$$\bar{n} = \frac{\cancel{\quad} e^{-\hbar\omega/kT}}{1 - e^{-\hbar\omega/kT}} = \frac{1}{e^{\hbar\omega/kT} - 1}$$

$$T \to \infty \qquad \bar{n} = \cancel{\quad}$$

$$\simeq \frac{1}{1 + \frac{\hbar\omega}{kT} - 1} = \frac{kT}{\hbar\omega}$$

$$\bar{E} = \cancel{\quad} 3NkT + \frac{3}{2}N\hbar\omega$$

$$C_V = \left(\frac{\partial E}{\partial T}\right) = 3Nk \quad : \quad \text{EQUIPARTITION}$$

IF $\hbar\omega \gg kT$

$$\bar{n} \approx e^{-\hbar\omega/kT}$$

$$\cancel{\quad} \quad \frac{C_V}{3Nk} \approx \left(\frac{\hbar\omega}{kT}\right)^2 e^{-\hbar\omega/kT}$$

BUT EXPERIMENT SHOWS

$C_V \sim T^3$ AT LOW TEMP

EINSTEIN MODEL

N OSCILLATORS IN THE SOLID EACH W/ FREQUENCY ω

$$E = \tfrac{1}{2}\hbar\omega, \quad \tfrac{3}{2}\hbar\omega, \quad \tfrac{5}{2}\hbar\omega \quad \ldots$$

$$E = \left(\tfrac{1}{2} + n\right)\hbar\omega$$

$$Z = \sum_{n=0}^{N} e^{-\left(\frac{1}{2}+n\right)\hbar\omega/kT}$$

N IS BIG THEN WE CAN TAKE A LIMIT AS $N \to \infty$

$$Z = e^{-\frac{\hbar\omega}{2kT}} \sum_{n=0}^{\infty} e^{-n\hbar\omega/kT}$$

$$Z = \frac{e^{-\frac{\hbar\omega}{2kT}}}{1 - e^{-\hbar\omega/kT}}$$