College Physics II section 2

Dr. Lee Chow
Department of Physics, UCF Office: PSB410

e-mail: chow@mail.ucf.edu

http:/www.physics.ucf.edu/~lc/Phy2054.html

Office hours: TuTh 4:30 – 5:20 pm
or by appointment

The Challenges of Physics 2054

1. Nearly zero direct experience with E&M in everyday life
   -for scientists and engineers, though, E&M is pervasive
2. The course is full of NEW concepts: a few per lecture!!
   -each concept has many different consequences
   -these concepts are interrelated
   -exercises / problems clarify these conceptions
3. The course "builds upon itself" sequentially
4. Math is the language of physics and here you need to learn to work with it

   -vectors: addition, decomposition

Electromagnetic force

Electromagnetic force is one of the fundamental forces in nature, and the dominant force in a vast range of natural and technological phenomena

- The electromagnetic force is solely responsible for the structure of matter, organic, or inorganic
  ➔ Physics, chemistry, biology, materials science
- The operation of most technological devices is based on electromagnetic forces. From lights, motors, and batteries, to communication and broadcasting systems, as well as microelectronic devices.
  ➔ Engineering

Outline of PHY 2054

How to Succeed in PHY 2054…..

1. Do the assignments and exercises *on time*
2. Attend class
3. Ask questions *early* if you don't understand things
   -in lecture ask me!! …also see me after class.
   -office hours
   -appointments outside of scheduled office hours
   -recitation sections
4. Use problem solving to develop conceptual understanding as well as computational skill
   - this is how you really learn science…

Chapter 18

Electric Forces and Electric Fields

Goals for Chapter 18

- To understand electric charge, conductors, and insulators.
- To understand Coulomb's law and solve some example problems.
- To understand electric fields.
- To calculate electrical forces.
- To be able to map out electric field lines

System of Units

We will use the SI system – SI ≡ International System of Units

Fundamental Quantities

Length ➔ meter [m]
Mass ➔ kilogram [kg]
Time ➔ second [s]

Other Units

Current ➔ ampere [A]

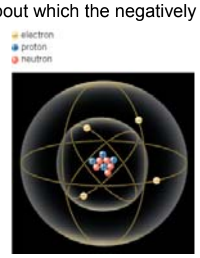
Derived Quantities

Force ➔ newton	**1 N = 1 kg m / s^2**
Energy ➔ joule	**1 J = 1 N m**
Charge ➔ coulomb	**1 C = 1 A s**
Electric Potential ➔ volt	**1 V = 1 J / C**
Resistance ➔ ohm	**1 Ω = 1 V / A**

18.1 The origin of electricity

The electrical nature of matter is inherent in atomic structure.

An atom contains a small positively charged nucleus, about which the negatively charged electrons move.



$$m_p = 1.673\times10^{-27}\,\text{kg}$$

$$m_n = 1.675\times10^{-27}\,\text{kg}$$

$$m_e = 9.11\times10^{-31}\,\text{kg}$$

$$e = 1.60\times10^{-19}\,\text{C}$$

Electric Charge

The nucleus contains protons and neutrons, while electrons form a diffuse cloud surrounding the nucleus.

	Mass	Charge
Proton	1.673×10^{-27} Kg	1.60×10^{-19} C
Neutron	1.675×10^{-27} Kg	zero
Electron	9.11×10^{-31} Kg	1.60×10^{-19} C

e = 1.60×10^{-19} C

Electric Charge

Atoms are usually electrically neutral, meaning that they contain equal number of protons and electrons.

When electrons are removed or added to atom, atom becomes ion. Ion can be positive ion, such as Li^+ ion, or negative ion, such as F^- ion.

Fundamental unit of charge is the charge of one electron (or one proton). Any charge of magnitude q is an integer multiple of e, that is q = Ne where N is an integer. This fact is called quantization of charge.

Electric Charge

History

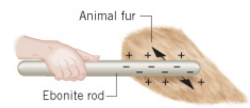
600 BC	**Greeks first discover attractive properties of amber when rubbed.**
1600 AD	**Electric bodies repel as well as attract**
1735 AD	**du Fay: Two distinct types of electricity**
1750 AD	**Franklin: Positive and Negative Charge**
1770 AD	**Coulomb: "Inverse Square Law"**
1890 AD	**J.J. Thompson: Quantization of electric charge - "Electron"**

Electric Charge

Summary of things we know:

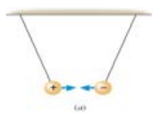
- There is a property of matter called electric charge. (In the SI system its units are Coulombs.)
- Charges can be negative (like electrons) or positive (like protons).
- In matter, the positive charges are stuck in place in the nuclei. Matter is negatively charged when extra electrons are added, and positively charged when electrons are removed.
- Like charges repel, unlike charges attract.
- Charges travel in conductors, not in insulators
- Force of attraction or repulsion ~ 1 / r^2

18.2 Charged objects and electric force

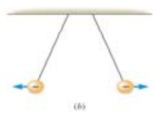


When an ebonite rod is rubbed against animal fur, electrons from atoms of the fur are transferred to the rod. This transfer gives the rod a negative charge (-) and leaves a positive charge (+) on the fur.

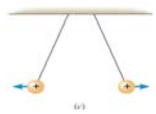
Experimentally we found that:



(a) A positive charge (+) and a negative charge (-) attract each other.



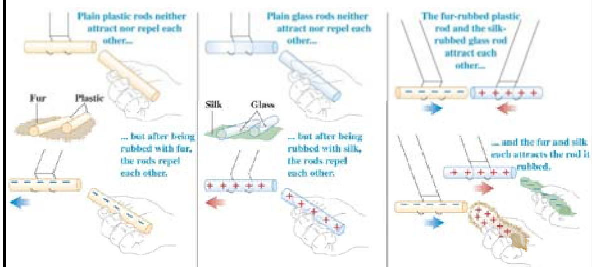
(b) Two negative charges repel each other.



(c) Two positive charges repel each other.

Exploring the nature of electric charge

Plastic, rubber, silk, glass, and fur can reveal fundamental behavior of charge



(a) Interaction between plastic rods rubbed on fur

(b) Interaction between glass rods rubbed on silk

(c) Interaction between objects with opposite charges

18.2 Charged Objects and the electric force

Law of conservation of electric charge

During any process, the net electric charge of an isolated system remains constant.

Like charges repel and unlike charges attract each other.

18.3 Conductors and Insulators

Not only can electric charge exist *on an object*, but it can also move *through an object*.

Substances that readily conduct electric charge are called ***electrical conductors***.

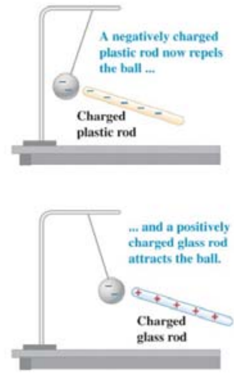
Materials that conduct electric charge poorly are called ***electrical insulators.***

(a) (b)

Conductors & Insulators

- Consider how charge is carried on macroscopic objects.
- For the time being we will make a simplifying assumption that there are only two kinds of objects in the world:
  - Insulators: In these materials, once they are charged, the charges ARE NOT FREE TO MOVE. Plastics, glass, and other "bad conductors of electricity" are good examples of insulators.
  - Conductors: In these materials, the charges ARE FREE TO MOVE.

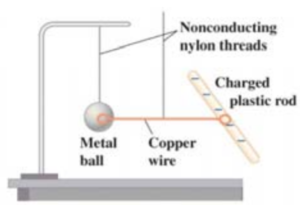
Charges in insulators



- Charges in insulators are not free to move

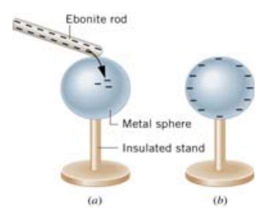
Charge movement in conductors

- Conductors: In these materials, the charges ARE FREE TO MOVE. Metals are good examples of conductors.



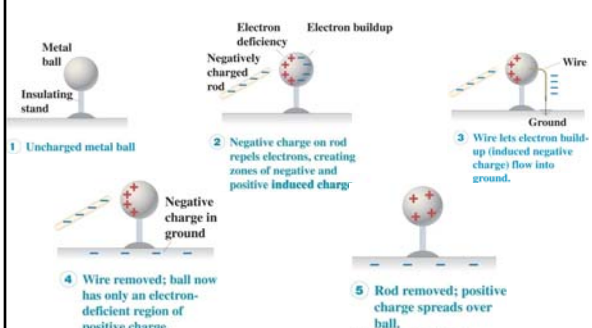
The wire conducts charge from the negatively charged plastic rod to the metal ball.

18.4 Charging by Contact

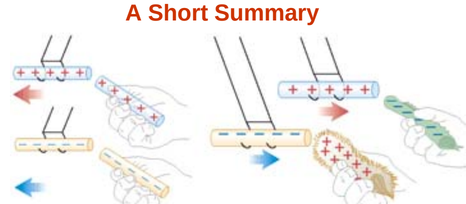


(a) Electrons are transferred by robbing the negatively charged rod on the metal sphere. (b) When the rod is removed, the electrons re-distribute themselves over the surface of the metal sphere.

18.4 Charging by Induction



A Short Summary



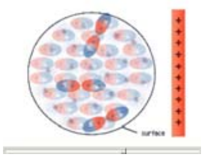
Electric Charge: quantized (electrons, protons)

Elementary charge $e = 1.6 \times 10^{-19}$ Coulombs

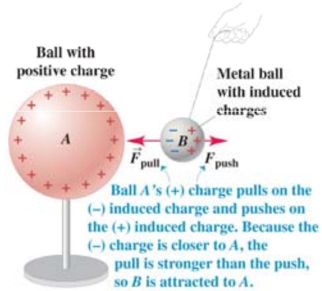
Charging an object can be done through (a) contact, and (b) induction.

Charge polarization (Induced charge effect)

A charged object can exert forces even on objects that are not charged themselves. (e.g. rub a balloon, may stick to wall)



- The charged object (above) induces charge on the surface of the insulator



18.5 Coulomb's Law

" The repulsive force between two small spheres charged with the same sort of electricity is in the inverse ratio of the squares of the distances between the centers of the spheres"

q_1 q_2 r

$$F \propto \frac{q_1 q_2}{r^2}$$

proportionality constant

$$k = 8.99 \times 10^9 \frac{\mathrm{N\,m}^2}{\mathrm{C}^2}$$

$$\boxed{F = k\frac{|q_1||q_2|}{r^2}}$$

Coulomb Law Qualitative

q_1 q_2 r

- **What happens if q_1 increases?**
  - ***F* (magnitude) increases**
- **What happens if q_1 changes sign (+ ⟶ -)?**
  - **The direction of $\vec{F}$ is reversed**
- **What happens if r increases?**
  - ***F* (magnitude) decreases**

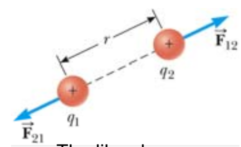
- **This force has same spatial dependence as the gravitational force, BUT there is NO mention of mass here!!**

Unit of charge: Coulomb 1 C = 1 A·s

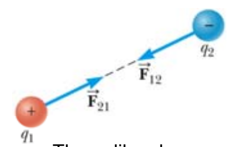
Vector Nature of Electric Forces

Remember that force is a vector quantity !

Two point charges are separated by a distance r



The like charges produce a **repulsive** force between them



The unlike charges produce an **attractive** force between them

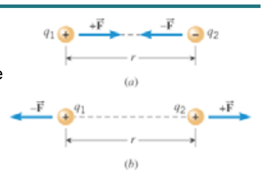
The force on q_1 is equal in magnitude and opposite in direction to the force on q_2

$$\vec{F}_{12} = -\vec{F}_{21}$$ **Newton's 3rd law**

18.5 Coulomb's Law

COULOMB'S LAW

The magnitude of the electrostatic force exerted by one point charge on another point charge is directly proportional to the magnitude of the charges and inversely proportional to the square of the distance between them.

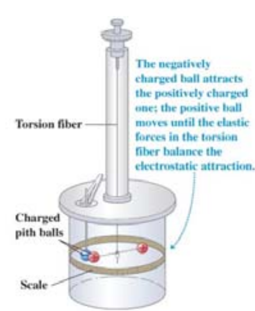


$$F = k\frac{|q_1||q_2|}{r^2}$$

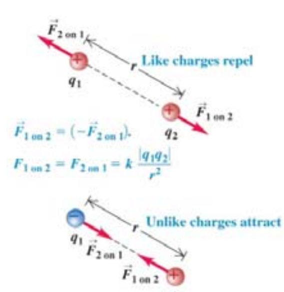
$$\varepsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/(N\cdot m^2)}$$

$$k = 1/(4\pi\varepsilon_o) = 8.99\times10^9\ \mathrm{N\cdot m^2/C^2}$$

How Coulomb measured the electric force

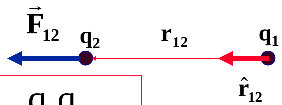


(a) A torsion balance of the type used by Coulomb to measure the electric force



(b) Interaction of like and unlike charges

Coulomb's Law



$$\vec{F}_{12} = k\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12}$$

Force on 1 due to 2

unit vector $\hat{\mathbf{r}}_{12} = \frac{\vec{r}_{12}}{r_{12}}$

with $r_{12} = |\vec{r}_{12}|$

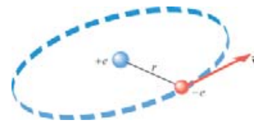
In SI units k is often written as:

$$k = \frac{1}{4\pi\varepsilon_o} = 8.99\times10^9\,\frac{\mathrm{Nm}^2}{\mathrm{C}^2}$$

ε_0 = permitivity of free space $\varepsilon_o = 8.66\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{Nm}^2}$

Example 3 A model of Hydrogen atom

In the Bohr model of the hydrogen atom, the electron is in orbit about the nuclear proton at a radius of 5.29×10^{-11}m. Determine the speed of the electron, assuming the orbit to be circular.



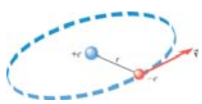
$$F = k\frac{|q_1||q_2|}{r^2}$$

Coulomb's Law

$$F_c = m\frac{v^2}{r}$$

Centripetal force

18.5 Coulomb's Law



$$F = k\frac{|q_1||q_2|}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(1.60\times10^{-19}\,\mathrm{C})^2}{(5.29\times10^{-11}\,\mathrm{m})^2} = 8.22\times10^{-8}\,\mathrm{N}$$

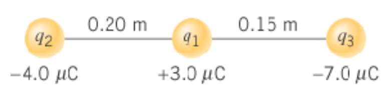
$$F = ma_c = mv^2/r$$

$$\Rightarrow \quad v = \sqrt{Fr/m} = \sqrt{\frac{(8.22\times10^{-8}\,\mathrm{N})(5.29\times10^{-11}\,\mathrm{m})}{9.11\times10^{-31}\,\mathrm{kg}}} = 2.18\times10^6\ \mathrm{m/s}$$

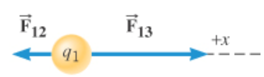
Example 4 Three Charges on a line

Example 4 **Three Charges on a Line**

Determine the magnitude and direction of the net force on q_1

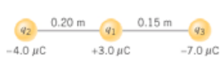


(a)

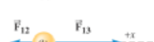


(b) Free-body diagram for q_1

Example 4 Three Charges on a line



(a)



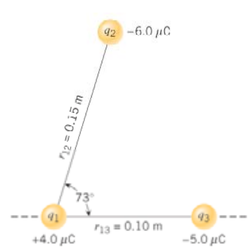
(b) Free-body diagram for q_1

$$F_{12} = k\frac{|q_1||q_2|}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.0\times10^{-6}\,\mathrm{C})(4.0\times10^{-6}\,\mathrm{C})}{(0.20\mathrm{m})^2} = 2.7\mathrm{N}$$

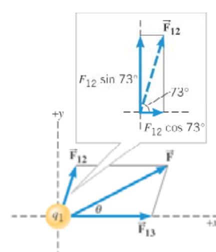
$$F_{13} = k\frac{|q_1||q_3|}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(3.0\times10^{-6}\,\mathrm{C})(7.0\times10^{-6}\,\mathrm{C})}{(0.15\mathrm{m})^2} = 8.4\mathrm{N}$$

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}_{12} + \vec{\mathbf{F}}_{13} = -2.7\mathrm{N} + 8.4\mathrm{N} = +5.7\mathrm{N}$$

Example 5 Three Charges on a Plane

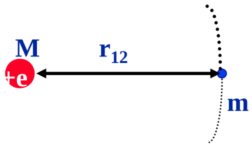


(a)



(b) Free-body diagram for q_1

Gravitational and Electric Forces of the Hydrogen Atom



m = 9.1x10^{-31} kg
M = 1.7x10^{-27} kg
r_{12} = 5.3x10^{-11} m
Q_p = Q_e = 1.6x10^{-19} As

Gravitational force

$$\boxed{\vec{F}_g = -G\frac{M\,m}{r_{12}^2}\hat{r}}$$

$$G = 6.7\times10^{-11}\frac{\text{Nm}^2}{\text{kg}^2}$$

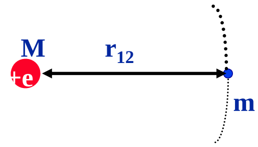
Electric Force

$$\boxed{\vec{F}_e = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q\,q}{r_{12}^2}\hat{r}}$$

$$\frac{1}{4\pi\varepsilon_0} = 9\times10^9\frac{\text{Nm}^2}{\text{C}^2}$$

afs 54sp09 L1

Gravitational and Electric Forces of the Hydrogen atom



m = 9.1x10^{-31} kg
M = 1.7x10^{-27} kg
r_{12} = 5.3x10^{-11} m
Q_p = Q_e = 1.6x10^{-19} As

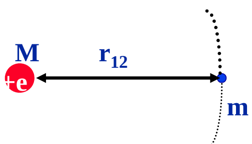
Gravitational force $\vec{F}_g = -G\frac{M\,m}{r_{12}^2}\hat{r}$

$$= 6.7\times10^{-11}\frac{\text{Nm}^2}{\text{kg}^2}\frac{(1.7\times10^{-27}\,\text{kg})\,(9.1\times10^{-31}\,\text{kg})}{(5.3\times10^{-11}\,\text{m})^2}$$

$$= 3.7\times10^{-47}\ \text{N}$$

afs 54sp09 L1

Gravitational and Electric Forces of the Hydrogen atom



m = 9.1x10^{-31} kg
M = 1.7x10^{-27} kg
r_{12} = 5.3x10^{-11} m
Q_p = Q_e = 1.6x10^{-19} As

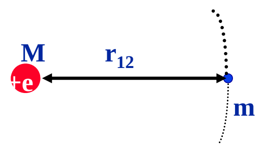
Electric Force $\vec{F}_e = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q\,q}{r_{12}^2}\hat{r}$

$$= 9\times10^9\frac{\text{Nm}^2}{\text{kg}^2}\frac{(1.6\times10^{-19}\,\text{C})\,(1.6\times10^{-19}\,\text{C})}{(5.3\times10^{-11}\,\text{m})^2}$$

$$= 8.2\times10^{-8}\ \text{N}$$

afs 54sp09 L1

Gravitational and Electric Forces of the Hydrogen atom



m = 9.1x10^{-31} kg
M = 1.7x10^{-27} kg
r_{12} = 5.3x10^{-11} m
Q_p = Q_e = 1.6x10^{-19} As

Gravitational force $\boxed{F_g = 3.6\times10^{-47}\,\text{N}}$

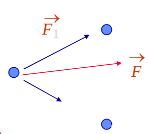
Electric Force $\boxed{F_e = 8.2\times10^{-8}\,\text{N}}$

$$\frac{F_e}{F_g} = \frac{8.2\times10^{-8}\,\text{N}}{3.6\times10^{-47}\,\text{N}} = 2.2\times10^{39}$$

afs 54sp09 L1

What happens when you consider more than two charges?

- **If q_1 were the only other charge, we would know the force on q due to q_1.**
- **If q_2 were the only other charge, we would know the force on q due to q_2.**
- **What is the force on q when both q_1 and q_2 are present??**



  - The answer: just as in mechanics, we have the **Law of Superposition:**
  - **The TOTAL FORCE on the object is just the VECTOR SUM of the individual forces.**

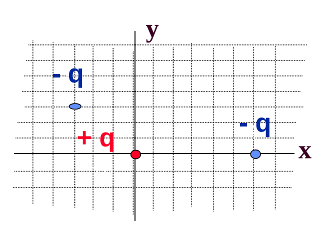
$$\boxed{\vec{F} = \vec{F}_1 + \vec{F}_2}$$

afs 54sp09 L1

Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?



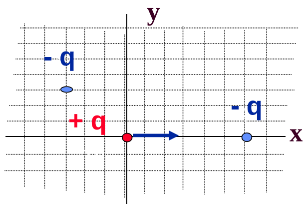
afs 54sp09 L1

Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Consider effect of each charge separately:

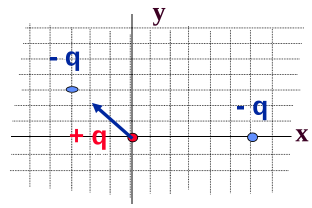


Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Take each charge in turn:

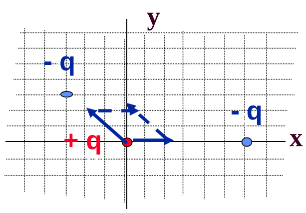


Superposition of forces from two charges

Blue charges fixed , negative, equal charge (-q)

What is force on positive red charge +q ?

Create vector sum:

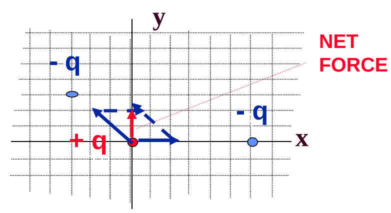


Superposition of forces from two charges

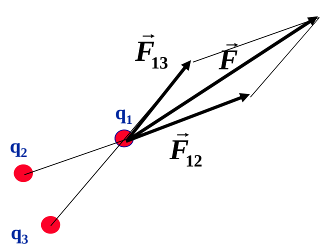
Blue charges fixed , negative, equal charge (-q)

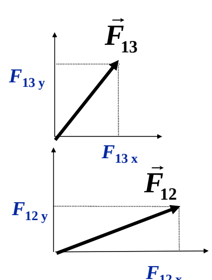
What is force on positive red charge +q ?

Find resultant:



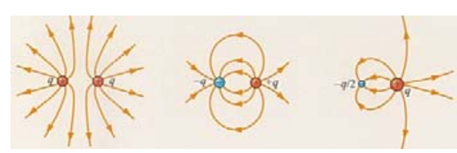
Superposition Principle





$$\vec{F} = \left(F_{12x} + F_{13x}\right)\,\hat{x} + \left(F_{12y} + F_{13y}\right)\,\hat{y}$$

The Electric Field



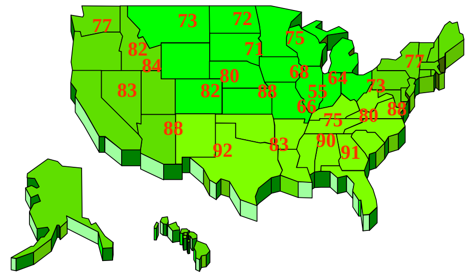
What is a Field?

A Field is a mathematical function representing physical quantities that can be defined in space everywhere.

•It can be a scalar field (e.g., Temperature field)
•It can be a vector field (e.g., Electric field)

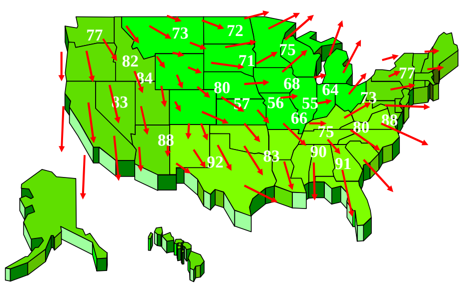
•Other examples of fields
•Gravitational field
•Ocean current field
•Pressure field
•Etc.

A Scalar Field



These isolated temperatures *sample* the scalar field (you only learn the temperature at the point you choose, but T is defined everywhere (x, y)

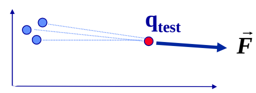
A Vector Field



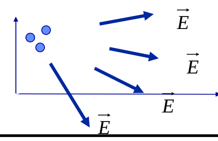
That would require a vector field (you learn both wind speed and direction)

The Electric Field

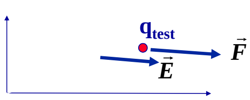
- A set of fixed charges exerts a force $\vec{F}$ given by Coulomb's law on a test charge q_{test} at position $\vec{r}$



This is action as a distance. It is a more difficult concept. So instead, we can introduce the "field" concept as follow:



The Electric Field



- The electric field is represented by the symbol $\vec{E}$, and is given in terms of this force by:

$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{\vec{\mathbf{F}}}{q_{\text{test}}}$$

This is a vector function of position.

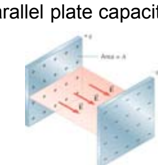
The electric field

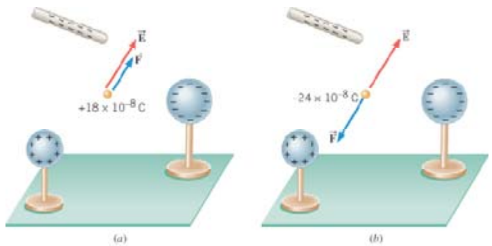
The electric field due to a point charge is given by

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$

The electric field due to a parallel plate capacitor,

$$\vec{E} = \frac{\sigma}{\varepsilon_o}\hat{x}$$

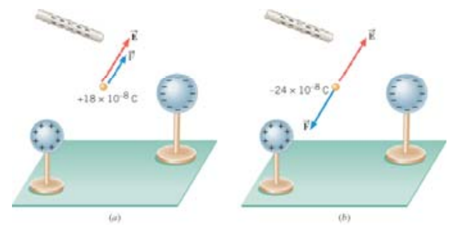


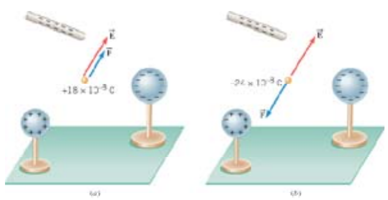


It is the surrounding charges that create the electric field at a given point.

***Example 7* An Electric Field Leads to a Force**

The charges on the two metal spheres and the ebonite rod create an electric field at the spot indicated. The field has a magnitude of 2.0 N/C. Determine the force on the charges in (a) and (b)



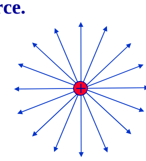


(a) $\vec{F} = q_o\vec{E} = (2.0\,\mathrm{N/C}\,\hat{E})(+18.0\times10^{-8}\,\mathrm{C}) = 36\times10^{-8}\,\mathrm{N}\hat{E}$

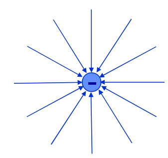
(b) $\vec{F} = q_o\vec{E} = (2.0\,\mathrm{N/C}\,\hat{E})(-24.0\times10^{-8}\,\mathrm{C}) = -48\times10^{-8}\,\mathrm{N}\hat{E}$

The Electric Field

- **The concept of electric fields was invented by Michael Faraday to describe his model of how charges interact.**
- **Charges interact by exerting forces on each other. Faraday thought of the fields as "lines of force" (sort of like strings) with a density in space proportional to the strength of the force.**



Positive charge *(Source)*



Negative charge *(Sink)*

Force due to an electric field

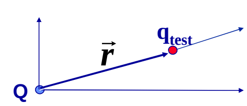
Just turn the definition of $\vec{E}$ around. If $\vec{E}(\vec{r})$ is known, the force on a charge q at point $\vec{r}$ is:

$$\boxed{\vec{\mathbf{F}} = q\vec{\mathbf{E}}(\vec{\mathbf{r}})}$$

The electric field at $\vec{r}$ points in the direction that a positive charge placed at $\vec{r}$ would be pushed.

Example: Point Charge

- **Find the electric field of a point charge Q located at the origin.**



$$\vec{\mathbf{F}} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_{test}}{r^2}\hat{\mathbf{r}}$$

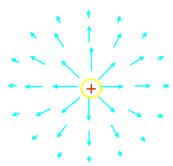
- Dividing out $\mathbf{q_{test}}$ gives the electric field at $\vec{r}$:

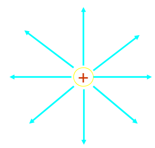
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$

Radially outward, falling off as 1/r^2

18.7 Electric field lines

Consider the E-field of a positive point charge at the origin





- Direction of arrow indicates direction of field
- Length of arrows ∝ local magnitude of

- Field lines leave (+) charges and return to (-) charges
- Number of lines leaving/entering charge ~ amount of charge
- Tangent of the field line = direction of *E*
- Local density of field lines ~ local magnitude of *E*

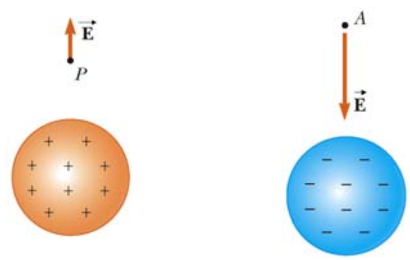
Example: Electric field in a hydrogen atom

In the Bohr model of the hydrogen atom, when the atom is in its lowest-energy state, the distance from the proton to the electron is 5.29×10^{-10} m.
Find the electric field due to the proton at this distance.

$$E = k\frac{|q|}{r^2}$$

$$= 8.99\times10^{9}\,\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}\,\frac{(1.60\times10^{-19}\,\mathrm{C})\,(1.6\times10^{-19}\,\mathrm{C})}{(5.29\times10^{-11}\,\mathrm{m})^2}$$

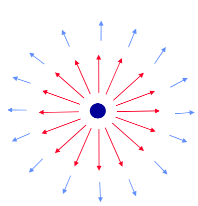
$$= 5.14\times10^{11}\,\frac{\mathrm{N}}{\mathrm{C}}$$



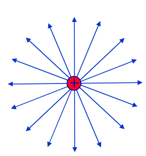
- The electric field produced by a positive charge is directed away from the charge
- The electric field produced by a negative charge is directed toward the charge

Example: Point Charge

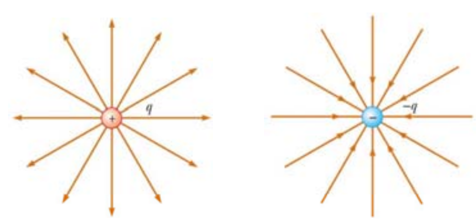
$$\vec{\mathbf{E}}(\vec{\mathbf{r}}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}}$$



Vector Field
(Show $\vec{E}$ at each $\vec{r}$)



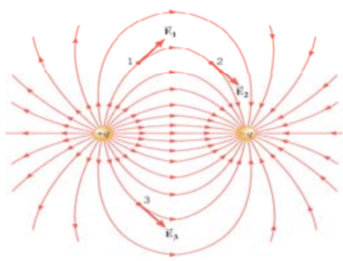
Field Lines
(Lines of force)



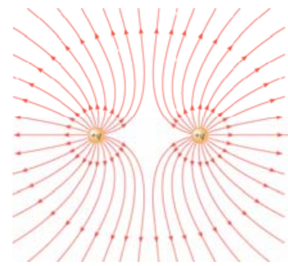
- Lines leave (+) charges and return to (-) charges
- Number of lines leaving/entering charge ∝ amount of charge
- Tangent of line = direction of *E*
- Local density of field lines ∝ local magnitude of *E*

18.7 Electric Field Lines

The number of lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.

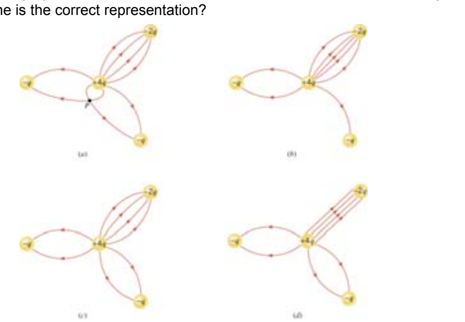


Field lines never cross each other



Conceptual Example 13 Drawing Electric Field Lines

The following figures show 4 possibilities of electric field lines between charges. Which one is the correct representation?

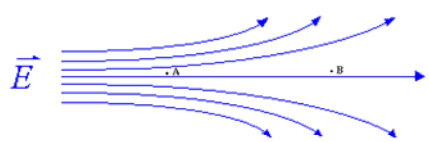


Concept test: Electric Field

A small object with a charge of magnitude q creates an electric field. At a point 1.0 m to the north of the charge, the field has a value of 2.0 N/C directed south. At a point 0.5 m to the west of the charge the field has a value of:

- **A. 4.0 N/C directed east**
- **B. 4.0 N/C directed west**
- **C. 8.0 N/C directed east**
- **D. 8.0 N/C directed west**

Concept test: Field lines

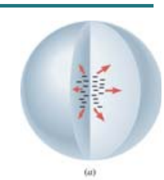


Compare the field strengths at points A and B.

a) $E_A > E_B$
b) $E_A = E_B$
c) $E_A < E_B$

18.8 The Electric Field Inside a Conductor: Shielding

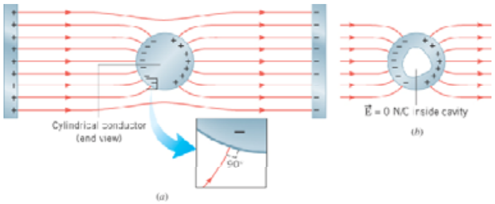
At equilibrium under electrostatic conditions, any excess charge resides on the surface of a conductor.



At equilibrium under electrostatic conditions, the electric field is zero at any point within a conducting material.

The conductor shields any charge within it from electric fields outside the condictor.

18.8 The Electric Field Inside a Conductor: Shielding



The electric field just outside the surface of a conductor is perpendicular to the surface at equilibrium under electrostatic conditions. The electric field inside the cavity of a conductor is zero.

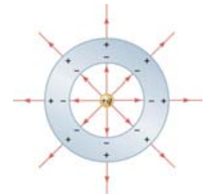
18.8 The Electric Field Inside a Conductor

Conceptual Example 14 **A Conductor in an Electric Field**

A charge is suspended at the center of a hollow, electrically neutral, spherical conductor. Show that this charge induces

(a)a charge of –q on the interior surface and

(b) a charge of +q on the exterior surface of the conductor.



18.9 Gauss' Law

From Coulomb law, we have

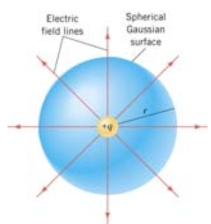
$$E = kq/r^2 = q/\left(4\pi\varepsilon_o r^2\right)$$

$$E = q/\left(A\varepsilon_o\right)$$

A is the surface of the sphere (Gaussian sphere)

$$\underbrace{EA}_{} = \frac{q}{\varepsilon_o}$$

Electric flux, $\Phi_E = EA$



$$\Phi_E = \oint \vec{E} \cdot \vec{dA} = E \cdot A$$

18.9 Gauss' Law

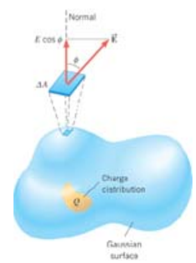
This is a remarkable result, namely the electric flux generated by a charge is independent to the choice of Gaussian surface.

The simple definition of flux is just (assume E perpendicular to the surface)

$$\Phi_E = E \cdot A$$

In more general cases, when E is not perpendicular to the surface

$$\Phi_E = \sum (E\cos\phi)\Delta A$$



18.9 Gauss' Law

Gauss' Law

The electric flux through a Gaussian surface is equal to the net charge enclosed in that surface divided by the permittivity of free space:

$$\sum (E\cos\phi)\Delta A = \frac{q}{\varepsilon_o}$$

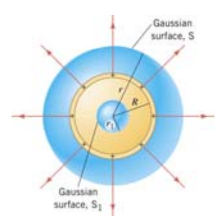
SI Units of Electric Flux: N·m²/C

18.9 Gauss' Law

Example 15 **The Electric Field of a Charged Thin Spherical Shell**

Positive charges are spread uniformly over the shell. Find the magnitude of the electric field at any point (a) outside the shell and (b) inside the shell.

$$\sum (E\cos\phi)\Delta A = \frac{Q}{\varepsilon_o}$$



Example 15 Electric field of a charged shell

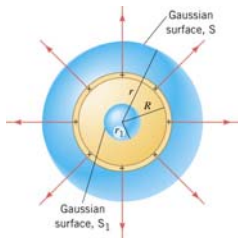
For a point outside the shell at r, we construct a Guassian sphere, S.

$$\Phi_E = \sum (E\cos\phi)\Delta A = \sum (E\cos 0)\Delta A$$

$$= E\sum \Delta A = E(4\pi r^2)$$

$$\Downarrow$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_o}$$



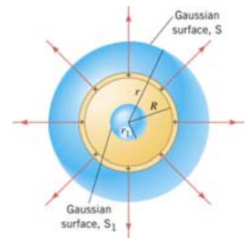
Example 15 Electric field of a charged shell

Inside the shell, at an arbitrary point r_1, we construct a Gaussian sphere, S_1; and use Gauss' law

$$E = \frac{q_{in}}{4\pi\varepsilon_o r^2}$$

Since inside S_1, there is no charge, $q_{in} = 0$, so

$$E = 0$$

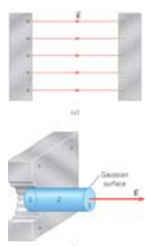


Example 16 Electric field inside a parallel plate capacitor

On page 548, we assume that the electric field inside a parallel plate capacitor is constant and equals to

$$E = \frac{q}{\varepsilon_o A} = \frac{\sigma}{\varepsilon_o}$$

Here we will try to use Guass law to derive the above result.



Example 16 Electric field inside a parallel plate capacitor

First construct a Gaussian surface like the blue cylinder in (b). This cylinder consists of three surfaces, S_1, S_2, and S_3. The flux passing through each surface are ϕ_1, ϕ_2, ϕ_3.

$$\Phi_1 = 0, \Phi_2 = 0, \Phi_3 = E\cdot A$$

$$\Phi_{total} = E\cdot A = \frac{q}{\varepsilon_o}$$

$$E = \frac{q}{\varepsilon_o A} = \frac{\sigma}{\varepsilon_o}$$