Exam #1

Name:

Question 1 [25 points]

Question 2 [15 points]

Question 3 [35 points]

Question 4 [25 points]

SOLUTION

Problem 1: A gas cylinder containing hydrogen molecules (2 grams per mole) and oxygen molecules (32 grams per mole) is in thermal equilibrium. [25 points total]

(a) Which molecules are moving faster on average? [5 points]

H_2

(b) By what factor? [10 points]

$$\sqrt{\frac{m_{O_2}}{m_{H_2}}} = \sqrt{\frac{32}{2}} = \sqrt{16} = \boxed{4}$$

(c) What is the average kinetic energy of hydrogen molecule at 300 K? [10 points]

$$E_{AVERAGE} = \frac{3}{2} k_B T = \frac{3}{2} \cdot 25\,meV = 37.5\,meV$$

Problem 2 Given a macrostate of an Einstein solid with 5 oscillators and 5 units of energy

(a) Calculate its multiplicity [10 points]

$$\Omega = \binom{N+q-1}{q} = \binom{5+5-1}{5} = \binom{9}{5}$$

$$= \frac{9!}{5!4!} = 3\,\frac{9\times 8\times 7\times 6}{4\cdot 3\cdot 2\cdot 1}$$

$$= 6\times 7\times 3 = 18\times 7 = \boxed{126}$$

(b) Calculate its entropy [5 points]

$$\boxed{S = k\ln 126} = S = k(4.83)$$

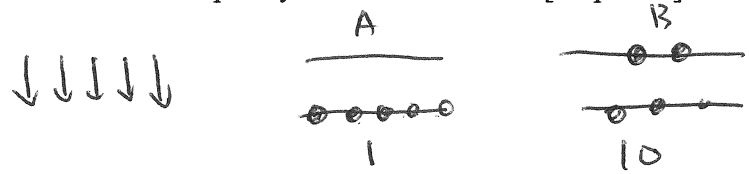
Problem 3 Given two two-state paramagnets, labeled A and B, (which exchange energy just with each other) each with 5 spins [total numbers of spins is 10] and spin energies given by $U = \mu \bullet B$ (that is $+\mu B$ for spins parallel to the magnetic field), answer the following questions.

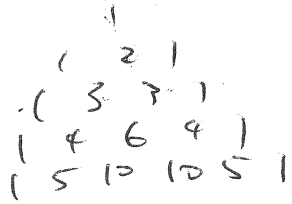
(a) At an extremely high temperature [thermal energy much larger than magnetic energy] How many spins will be pointing up (count up all "up" spins in A and B)? [5 points]

5 SPINS

Now we prepare an initial macrostate with paramagnet A having 5 "down" spins and Paramagnet B having 3 "down" spins

(b) Calculate the multiplicity of this initial state [10 points]





$\Omega_{INITIAL} = 10$

(c) Let the system reach thermal equilibrium. What is the final state of paramagnet A and B? [5 points]

4 SPINS DOWN 1 SPIN UP

(d) Which direction did the heat flow? [to reach the final state from the initial state] [5 points]

HEAT FROM B TO A

(e) Find the multiplicity of the final state in thermal equilibrium. [10 points]

$5 \times 5 = 25$

Problem 4 Ideal monoatomic gas is compressed to ½ its initial volume isothermally.

(a) Calculate work done to the gas [12.5 points]

ISOTHERMAL COMPRESSION

$\Delta U = 0 = Q + W \qquad Q = -W$

$W = -p\,dV = -\frac{NkT}{V}\,dV$

$W_{TOTAL} = \int_{V}^{V/2} -\frac{NkT\,dV}{V} = \boxed{NkT \ln 2}$

$= NkT(0.69)$

(b) Calculate change in entropy [12.5 points]

ISOTHERMAL $\quad dS = \frac{Q}{T} \Rightarrow \boxed{dS = -Nk \ln 2}$

$dS = -NK(0.69)$