Name: SOLUTION

Problem 1 [20 points]

Problem 2 [30 points]

Problem 3 [30 points]

Problem 4 [20 points]

The multiplicity for q units of energy among N oscillators is given by the expression:

$$\Omega(N,q) = \binom{q+N-1}{q} = \frac{(q+N-1)!}{q!(N-1)!}$$

| $n$ | $\binom{n}{0}$ | $\binom{n}{1}$ | $\binom{n}{2}$ | $\binom{n}{3}$ | $\binom{n}{4}$ | $\binom{n}{5}$ | $\binom{n}{6}$ | $\binom{n}{7}$ | $\binom{n}{8}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | | | | | | | | |
| 1 | 1 | 1 | | | | | | | |
| 2 | 1 | 2 | 1 | | | | | | |
| 3 | 1 | 3 | 3 | 1 | | | | | |
| 4 | 1 | 4 | 6 | 4 | 1 | | | | |
| 5 | 1 | 5 | 10 | 10 | 5 | 1 | | | |
| 6 | 1 | 6 | 15 | 20 | 15 | 6 | 1 | | |
| 7 | 1 | 7 | 21 | 35 | 35 | 21 | 7 | 1 | |
| 8 | 1 | 8 | 28 | 56 | 70 | 56 | 28 | 8 | 1 |

Problem 1 [equipartition] [20 points]

Consider a gas cylinder containing N molecules at temperature T.

a) Assuming that gas molecules are monoatomic, how many quadratic degrees of freedom does argon have (you may assume the temperature of the gas is near the room temperature)? [5 points]

3

b) What is the total energy of the container if the gas is monoatomic? [5 points]

$\frac{3}{2} NkT$

Consider a 1D simple harmonic oscillator

c) How many quadratic degrees of freedom does it have? [5 points]

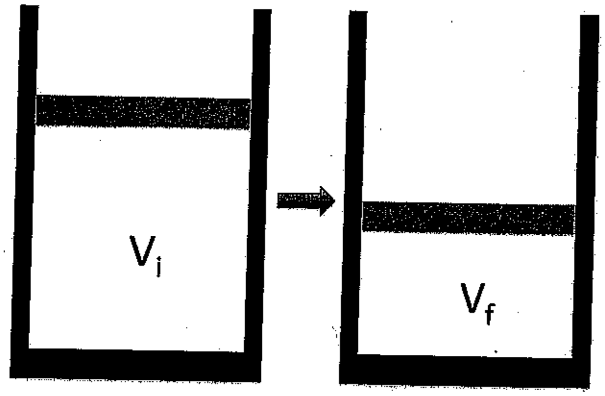
2

d) What is its average energy at temperature T? [5 points]

$\underline{kT}$

Problem 2 [compression of ideal gas] [30 points]

Ideal gas (monoatomic) with number of molecules/atoms, N, and temperature, T, is compressed as shown in the figure.




A) Assuming that the pressure of the gas remained constant through out the compression process, what is the work done to the gas? [5 points]

$P(V_i - V_f)$ ∴ NO PARTIAL CREDIT FOR SIGN ERROR

B) Assuming the temperature of the gas remains constant:

- What is the final pressure of the gas (express using $V_i$, $V_f$, $P_i$, N or T) [6 points]

$P_i V_i = P_f V_f$ $P_f = \frac{P_i V_i}{V_f}$

- What is the work done to the gas? [6 points]

$dW = -p\,dV = -\frac{NkT}{V}dV$

$W = \int_{V_i}^{V_f} -\frac{NkT}{V}dV = -NkT \ln\frac{V_f}{V_i} = \boxed{NkT \ln\frac{V_i}{V_f}}$

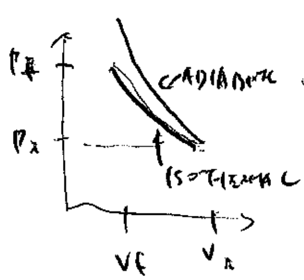
C) Assuming the compression is adiabatic

- What remains constant or remains zero in this case? [5 points]

HEAT FLOW OR $Q = 0$

- Is the amount of the work done to the gas same as in B)? If not, is it larger or smaller than B)? [8 points]

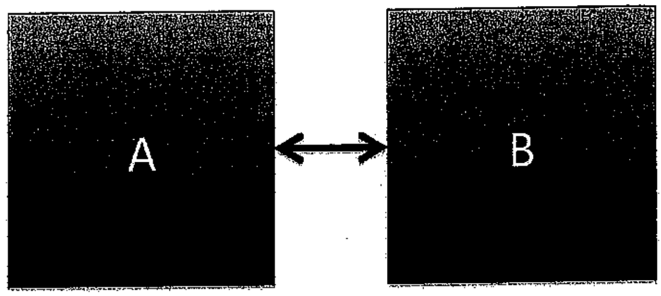
NO




WORK IS MORE

Problem 3: [Einstein Solid] [30 points]

Einstein solids A and B exchange energy with each other. Both solids contain 3 oscillators.




These solids are first cooled to 0 K. in a refrigerator.

a) At this temperature what is the total energy (energy of A and B combined) in multiples of hf? [3 points]

0

b) What is the total multiplicity at 0 K? [3 points]

1

c) What is the entropy at 0 K? [4 points]

0

Now the solids are given $q_A$=2 and $q_B$=4.

d) What is the total multiplicity (you can use binomial look up table given)? [5 points]

$$\Omega_A = \binom{2+3-1}{2} = \binom{4}{2} = 6$$

$$\Omega_B = \binom{4+3-1}{4} = \binom{6}{4} = 15$$

$6 \times 15 = \boxed{90}$

e) What are $q_A$ and $q_B$ at t=infinity? [5 points]

$q_A = q_B = 3$

f) What is the total multiplicity at t=infinity? [5 points]

$$\Omega_A = \Omega_B = \binom{3+3-1}{3} = \binom{5}{3} = 10$$

$\boxed{100}$

g) Does the multiplicity increase or decrease? [5 points]

INCREASES

Problem 4 [Entropy] [20 points]

Multiplicity of a gas trapped in a rigid cylinder is given by $\Omega = CV^N U^{\frac{3N}{2}} h^3$, where C is some constant, V is volume, U is energy, and h is the Plank's constant. Furthermore, you know that the heat capacity at constant volume is given by $C_V = 5NkT$, where k is the Bolzmann constant.

What is the change of the entropy if the temperature is suddenly raised by a factor of 2? Remember that volume remains constant.

$$S = k \ln \Omega = k \ln \left( C V^N U^{3N/2} h^3 \right)$$

$$= k \left[ \ln C + N \ln V + \frac{3N}{2} \ln U + \ln h^3 \right)$$

$$\boxed{\Delta S = \frac{3Nk}{2} \ln 2}$$