Physics 3513

Exam 2

Name: SOLUTION

Problem 1

Problem 2

Problem 3

Problem 1 Given dU = TdS-PdV+μdN, F=U-TS. F is Helmholtz free energy.
a) What is dF? (5 points)

$$dF = dU - TdS - SdT$$
$$dF = TdS - pdV + \mu dN - TdS - SdT$$
$$\boxed{dF = -SdT - pdV + \mu dN}$$

b) An ideal gas is isothermally expanded from V_0 to $2V_0$. Number of particles is N and temperature is T. Calculate dF for this isothermal expansion. (10 points)

$dT = 0$

$$dF = -pdV = -\frac{NkT}{V}dV$$
$$\Delta F = \int_{V_0}^{2V_0} -\frac{NkT}{V}dV = -NkT\ln 2$$

c) what does dF, calculated in b), correspond to? (5 points)

WORK DONE TO THE GAS

Problem 2 Consider a Carnot engine in which heat input and output are Q_h and Q_c, and thermal reservoirs are at T_h and T_c.

(a) What is the efficiency of the engine in Q_h and Q_c? (5 points)

$$\text{EFFICIENCY} = \frac{\text{WORK}}{\text{HEAT}} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

(b) What is the entropy change for one cycle in the Carnot engine? Note that the cycle is completely reversible process (5 points)

$$\Delta S = 0$$

(c) What is the efficiency of the engine in terms of T_h and T_c? If not work is shown here, then no credit will be given. (10 points)

$$\Delta S = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C} = 0$$

$$\frac{Q_H}{T_H} = \frac{Q_C}{T_C} \qquad \frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

$$\text{EFFICIENCY} = 1 - \frac{T_C}{T_H}$$

Problem 3 Consider a system composed of N spins in which energy levels are defined to be -μB, 0, μB and magnetic moment is for these levels are given by μ, 0, -μ.

(a) What is the partition function? (5 points)

$$Z = e^{\mu B \beta} + 1 + e^{-\mu B \beta}$$

$$\beta = \frac{1}{kT}$$

(b) What is the probability of finding a spin with magnetic moment of 0 at infinite temperature? (5 points)

$$P(0) = \frac{1}{Z} \qquad Z(T=\infty) = Z(\beta = 0) = 3$$

$$\boxed{P(0) = \frac{1}{3}}$$

(c) What is the average energy for this system? (10 points)

$$\bar{E} = \frac{N}{Z}\left[-\mu B\, e^{\mu B \beta} + \mu B\, e^{-\mu B \beta}\right]$$

(d) What happens to the average energy as T approaches infinity? (10 points)

$$T = \infty \qquad \beta = 0$$

$$\boxed{\bar{E} = 0}$$

See next page for more problems

(e) What is the average magnetic moment for this system? (10 points)

$$\bar{\mu} = \frac{N}{Z}\left[\mu e^{\mu B\beta} - \mu e^{-\mu B\beta}\right]$$

(f) What is the average magnetic moment at T=0? (10 points)

$T \to 0 \quad \beta = \infty \quad Z \sim e^{\mu B\beta}$

$$\text{So } \boxed{\bar{\mu} = N\mu}$$