



The capacitor consists of plates of positive and negative charge

The total electric field between the plates is given by

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$



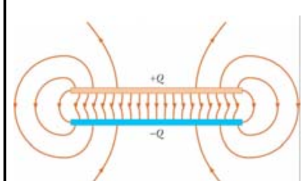
The field outside the plates is zero

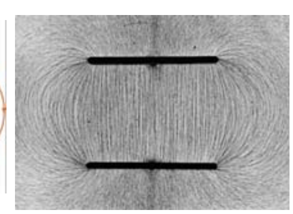
$$\Delta V = V_b - V_a = -\sum E\,\Delta r = \frac{Q}{A\varepsilon_0} d$$

$$\Rightarrow C \equiv \frac{Q}{\Delta V} = \frac{A\varepsilon_0}{d}$$

capacitance depends only on capacitor's geometry (*A*, *d*).

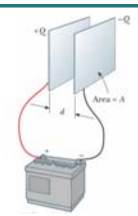
Parallel Plate Capacitor: Field Lines



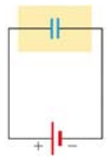


Capacitors in Circuits

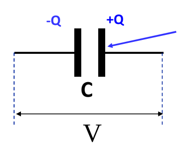
- A *circuit* is a collection of objects usually containing a source of electrical energy (such as a battery) connected to elements that convert electrical energy to other forms

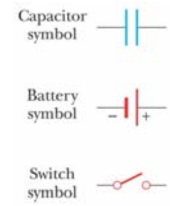


- A *circuit diagram* can be used to show the path of the real circuit



Capacitors in Circuits





A piece of metal in equilibrium has a constant value of potential.

Thus, the potential of a plate and attached wire is the same.

The potential difference between the ends of the wires is V, the same as the potential difference between the plates.

What Does a Capacitor Do?

- Stores electrical charge.
- Stores electrical energy.

Capacitors are basic elements of electrical circuits both macroscopic (as discrete elements) and microscopic (as parts of integrated circuits).

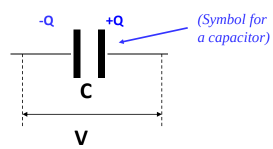
Capacitors are used when a sudden release of energy is needed (such as in a photographic flash).

Electrodes with capacitor-like configurations are used to control charged particle beams (ions, electrons).

What Does a Capacitor Do?

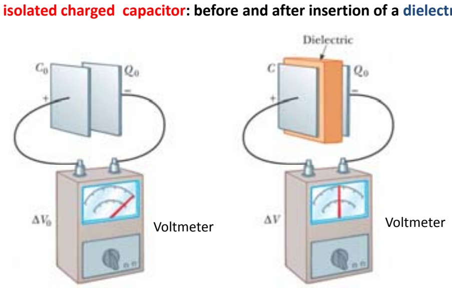
- Stores electrical charge.
- Stores electrical energy.

The charge is easy to see. If a certain potential, V, is applied to a capacitor C, it must store a charge **Q=CV**:



Dielectrics

An isolated charged capacitor: before and after insertion of a dielectric



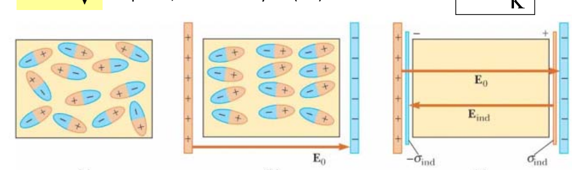
The charge on the plates remains unchanged, but the potential difference decreases

Atomic Description of Dielectrics

$$C \equiv \frac{Q}{V}$$

In the presence of a dielectric, the electric field, and thus the voltage between the plates, is reduced by κ (> 1)

$$\vec{E} = \frac{\vec{E}_o}{\kappa}$$



(a) Polar molecules are randomly oriented in the absence of an external electric field

(b) When an external electric field is applied, the molecules partially align with the field

(c) The charged edges of the dielectric can be modeled as an additional pair of parallel plates establishing an electric field $\mathbf{E}_{ind}$ in the direction opposite to that of $\mathbf{E}_o$

Effect on Capacitance

- A dielectric reduces the electric field by a factor κ
- $E = \frac{E_o}{\kappa} = \frac{v}{d}$
- Hence V = E d is reduced by κ [$V = (E_0/\kappa)\, d = V_0/\kappa$]
- and C= Q/V is increased by κ [$C = (Q\,\kappa) / V_0 = C_0\,\kappa$]

$\therefore\ C = \frac{\varepsilon_o \kappa A}{d}$ parallel plate capacitor with dielectric

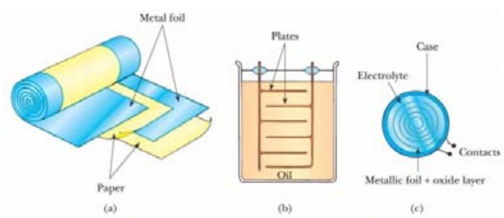
Adding a dielectric *increases* the capacitance.

$\kappa = \frac{E_o}{E}$ Dielectric constant

Approximate Dielectric Constants and Dielectric Strengths of Various Materials at Room Temperature

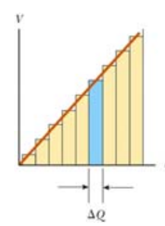
Material	Dielectric Constant κ	Dielectric Strength* (10^6 V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	3.78	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	12
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	—
Water	80	—

* The dielectric strength equals the maximum electric field that can exist in a dielectric without electrical breakdown. Note that these values depend strongly on the presence of impurities and flaws in the materials.



Energy sorage in a Capacitor

- How much energy is stored in a charged capacitor?
  - Calculate the work provided (usually by a battery) to charge a capacitor to +/- Q:



Calculate incremental work ΔW needed to add charge ΔQ to capacitor at voltage V (there is a trick here!):

$$\Delta W = V_{av}(q)\cdot \Delta Q$$

- The total work W to charge to Q is then given by:

$$U = W_{total} = \frac{V}{2}Q$$

- In terms of the voltage V: $W = \frac{1}{2}CV^2$

Energy stored in a Capacitor

- The total work to charge to Q equals the energy U stored in the capacitor:

$$U = \frac{V}{2}Q = \frac{1}{2}\frac{Q^2}{C}$$

Look at this!
Two ways to write U

- In terms of the voltage V:

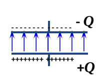
$$U = \frac{1}{2}CV^2$$

- You can do one of two things to a capacitor :
- hook it up to a battery, specify V and Q follows
- put some charge on it, specify Q and V follows

$$Q = CV$$
$$V = \frac{Q}{C}$$

Where is the Energy Stored ?

- Claim: energy is stored in the electric field itself. Think of the energy needed to charge the capacitor as being the energy needed to create the field.
- To calculate the energy density in the field, first consider the constant field generated by a parallel plate capacitor, where



$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{Q^2}{(A\varepsilon_0/d)}$$

- The electric field is given by:

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A} \quad \Rightarrow \quad U = \frac{1}{2}E^2\varepsilon_0 Ad$$

Energy Density of electric field

$$U = \frac{1}{2} E^2 \varepsilon_0 Ad$$

- The energy density u in the field is given by:

$$u = \frac{U}{volume} = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

Claim: the expression for the energy density of the electrostatic field

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Energy stored in a capacitor

If the capacitor contains dielectric material, the energy density inside is given by

$$u = \frac{1}{2}\kappa\varepsilon_o E^2$$

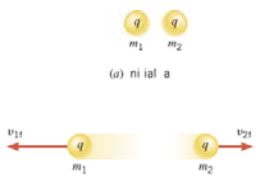
We can also define the permittivity of a material as

$$\varepsilon = \kappa\varepsilon_o$$

ε_o Permittivity of free space
ε Permittivity of a material

Example 13 Conservation of Energy and Momentum

Particle 1 has a mass of $m_1 = 3.6\text{x}10^{-6}$ kg, while particle 2 has a mass of $m_2 = 6.2\text{ x}10^{-6}$ kg. Each has the same electric charge. The particles are held at rest initially and has an electric energy of 0.150 J. Suddenly, the particles are released and fly apart because of the repulsive electric force. At one instance, the speed of particle is 170 m/s. What is the electric potential energy of the two-particle system?



(a) ni ial a

(b) Final

First use conservation of momentum to find the velocity of particle 2.

$$\begin{aligned} m_1 v_{01} + m_2 v_{02} &= m_1 v_{f1} + m_2 v_{f2} \\ 0 &= m_1 v_{f1} + m_2 v_{f2} \\ v_{f2} &= -(3.6/6.2)(-170) \\ v_{f2} &= 98.7\ m/s \end{aligned}$$

Next, use conservation of energy to find the electric potential energy

$$\begin{aligned} KE_f + U_f &= KE_o + U_o \\ U_f &= U_o - KE_f \qquad (KE_o = 0) \\ U_f &= 0.15J - (\frac{1}{2}m_1 \cdot 170^2 + \frac{1}{2}m_2 \cdot 98.7^2) \\ U_f &= 0.068J \end{aligned}$$