2049H Spring 2010 Exam 1

Name:

Grading

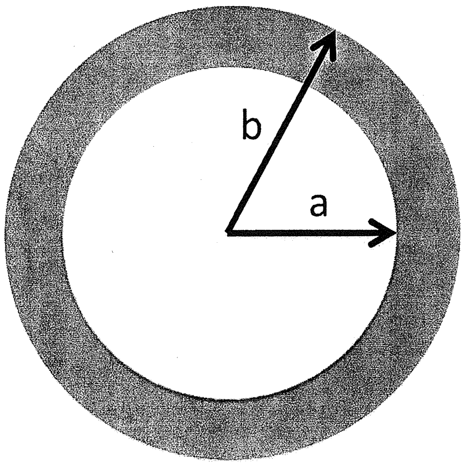
Problem 1

Problem 2

Problem 3

Problem 1 [54 points, 9pts each]

Consider an infinite uniformly charged pipe with cross section as depicted below. The volume charge density is ρ (C/m^3).



Calculate electric field (as a function of r: radial position with respect to the center of the pipe) for

(a) r > b
(b) a<r<b
(c) r<a

Taking voltage at the center of the pipe to be zero and calculate voltage (as a function of r: radial position with respect to the center of the pipe) for

d (a) r > b
e (b) a<r<b
f (c) r<a

a)

$$2\pi r l E = \frac{Q_{ENCLOSED}}{\varepsilon_0} = \frac{\pi(b^2-a^2) l \rho}{\varepsilon_0}$$

$$2\cancel{\pi} r \cancel{l} \quad E = \frac{\cancel{\pi}(b^2-a^2)\cancel{l}\rho}{\varepsilon_0}$$

$$E = \frac{(b^2-a^2)\rho}{2r\varepsilon_0}$$

b)

$$2\cancel{\pi} r \cancel{l} E = \frac{\cancel{\pi}(r^2-a^2)\cancel{l}\rho}{\varepsilon_0}$$

$$E = \frac{(r^2-a^2)\rho}{2r\varepsilon_0}$$

c)

$$E = 0$$

f) $r<a$ $\quad V=0$

e) $a<r<b$ $\quad V(a) - V(r) = \int_a^r \frac{(r^2-a^2)}{2r\varepsilon_0}\rho \, dr$

$$= \frac{\rho}{2\varepsilon_0}\left[\frac{r^2}{2} - a^2 \ln r\right]_a^r$$

$$= \frac{\rho}{2\varepsilon_0}\left[\frac{r^2-a^2}{2} - a^2 \ln\frac{r}{a}\right]$$

f)

$$V(r) - V(b) = \int_r^b \frac{(b^2-a^2)\rho}{2\varepsilon_0 r}\, dr$$

$$V(r) = \frac{(b^2-a^2)\rho}{2\varepsilon_0} \ln\frac{b}{r} + V(b)$$

$V(b) =$

e)

$$-V(r) = \frac{\rho}{2\varepsilon_0}\left[\frac{r^2-a^2}{2} - a^2 \ln \frac{r}{a}\right]$$

$$\boxed{V(r) = \frac{\rho}{2\varepsilon_0}\left[\frac{a^2-r^2}{2} + a^2 \ln \cancel{b}\frac{r}{a}\right]}$$

d)

$$V(r) = \frac{\rho}{2\varepsilon_0}\left[\frac{a^2-b^2}{2} + a^2 \ln \frac{b}{a}\right]$$

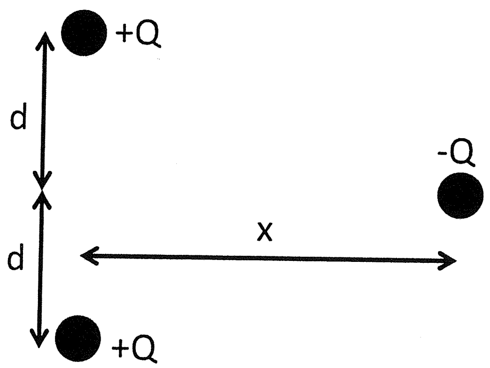
$$+ \frac{b^2-a^2}{2\varepsilon_0}\rho \ln \frac{b}{r}$$

$W_{x\infty} = U(x) - U(\infty)$

$\Rightarrow$ WORK HAND Dh. $w_{x\infty}$

$= W_{\infty x}$

Problem 2 [32 points, equally distributed]



We have equal charges +Q distance 2d away from each other as shown in the figure below. A charge, -Q is brought in from infinite distance away to the position shown.

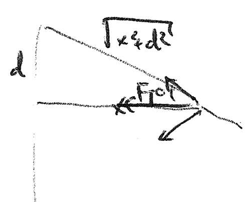
(a) Calculate the force on –Q by the +Q charges

(b) Calculate the electric potential of -Q

(c) Calculate the electric potential energy of -Q

(d) Calculate the work required to move -Q into the position shown in the figure

a)



$$\left|F_{\text{BY ONE CHARGE}}\right| = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(x^2+d^2)}$$

ONLY X DIRECTIONS REMAIN

$$F_x = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(x^2+d^2)}\cdot\frac{x}{\sqrt{x^2+d^2}}$$

$$\boxed{\vec{F}_T = \frac{1}{2\pi\varepsilon_0}\frac{Q^2 x}{(x^2+d^2)^{3/2}}\,-\hat{x}}$$

b) POTENTIAL

$$V_1(r)+V_2(r) = 2(V(r)) = 2\cdot\frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+d^2}}$$

$$\boxed{V(x) = +\frac{1}{2\pi\varepsilon_0}\frac{Q}{\sqrt{x^2+d^2}}}$$

c)

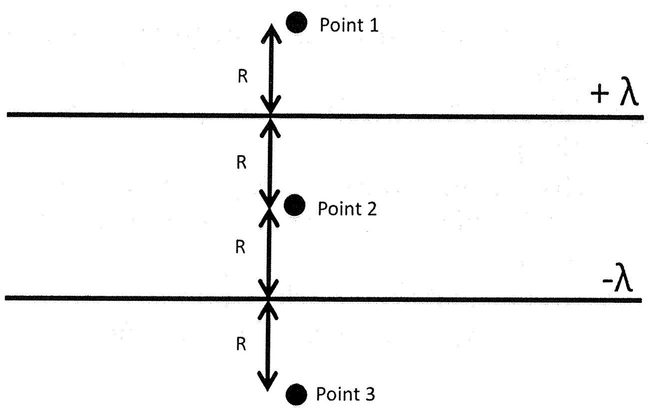
$$U(x) = -QV(x) = -\frac{Q^2}{2\pi\varepsilon_0\sqrt{x^2+d^2}}$$

d)

WORK $\dfrac{-Q^2}{2\pi\varepsilon_0\sqrt{x^2+d^2}}$

Problem 3 [14 points]

Consider two infinite line charge distribution as shown below with opposite charges.



Find the direction and the magnitude of electric field at point 1,2, and 3

$$\vec{E} \text{ by } +\lambda \text{ line} = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}\hat{r}$$

$$\vec{E} \text{ by } -\lambda \text{ line} = \frac{1}{2\pi\epsilon_0}\frac{-\lambda}{r}\hat{r}$$

Point 1) $\vec{E} = \frac{1}{2\pi\epsilon_0}\left(\frac{\lambda}{R} - \frac{\lambda}{3R}\right) = \frac{1}{2\pi\epsilon_0}\lambda \frac{\cancel{2R}\,2}{3R}$

$$= \frac{2}{3}\frac{1}{2\pi\epsilon_0}\frac{\lambda}{R} + \hat{j}$$

Point 2) $\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{2\lambda}{r} - \hat{y})$

Point 3) $\vec{E} = + \frac{2}{3}\frac{1}{2\pi\epsilon_0}\frac{\lambda}{R}\hat{y}$