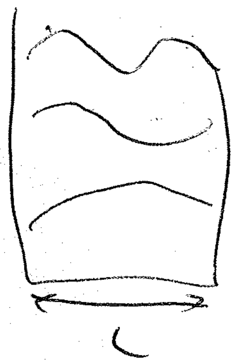
LAST TIME: EINSTEIN MODEL OF SOLIDS

BOUNDARY CONDITIONS DISCUSSED SO FAR



$\sin \frac{3\pi}{L} x \qquad 2$

$\sin \frac{2\pi}{L} x \qquad 2$

$\sin \frac{\pi}{L} x \qquad 2$

L

$\sin k x \qquad k: \quad \underline{\hbar k = p}$

$\frac{n\pi}{L} \qquad N = \left(\frac{L}{2\pi}\right)^3 \frac{4\pi k^3}{3}$

$$\frac{\partial N}{\partial \omega} = \frac{\partial N}{\partial k} \frac{\partial k}{\partial \omega} \qquad \omega = v k, \quad k = \frac{\omega}{v}$$

$$= \left(\frac{L}{2\pi}\right)^3 \cdot \frac{4\pi k^2}{v} \qquad \frac{\partial k}{\partial \omega} = \frac{1}{v}$$

$$D(\omega) = \frac{V \omega^2}{2\pi^2 v^3}$$

λ_{MIN} =

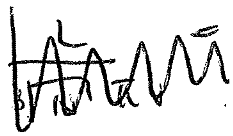
$$\lambda = \frac{2L}{n}$$

$$\lambda_{MIN} = \frac{2L}{\sqrt[3]{N}} \qquad \lambda_{MIN} f_{MAX} = v$$

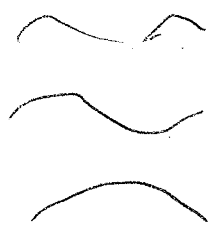
$$n_{MAX} = \sqrt[3]{N}$$

$\frac{2\pi}{w} = T \qquad \frac{w}{2\pi} = f$

$\frac{2L}{\sqrt[3]{N}} \qquad \frac{w_{MAX}}{2\pi} = v$



$w_{MAX} = \frac{\pi v \sqrt[3]{N}}{L}$



$N_{MAX} = N$

$N = \frac{V w_D^2}{2\pi^2 v^3}$

$w_D^3 =$

$w_D^3 = 6\pi^2 v^3 \frac{N}{V}$

$\frac{\hbar w_D}{k} = \Theta$: DEBYE TEMPERATURE

$D(w) = \frac{V}{2\pi^2}\frac{w^2}{v^3}$

$U = \int dw \; D(w) \langle n \rangle \hbar w \, dw = \int_0^{w_D} \left(\frac{V w^2}{2\pi^2 v^3} \right) \frac{\hbar w}{e^{\hbar w / kT} - 1}$

$U = \frac{3 V k_B^4 T^4}{2\pi^2 v^3 \hbar^3} \int_0^{x_D} \frac{x^3}{e^x - 1}$

$x_D = \frac{\hbar w_D}{k_B T} = \frac{\Theta}{T}$

Θ = DEBYE TEMP

$$\left[\; U = 9 N k_B T \left(\frac{T^3}{\theta}\right) \int_0^{x_D} dx \, \frac{x^3}{e^x - 1} \;\right]$$

$$U = \frac{3 V k_B^4 T^4}{2\pi^2 v^3 \hbar^3} \int_0^{x_D} dx \, \frac{x^3}{e^x - 1}$$

$T \to 0$

IF $x_D \to \infty$

$$\int_0^{x_D} dx \, \frac{x^3}{e^x - 1} = \int_0^{\infty} dx \, \frac{x^3}{e^x - 1}$$

$$= \int_0^{\infty} dx \, x^3 \sum_{s=1}^{\infty} \exp(-s x)$$

$$\downarrow$$

$$\frac{\pi^4}{15}$$

$$U = \frac{3 V k_B^4 T^4 \pi^4}{2\pi^2 v^3 \hbar^3 15} \sim \underline{T^4}$$

$C_V \simeq T^3$

$3 V k_B$

IF $T \to \infty$ $x_D \to 0$

$$e^x - 1 = 1 + x - 1 = 1$$

$$U = \underbrace{\frac{3 V k_B^4 T^4}{2\pi^2 v^3 \hbar^3}} \int_0^{x_D} \frac{x^3}{x} dx$$

$$= \frac{\cancel{3} V k_B^4 T^4}{2\pi^2 v^3 \hbar^3} \left. \frac{x^3}{\cancel{3}} \right]_0^{x_D}$$

$$x_D = \frac{\hbar \omega_D}{kT} \qquad \omega_D = \left(6\pi^2 v^3 \frac{N}{V}\right)^{1/3}$$

$$= \frac{\cancel{V} k_B^{\cancel{4}} T^{\cancel{4}}}{\cancel{2\pi^2} \cancel{v^3} \cancel{\hbar^3}} \frac{\cancel{\hbar^3}}{\cancel{k^3}} \frac{\cancel{6\pi^2}\, v^3}{\cancel{T^3}} \frac{N}{\cancel{V}}$$

$$U = \underline{3 N k T}$$

DEBYE TEMPERATURE

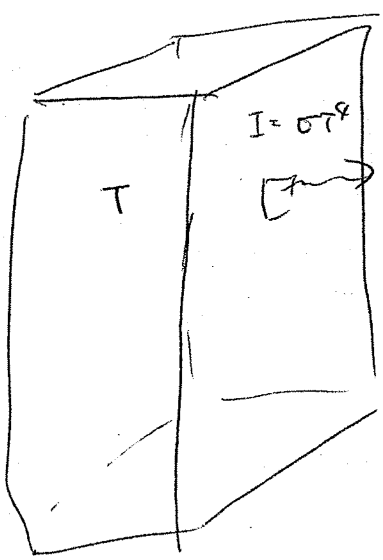
CESIUM 38 K

ZINC 327 K

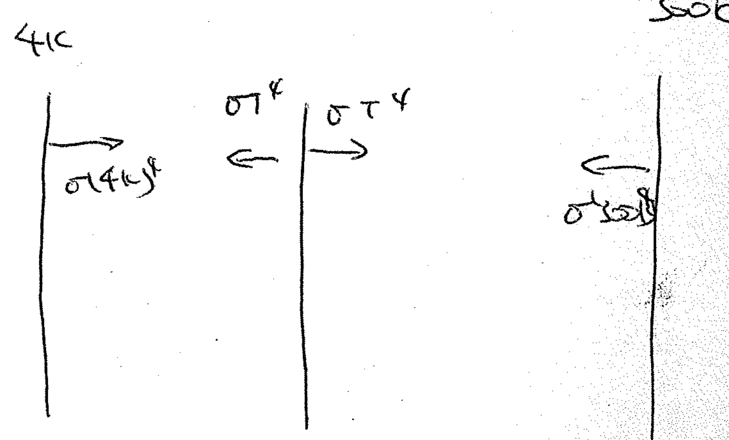
BERYLLIUM 1440 K

GOLD 170 K

A WORD ON BLACK BODY

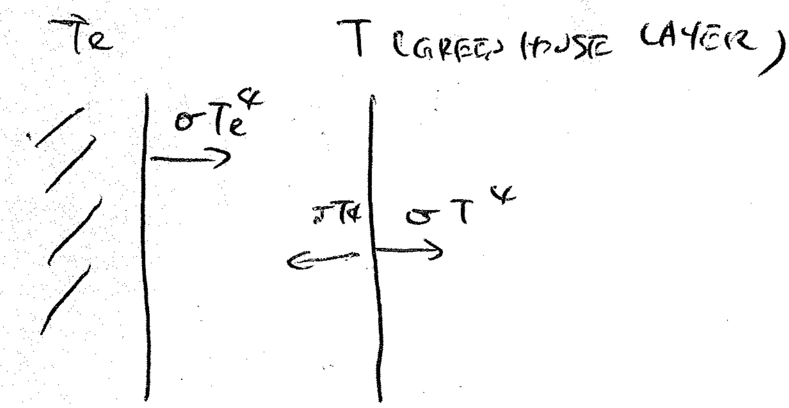


i.e.



$$\sigma(300K)^4 - \sigma T^4 = \sigma T^4 - \sigma(4K)^4$$

$$2\sigma T^4 = \sigma(300K)^4 - \sigma(4K)^4$$



$$\sigma T_e^4 - \sigma T^4 = \sigma T^4$$

$$\sigma T_e^4 = 2\sigma T^4$$

RESULT IN THE BOOK