Electricity and Magnetism I

MWF 9:30 – 10:20
MSB306

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Why taking this course?

- E&M is a well-established field. A complete theory is available. Through E&M, we can learn the approach to a problem from a physicist's point of view.
- It is important. It can be applied to other fields. The concept of potential can be used in gravitational field. The field concept leads to QED, and the unification leads to the Grand Unification Theory and superstring theory.
- It is useful. Almost all the « Everyday » life experience of forces deal with E&M.

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What we will learn from E&M I ?

Chap. 1 Vector analysis and Integral Calculus

- Vector algebra
- Differential calculus
- Integral Calculus
- Curvilinear Coordinates
- Delta Function

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Chap. 2 Electrostatics

Coulomb law (point charge)	$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$
E field due to charge density	$\vec{E} = \frac{1}{4\pi\epsilon_o}\iiint \frac{\rho dV}{r^2}\hat{r}$
Gauss Law	$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_o}$
E field is conservative	$\oint \vec{E} \cdot \vec{d\ell} = 0 \Rightarrow \nabla \times \vec{E} = 0$
Scalar potential	$\vec{E} = -\nabla V$
Poisson's equation	$\nabla^2 V = -\frac{\rho}{\epsilon_o}$

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Chap. 3 Special Techniques

Mainly mathematics

- Image method
- Separation of Variables
- Spherical Coordinates
- Multiple Expansion

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Chap. 4 Electrostatic field in matter

• Polarization	$\vec{P} = \alpha\vec{E}$
• Surface charge (bound)	σ_b
• Volume charge (bound)	ρ_b
• Electric displacement	$\vec{D} = \epsilon_o\vec{E} + \vec{P}$
• Relation between $\vec{D}$ & ρ_f	$\nabla \cdot \vec{D} = \rho_f$

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Chap 5 Magnetostatics

- Lorentz force law $\vec{F} = q[\vec{E} + (\vec{v} \times \vec{B})]$
- Biot-Savart law $\vec{B} = \frac{\mu_o I}{4\pi} \int \frac{d\vec{\ell} \times \hat{r}}{r^2}$
- Ampere's law $\nabla \times \vec{B} = \mu_o \vec{J}$
- No magnetic monopole $\nabla \cdot \vec{B} = 0$
- Vector potential $\vec{B} = \nabla \times \vec{A}$
- $\nabla \cdot \vec{A} = 0$ (Gauge selection)
- $\nabla^2 \vec{A} = -\mu_o \vec{J}$ (Poisson's Equation)

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Chap 6 Magnetic Field in Matter

- Magnetization
- The field of a magnetized object
- The Auxiliary field H
- Linear and Nonlinear Media

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Review of 2049

- Coulomb's Law $\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r}$
- Electric field $\vec{E} = k\frac{q}{r^2}\hat{r}$
- Gauss Law $\oint \vec{E} \cdot \vec{dA} = \frac{q}{\epsilon_o}$
- Electric Potential $V = k\frac{q}{r}, \quad k = \frac{1}{4\pi\epsilon_o}$
- Potential $\int_a^b \vec{E} \cdot \vec{ds} = -(V_b - V_a)$
- Potential relation $\vec{E} = -\nabla V$

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- Capacitance $q = CV, C = \frac{q}{V}$
- Parallel plate capacitor, $C = \epsilon_o \frac{A}{d}$
- Dielectric materials, $C = \epsilon \frac{A}{d}$
- Energy density in E field, $u = \frac{1}{2}\epsilon_o E^2$
- Ohm's Law V=IR
- Current and Resistance
- Circuits (AC & DC)

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- Ampere's Law $\oint \vec{B} \cdot \vec{ds} = \mu_o i$
- Biot-Savart Law $\vec{B} = \frac{\mu_o}{4\pi} \int \frac{i\vec{ds} \times \hat{r}}{r^2}$
- Faraday's Law $\varepsilon = -\frac{d\Phi_B}{dt}$
- Magnetic Flux $\Phi_B = \int \vec{B} \cdot \vec{dA}$
- Lorentz Force $\vec{F} = q\vec{v} \times \vec{B}$

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Maxwell Equations

- $\oint \vec{E} \cdot \vec{dA} = \frac{q}{\epsilon_o}$ $\quad \nabla \cdot \vec{E} = \frac{\rho}{\epsilon_o}$
- $\oint \vec{B} \cdot \vec{dA} = 0$ $\quad \nabla \cdot \vec{B} = 0$
- $\oint \vec{E} \cdot \vec{ds} = -\frac{d\Phi_B}{dt}$ $\quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$
- $\oint \vec{B} \cdot \vec{ds} = \mu_o i + \mu_o \epsilon_o \frac{d\Phi_E}{dt}$ $\quad \nabla \times \vec{B} = \mu_o j + \mu_o \epsilon_o \frac{d\vec{E}}{dt}$

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Major Laws in E&M

1. Coulomb law

$$\vec{F} = \frac{1}{4\pi\epsilon_o}\frac{qQ}{r^2}\hat{r}$$

Concept of the Field $\vec{F} = q\cdot\vec{E}$

$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{Q}{r^2}\hat{r}$ (Point charge)

$\vec{E} = \frac{1}{4\pi\epsilon_o}\int\frac{\rho dV}{r^2}\hat{r}$ (Charge distribution)

2. Gauss Law

$$\oint \vec{E}\cdot\vec{da} = \frac{Q_{Enc.}}{\epsilon_o}$$

From divergence theorem,

$$\oint_A \vec{E}\cdot\vec{da} = \int_V(\nabla\cdot\vec{E})dV = \int_V\frac{\rho}{\epsilon_o}dV$$

$\nabla\cdot\vec{E} = \frac{\rho}{\epsilon_o}$ (differential form)

Relationship between Coulomb law and Gauss law

Gauss law and Coulomb law are closely related.

$$\oint \vec{E}\cdot\vec{da} = \oint\frac{1}{4\pi\epsilon_o}\cdot\frac{q}{r^2}\hat{r}\cdot(r^2 sin\theta d\theta d\varphi\hat{r}) = \frac{q}{\epsilon_o}$$

Gauss law is easier to solve when symmetry is involved.

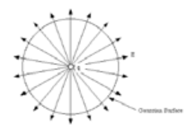
$$\nabla\cdot\vec{E} = \frac{1}{4\pi\epsilon_o}\int\nabla\cdot\left(\frac{\rho}{r^2}d\tau\right)\hat{r}$$

$$=\frac{\rho}{4\pi\epsilon_o}\int\nabla\cdot\left(\frac{\hat{r}}{r^2}\right)d\tau = \frac{\rho}{\epsilon_o}$$

3. Divergence of $\vec{E}$ field

The divergence of $\vec{E}$, measures the spread out of the electric field,

$$\nabla\cdot\vec{E} = \frac{q}{\epsilon_o}$$



In Cartesian Coordinates,

$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

4. Curl of the $\vec{E}$ field.

In electrostatic, the curl of $\vec{E}$ field is equal to zero. We can start with Coulomb law

$$\int_a^b \vec{E}\cdot\vec{ds} = \frac{1}{4\pi\epsilon_o}\int_a^b\frac{q}{r^2}dr = \frac{1}{4\pi\epsilon_o}\left(\frac{q}{r_a} - \frac{q}{r_b}\right)$$

The integral does not depend on the path

$$\oint \vec{E}\cdot\vec{ds} = \int_S(\nabla\times\vec{E})\cdot\vec{da} = 0$$

Therefore $\nabla\times\vec{E} = 0$ is always true.

Since the curl of a gradient is always equal to zero ($\nabla\times\nabla f = 0$), we can see from last equation that the electric field, $\vec{E}$ can be expressed as a gradient of a scalar potential:

$$\vec{E} = -\nabla V$$

This also implied that the $\vec{E}$ is a conservative field,

5. **Lorentz force law.**

$$\vec{F} = q(\vec{v} \times \vec{B})$$

There is no work done by the B field because $\vec{F}$ is perpendicular to $\vec{B}$.

6. Biot-Savart Law

$$\vec{B} = \frac{\mu_o}{4\pi}\int \frac{\vec{I}\times\vec{dr}}{r^2} \qquad \text{(Magneto-statics)}$$

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7. Continuity equation

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

8. Ampere's Law

$$\oint \vec{B} \cdot \vec{ds} = \mu_o I$$

This is the magnetic analogy of Gauss Law.

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9. Divergence of the B field

$$\nabla \cdot \vec{B} = 0$$

Because there is no magnetic monopole.

10. Curl of B

From Ampere's law, $\oint \vec{B} \cdot \vec{ds} = \mu_o I$

$$\oint \vec{B} \cdot \vec{ds} = \int (\nabla \times \vec{B}) \cdot \vec{da} = \mu_o \int \vec{J} \cdot \vec{da}$$

$$\nabla \times \vec{B} = \mu_o \vec{J}$$

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